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Curve-based Approach for Optimal Trajectory Planning with Optimal Energy Consumption: application to Wheeled Mobile Robots

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Abstract: Wheeled transportation systems have been used since the dawn of civilization. The great amount of experience we have with wheeled transportation systems gives the Wheeled Mobile Robots (WMRs) an edge over other types of robots. This has allowed WMRs to be the prime candidates for many applications including, but not limited to, military and space applications. Sometimes these vehicles have to work in complex environments with complicated constraints to finish their assigned tasks. This paper presents a methodology to generate a smooth feasible trajectory for WMR using Pythagorean Hodograph (PH) curves. Generated trajectory not only avoids the static obstacles but also conforms to minimum energy consumption and travel time possible within the kinematic constraints of the WMR. Simulations are presented to check the performance of the proposed approach. The proposed method is experimentally validated using a WMR 'Robotino'.

1. INTRODUCTION

Accurate trajectory planning is a crucial task for WMRs to navigate safely and efficiently from the start point to the finish point autonomously. The path undertaken by them should be smooth, short, and collision-free. The selected velocity profile on the generated path should assure the minimum energy consumption to increase the availability of the WMR.

1.1 Related Work

Path planning of wheeled mobile robots has been done using various algorithms like Dijkstra's algorithm, A* algorithm, Rapidly exploring random tree (RRT) algorithm, Probabilistic road map (PRM) algorithm, Potential fields algorithm etc.. (Yang et al., 2016).
Most of the methods generate a path consisting of straight lines with sharp turns. Therefore, the path is unnatural for wheeled robots to follow as they can not take sudden sharp turns. To avoid this problem, curve based approaches are used to make a smooth path. Ravankar et al. (2018) gives a review of different curve based approaches: Bezier curves, cubic splines, B-spline, NURBS, etc. In 1990, Pythagorean Hodograph (PH) curves were introduced by Farouki and Sakkalis (Farouki and Sakkalis, 1990) to overcome the various drawbacks of the above-mentioned curves. Therefore, in Bruyninckx and Reynaerts (1997), PH curves are used for path planning of mobile robots.
Most of the PH based path planning has been done by varying the curvature of the PH curve to avoid the obstacles. However, the curvature variation leads to a significant increase in the path length. Therefore, Shah et al. (2010) proposed a methodology to avoid obstacles using PH curves by taking a different combination of direction vectors at the start and endpoint. The method considered only some combinations of direction vectors from four different quadrants. This paper overcomes this limitation and proposes an approach to optimize the direction vectors at start and destination (end) points to create an obstacle-free path.
Further, motion planning of WMR is needed on the planned path so that it should consume minimum energy to finish its task in minimum travel time. For the energy consumption of autonomous vehicles, most of the works use the A* algorithm to generate a path on which minimum energy consumption of the vehicle is considered (Datouo et al., 2017). In Ondrúška et al. (2015), the minimum energy consumption technique is presented for a known path. In this work, parameters of the PH curve are optimized to generate a trajectory with minimum energy consumption in minimum travel time. In the present work, we propose an optimization approach:

- to optimize the direction vectors at the start and endpoints of the PH curve to avoid known obstacles.
- to generate a trajectory using PH curves by consideration of minimum energy consumption and travel time of WMR.

1.2 Outline

The paper is organized as follows:
Section 2 discusses the formulation of a quintic PH curve for path planning. The energy consumption of wheeled robots is discussed in section 3. Optimization methodology to generate a smooth and collision-free path has been

discussed in section 4. Section 5 presents the discretization of a path into more than one PH curve. Section 6 provides simulation as well as experimental results for the proposed methodology. Finally, section 7 provides the conclusions and future perspectives of the work.

