Outlier Robust State Estimation Through Smoothing on a Sliding Window ⋆

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Abstract: Measurement outliers can severely impact on the performance of conventional state estimators. The design of state estimators exhibiting enhanced robustness to measurement outliers is of interest in many areas of systems and control engineering. In marine robotics applications the issue is particularly relevant for navigation and model identification tasks exploiting acoustic based positioning and velocity sensors that are subject to relatively high rates of outliers. A sliding window state estimator is designed by minimizing the Least Median of Squares cost function evaluated by running a Rauch-Tung-Striebel smoother on the current window. The resulting estimator is tested on Doppler Velocity Log navigation data acquired on an underwater robot. Although these are only preliminary results, they confirm that the approach can be successfully used online.

Keywords: Kalman filtering techniques in marine systems control, Marine system navigation, guidance and control, Marine system identification and modelling, Filtering and smoothing, Estimation and filtering, Recursive identification.

1. INTRODUCTION

State estimation in the presence of measurement outliers is a relevant and challenging issue in many applications. Particularly so in marine robotics and oceanic engineering scenarios where acoustic measuring devices are most common. Multi-path and multi-reflection phenomena in underwater acoustic propagation may generate outliers in range and velocity measurements used for marine system navigation and localization applications. Possible approaches to cope with outliers in Ultra-Short Base Line and Long Base Line acoustic positioning systems are addressed by many authors including, by example, Vaganay et al. (1996), Bingham and Seering (2006), A. Alcocer (2006), Vasilijevic et al. (2012), Morgado et al. (2015), and Leonard and Bahr (2016). Similar problems arise when processing Doppler Velocity Logger (DVL) data for measuring velocity in marine vehicles in navigation or identification scenarios. Specific examples in this area include results by Martin and Whitcomb (2014), van de Ven et al. (2007), and Lekkas et al. (2015). Outliers may significantly impact also on cooperative robotics marine systems as accounted by Bahr et al. (2009) and Soares et al. (2013) as well as on underwater vehicle control (Caccia et al. (2003)) or diver tracking applications (Mišković et al. (2015)). It should be noticed that measurement outliers in underwater applications are not limited to acoustic transducers. Indeed, vision related processing is also potentially affected by outliers as discussed by Horgan and Toal (2006), Leone et al. (2006), and Distante and Indiveri (2011) just to name a few examples.

Fitting data to models in the presence of outliers as well as outlier identification are central topics in statistics. The related literature is extremely wide and interesting: significant textbooks addressing outlier related issues for statistical models include the ones by Hawkins (1980), Rousseeuw and Leroy (2003), and Huber and Ronchetti (2009). Although many results and ideas derived in the statistics literature are of paramount relevance in engineering applications, most of these results refer to static models and batch data processing approaches: indeed recursive algorithms as applied to dynamic (state space) models are basically absent in the statistics literature. An exception is the paper by Ruckdeschel et al. (2014) suggesting to saturate the correction term of the Kalman filter in order to prevent outliers from arbitrarily affecting the Kalman state estimate. The idea of limiting (i.e. saturating) the correction term in prediction - correction state estimation filters to limit the impact of potential measurement outliers is a quite common and simple. In marine applications a similar approach was used by Vike and Jouffroy (2005) designing a nonlinear diffusion based prediction - correction filter having an intrinsically limited correction term. The problem with such approaches as applied to linear state space estimation (like for the Kalman or Luenberger filters) is that a saturated correction term can jeopardize the filter stability, in particular when the state transition matrix is not stable. This problem was recently studied and solved by Alessandri and Zaccarian (2018) in the linear time-invariant system case, both in continuous and discrete time. In particular they designed an estimator with

a correction term with an adaptive saturation threshold granting global exponential stability to the origin for the error dynamics.

In this paper a sliding windows approach is discussed: similarly to the solution we previously derived in (De Palma and Indiveri (2017)), the estimate is computed by minimizing an outlier robust cost function on the moving window. The novelty here is twofold: the considered cost function is the Least Median of Squares (LMS); moreover, such cost function is evaluated using a Rauch-Tung-Striebel (RTS) smoother (Rauch et al. (1965)) on the window rather than the pure prediction of the state estimate. Numerical simulation results suggest that this significantly improves the performances of the filter as compared to an implementation computing the cost function on the prediction only. Notice that the proposed filter structure could be also implemented using alternative outlier robust cost functions on the sliding window as, by example, the Least Trimmed Squares (LTS) one. Indeed the idea of exploiting the structure of the LTS parameter identification approach for state estimation is also exploited by Alessandri and Awawdeh (2016) where smoothing is not performed on the moving window.

2. BACKGROUND AND SETTING

Consider the dynamic linear model given by

$$\mathbf{x}_k = A_{k-1}\,\mathbf{x}_{k-1} + B_{k-1}\,\mathbf{u}_{k-1} + \mathbf{w}_{k-1} \qquad (1)$$

$$\mathbf{y}_k = C_k\,\mathbf{x}_k + \boldsymbol{\epsilon}_k \qquad (2)$$

where $\mathbf{x}_k \in \mathbb{R}^n$ is the state vector, $\mathbf{u}_k \in \mathbb{R}^q$ is the known input vector, $\mathbf{y}_k \in \mathbb{R}^p$ is the observation vector, $\mathbf{w}_k \in \mathbb{R}^n$ is the state noise, $\boldsymbol{\epsilon}_k \in \mathbb{R}^p$ is the observation noise, $A_k \in \mathbb{R}^{n \times n}$ is the state transition matrix, $B_k \in \mathbb{R}^{n \times q}$ is the input matrix and $C_k \in \mathbb{R}^{p \times n}$ is the observation matrix at time step k. Assume $\mathbf{w}_k$ and $\boldsymbol{\epsilon}_k$ to be zero-mean, Gaussian, white and uncorrelated noise, that is, $\mathbf{w}_k \sim \mathcal{N}(0, Q_k)$ and $\boldsymbol{\epsilon}_k \sim \mathcal{N}(0, R_k