2. PYTHAGOREAN HODOGRAPH CURVES FOR PATH PLANNING

Quintic PH curves are used for path planning because they are a more appropriate compromise between complexity and smoothness. Formulation of the planar PH-curve used for path planning, originating from an initial position to the final (destination) position is discussed using normal polynomial parametric curves as (Singh et al., 2018b,a):
Let $r(h)$ be the polynomial curve,

$$r(h) = (x(h), y(h)); \qquad 0 \leq h \leq 1 \tag{1}$$

where h is the curvilinear coordinate of the curve.
It means start and final points of the curve are:

$$\begin{aligned} P_s &= r(0) = (x(0), y(0)) \qquad at \quad h = 0 \\ P_f &= r(1) = (x(1), y(1)) \qquad at \quad h = 1 \end{aligned} \tag{2}$$

The curve joining derivatives of the polynomial curve is defined as the 'hodograph' of the curve. This is parallel to the tangent to the curve and is given as follows:

$$r'(h) = (x'(h), y'(h)) \tag{3}$$

Let $L(h)$ represents the length of the path constructed using the curve $r(h)$:

$$L(h) = \int_0^1 |r'(h)| \, \mathrm{d}h = \int_0^1 \sqrt{x'(h)^2 + y'(h)^2} \, \mathrm{d}h \tag{4}$$

From eq. (4), the square root sign can be eliminated, if:

$$x'(h)^2 + y'(h)^2 = \sigma(h)^2 \tag{5}$$

Eq. (5) is known as Pythagorean law or condition. Therefore, if the first derivative (hodograph) of parametric polynomials, satisfies the pythagorean condition at all the points, the curve is known as Pythagorean Hodographs.
Now eq. (4) can be written as:

$$L(h) = \int_0^1 |\sigma(h)| \, \mathrm{d}h \tag{6}$$

This also allows the closed form solution of the length of the path.
For the construction of a PH curve r with minimum bending energy, four boundary conditions are required (Singh et al., 2018b):

- $P_s(x_s, y_s)$, start point of the curve.
- $d_s(\theta_s)$, unit direction vector at the start point P_s.
- $P_f(x_f, y_f)$, end point of the curve.
- $d_f(\theta_f)$, unit direction vector at the end point P_f.

Using these four boundary conditions, six control points of a quintic PH curve are computed to generate a curve with minimum bending and is explained in Singh et al. (2018b). A basic quintic PH is shown in Fig. 1(a) with its four boundary conditions and six control points from P_0 to P_5. Four boundary conditions (P_s, P_f, d_s, d_f) fix the four control points of a quintic PH curve and there are two free control points (Fig. 1(a)). Due to these two free control points there are multiple solutions to generate quintic PH curve for four fixed boundary conditions. Formulation of a quintic PH uses (Singh et al., 2018b; Farouki and Sakkalis, 1990) a condition to generate a unique curve with minimum bending energy for the above mentioned four boundary conditions. Hence, a direct solution for minimum energy curve is possible instead of an iterative solution.
The general equation of quintic PH curve $r(h)$ with control points is as follows:

$$r(h) = \begin{bmatrix} 1 \\ h \\ h^2 \\ h^3 \\ h^4 \\ h^5 \end{bmatrix}^T \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -5 & 5 & 0 & 0 & 0 & 0 \\ 10 & -20 & 10 & 0 & 0 & 0 \\ -10 & 30 & -30 & 10 & 0 & 0 \\ 5 & -20 & 30 & -20 & 5 & 0 \\ -1 & 5 & -10 & 10 & -5 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} \tag{7}$$

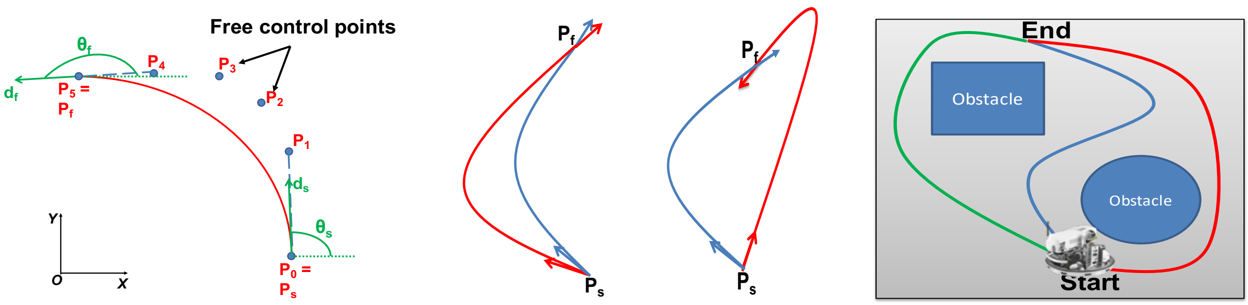
The curvature of curve $r(h)$ is given as:

$$\kappa = \frac{|\boldsymbol{r}' \times \boldsymbol{r}''|}{|\boldsymbol{r}'|^3} \tag{8}$$

Start (P_s) and end (P_f) points of a path are fixed but direction vectors ($d_s(\theta_s)$ and $d_f(\theta_f)$) at these points can vary. Figures 1(b),1(c) shows the effect of direction vectors on the path. Also, Fig. 1(d) illustrates that by variation of direction vectors, different possible paths can be achieved to avoid obstacles. Section 4 explains the method to select optimal direction vectors according to our requirements.

3. ENERGY CONSUMPTION OF WHEELED ROBOTS

The total energy consumption of a WMR is a sum of the total mechanical (kinetic + potential) energy of the WMR and the various losses. For the current formulation, the path is assumed as planar, hence the potential energy is unchanged. The losses associated with WMR include the frictional losses at the wheel and electrical losses.



(a) (b) (c) (d)

Fig. 1. Quintic PH curve (a) Basic quintic PH, (b) Effect of direction vectors on length of PH, (c) Effect of direction vectors on curvature of PH and (d) Role of direction vectors in obstacle avoidance

The energy estimation model of an n-wheeled WMR can be expressed as follow (Liu and Sun, 2014):
$E_{\text{Total}} = E_{\text{Kinetic}} + E_{\text{Loss}}$
$=$ (Linear Kinetic Energy + Rotational Kinetic Energy) + (Traction loss + Electrical Loss)

$$= \int\limits_t \begin{pmatrix} m\max\{V(t)\dot{V}(t), 0\} + I_{zz}\max\{\omega(t)\dot{\omega}(t), 0\} \\ +n\mu mg\max\{|V(t)|, |b\omega(t)|\} + P_s \end{pmatrix} dt \tag{9}$$

where E_{Total}, E_{Kinetic}, and E_{Loss} denote the total energy of the robot, total kinetic energy and total energy loss respectively. m, I_{zz}, μ, and b represent the mass, Moment of Inertia, friction coefficient, and the distance between the wheel & centre of gravity, of the WMR respectively. V and $\dot{V}$ are the linear velocity and acceleration of the WMR. ω and $\dot{\omega}$ represent the angular velocity and angular acceleration of the WMR while steering. P_s represents the static power consumption of the embedded computer and different sensors. g is the standard acceleration due to gravity.

4. OPTIMIZATION STRATEGY FOR WMR

The optimization problem in this paper is recognized as a bounded, constrained, multi-variable, multi-objective, feasibility study. Hence, a multi-objective Genetic Algorithm is used to solve the problem.

4.1 Objective Functions and Constraints

The objective functions to be minimized for the trajectory generation are generally expressions containing significant physical parameters related to the WMR's behavior and to the efficiency of the system. Hereafter, we particularly consider the following functions:

$$F_1 = E_{Total}, \qquad F_2 = T \tag{10}$$

where, T is the travel time, E_{Total} represents the total energy consumption.
As discussed earlier PH based path generation depends only on start and end direction vectors:

$$r = f_1(\theta_s, \theta_f) \tag{11}$$

From eq. 9, energy consumption of a WMR depends on the linear and angular velocity:

$$E_{Total} = f_2(V, \omega) \tag{12}$$

but for a selected path, angular velocity of the WMR is the function of linear velocity and the path.

$$\omega = f_3(r, V) \tag{13}$$

Therefore, from eqs. 11-13, the energy consumption depends on start and end direction vectors (θ_s, θ_f) of the path (r) and the velocity profile (V) as:

$$E_{Total} = f_4(V, \theta_s, \theta_f) \tag{14}$$

Therefore, to minimize energy consumption, both the optimal direction vectors (θ_s, θ_f) and optimal velocity profile (V) is required for the generated path.
Physical limitations: The search space for the variables (start and end direction vectors) is not continuous because some combinations of variables give a path that passes through the obstacles. Therefore these solutions are considered infeasible. The solution from the optimization problem must be feasible. This is modeled as a constraint given below:

$$\{r \cap O\} = \{\phi\} \tag{15}$$

where r is the PH based path and O is the set of obstacles.

Although due to the cyclic nature of angles there are no bounds on the direction vectors, but, the search space for direction vectors is bounded to one cycle.

$$\begin{aligned} 0^\circ \le \theta_s \le 360^\circ \\ 0^\circ \le \theta_f \le 360^\circ \end{aligned} \tag{16}$$

The velocity profile and acceleration evaluated at the center of gravity are bounded by the dynamic limitations of the robot and also the path:

$$\begin{aligned} V_{\min} \le V \le V_{\max} \\ \dot{V}_{\min} \le \dot{V} \le \dot{V}_{\max} \end{aligned} \tag{17}$$

In this work, Genetic Algorithm (GA) is used to find the global optimal solution.

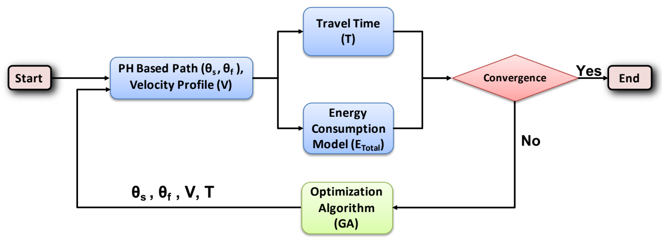


Fig. 2. Generation of Pareto Front in GA

4.2 Methodology

The method includes the calculation of an optimal path and an optimal velocity profile for the path. This is achieved using the GA algorithm, to find multiple Pareto-optimal solutions in a multi-objective optimization problem. From the generated Pareto front, a solution (objective point) is selected (Fig. 4) according to our requirement regarding energy consumption and travel time.

Pareto front generation: The procedure for generating optimal Pareto front for WMR path and velocity profiles is described in Fig. 2. The algorithm aims to find the solution of the path and velocity profiles ensuring the compromise between the travel time T and the energy consumption E_{Total}. A PH path is created using the information of direction vectors as discussed in section 2. Further, a velocity profile is generated using a cubic spline interpolating a set of intermediate control points uniformly distributed along the time scale $[0, T]$, as shown in Fig. 3. The positions of these control points are investigated within the limits imposed by kinematic constraints in eq. 17.

Compromise point selection: The optimization process provides Pareto front. A compromise point is selected from Pareto front in regards of the ideal point whose coordinates are minimum energy consumption and minimum travel time $[min(T), min(E)]$. The selected objective point corresponds to the nearest point to the ideal point as shown in Fig.4. As the range of energy consumption and travel time is different and can not be compared, therefore both

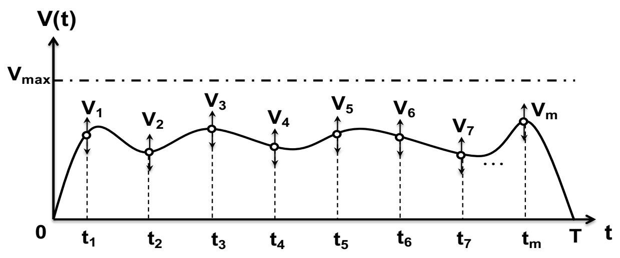


Fig. 3. Generation of optimal velocity profile

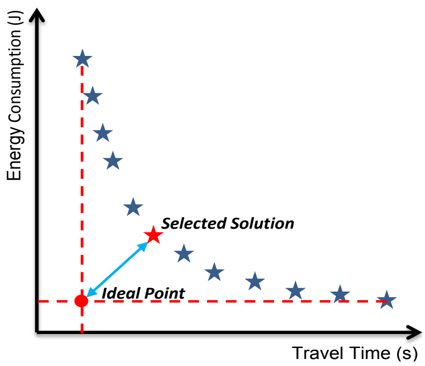


Fig. 4. Selection of objective point from Pareto front

of them are normalised such that the range of energy consumption and travel time are equal.

Figure 2 represents the block diagram of optimization algorithm. This depicts that, using GA, direction vectors (θ_s, θ_f) and velocity profile (V) are optimized to generate a trajectory considering minimum energy consumption with minimum travel time by WMR.

5. DISCRETIZATION OF PATH

The PH generation method discussed in Section 2 gives a PH curve of order 5. This is a major limitation because, for a particular set of obstacles, a fifth-order PH might not be flexible enough to generate a feasible path. In such a condition, it is proposed that the curve is disjointed to form multiple curves. To achieve this, intermediate points are created. An intermediate point is initiated at the endpoint of the path and shifted using the optimization technique. The need can be explained using Fig. 5. In the figure, Path 1 is generated using a single PH curve but Path 2 which requires a sharp turn is a combination of two PH curves joined at the Intermediate point.

Hence, the problem becomes a bi-level optimization problem. In this problem, a search algorithm for optimal path and the velocity profile is embedded inside a search algorithm for the optimum location of an intermediate point. The structure of the optimization problem is discussed in Fig. 6.

Initially, a new intermediate point is initialized. PH curves are generated, first between the start and intermediate point and second between the intermediate and endpoint. The individual optimization function is evaluated for each curve as discussed previously in section 4.1. These objective functions are used to find optimal direction vectors and a velocity profile.

The objective function to find the optimum location of the intermediate point is evaluated by adding individual objective functions of each curve. A constraint is added to assure the smoothness of the path. This is done by equating the direction vectors at the intermediate point for both the PH. The algorithm tries to find the best combination of direction vectors i.e. at the start, intermediate and endpoint. If a feasible combination is not found then the search algorithm for intermediate point continues.

If the search for optimum intermediate point location exhausts without a feasible solution, another intermediate point must be added, thereby increasing the number of curves. The solution achieved before the introduction of a new curve is considered as the best possible solution using lesser number of curves. Therefore, this acts as an initial solution for the search for a new intermediate point problem. The new intermediate point must be placed to split the worst curve i.e. with the highest individual value of the objective function. The optimum position of the new intermediate point and the associated direction vectors are found. The process is continued and new intermediate points are added till a collision-free path is created.

6. RESULTS AND DISCUSSIONS

The presented methodology is applied on mobile robot Robotino. The robot is powered by two $12V$ batteries capable for powering the robot for upto two hours. The mechanical specifications of the robot are given in table 1. An environment is created with obstacles, shown in

Table 1. Robotino Parameters

Parameter	Value
Robot radius	175 mm
Height	210 mm
Battery voltage	24 V
Robot mass	11 Kg
Max velocity	1.325 ms^{-1}
Moment of inertia	0.16245 Kgm^{-2}

Fig. 7 to plan a planar trajectory of the Robotino from start to the destination point. The algorithm provides the trajectory of the center of mass of the Robotino. This trajectory must always maintain a specific distance from any obstacle. In current case, this distance is equal to the radius of Robotino. Hence, a clearance equal to this radius is provided around the obstacles represented by dotted lines. The available work environment after consideration of clearance is provided in Fig. 7.
The bounds for Robotino are as follows:

$$\begin{aligned} 0.01 \leq V \leq 1.325(ms^{-1}) \\ 0^\circ \leq \theta_s \leq 360^\circ \\ 0^\circ \leq \theta_f \leq 360^\circ \end{aligned} \tag{18}$$

Also, the linear acceleration constraint for robotino is given as:

$$-0.05 \leq \dot{V} \leq 1.02(ms^{-2}) \tag{19}$$

A trajectory is planned from start to the destination point using the proposed approach (Fig. 7). This simulated trajectory ensures the minimum energy consumption of the Robotino in the minimum travel time. Here, trajectory is made up of two PH curves. Also, as per discritization algorithm, continuity between the two PH curves is ensured. The final values of the optimization variables are: $\theta_s =$

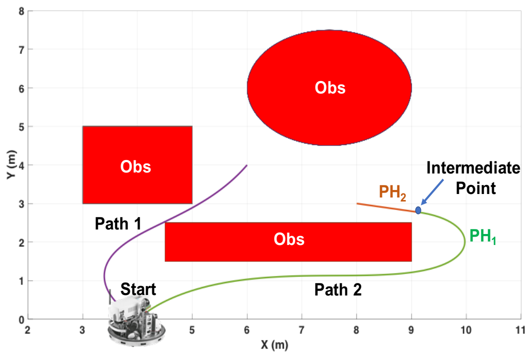


Fig. 5. Need for discretization of path

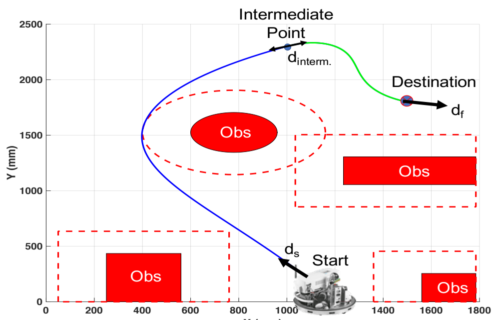


Fig. 7. Simulated optimal trajectory

122.47^o, $\theta_{intermediate} = 35.92^o$ and $\theta_f = 348.52^o$. Optimal velocity profile (V) is shown in Fig. 10(a). The elements of the objective function (energy consumption and travel time) are given in Table 2.

The same environment is created as shown in Fig. 8 for experimental validation. In this environment, a trajectory is planned using the proposed PH based approach as well as the widely used A-star approach. For the A-star approach, the optimal velocity profile is calculated using the same method as used for PH based approach discussed in Section 4.2. Figure 9 shows both of the planned trajectories in the same environment. Robotino is used to follow both of these trajectories. The energy consumption of Robotino is computed using its battery current and voltage. The comparison of data is given in table 2. Figure 10 shows the comparison of the optimal velocity profile as well as power consumption for the

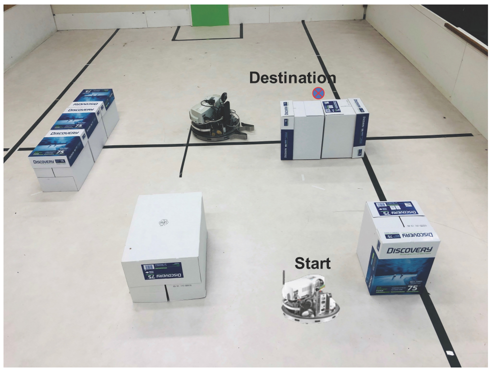


Fig. 8. Experimental set-up

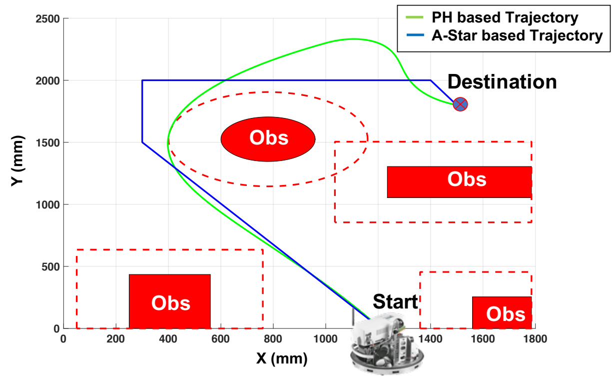


Fig. 9. Experimental comparison of performances of trajectory planning approaches

planned trajectory for the presented scenario (Fig. 7). For the PH based path the high energy consumption is observed during high velocities, whereas for A-star based path, high energy consumption is observed during low velocity. It can be interpreted that for the PH curve most of the energy is consumed for the propulsion of the robot along the path but most of the energy for A-star trajectory is consumed when the robot turns to shift from one portion of the curve to the other. This shows that the vehicle is better able to utilize the available energy while following the PH based path. Therefore, for straight line and simple paths with negligible steering requirements A-star trajectory is expected to be energy efficient but for a complex path PH based trajectory is expected to be more energy-efficient. From the results, it is observed that the

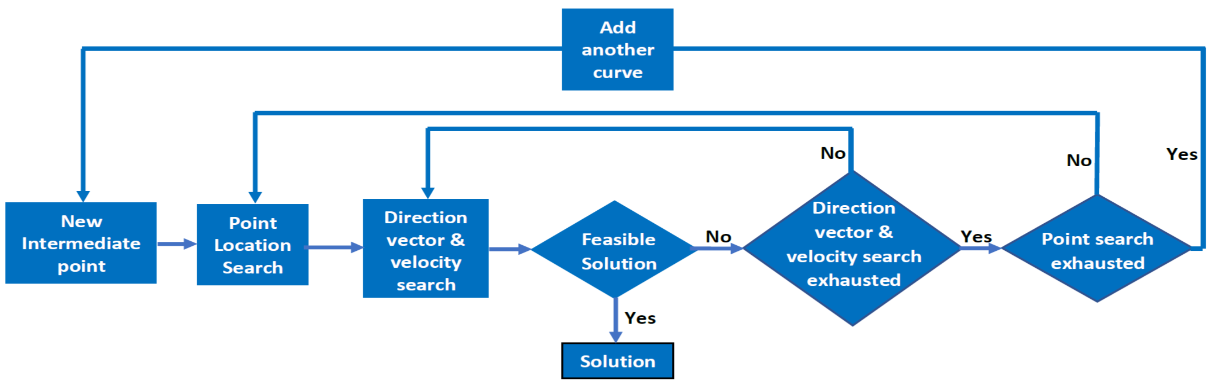


Fig. 6. Block diagram of algorithm of discretization of path

Table 2. Comparison of proposed approach with simulation and A-star algorithm

Trajectory	Path Length (mm)	Energy Consumption (J)	Travel Time (sec)
PH based (Experiment)	3612.44	136.81	23.89
PH based (Simulation)	3592.10	133.01	22.40
A-star based (Experiment)	3578.10	161.67	32.65

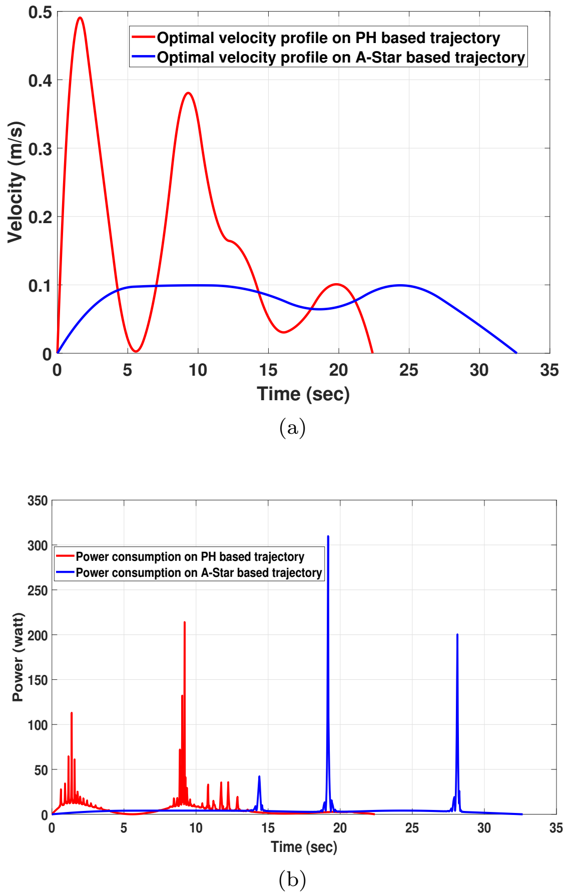


Fig. 10. Comparison between PH and A-Star based trajectories: (a) Optimal velocity profile, (b) Power consumption

energy consumption and the travel time is significantly less when robot moves on the path generated using the proposed approach in-spite of a slight increase in path length.

As discussed in section 4.2, one solution is selected from pareto front considering an equal compromise between minimum energy and minimum travel time. However, according to the need, preference can be given to a point for either a lower energy consumption or a lower travel time.

7. CONCLUSIONS

This paper presents a curve-based approach to plan a smooth and power-efficient trajectory for a WMR. On account of the possibility of generating a higher-order curve, using lesser boundary conditions and availability of direct condition for minimum bending energy, PH curves are chosen to generate a path. The energy consumption and travel time of the robot are minimized by optimizing the PH based path and velocity profile. The paper also discusses the limitations of PH based path in case of using a single PH and a solution is proposed to overcome this limitation by including a serial of PH curves. Experiments are performed using Robotino to check and validate the performance of the proposed trajectory planning methodology. Our future work includes adapting this curve-based approach to the case of a dynamic environment.

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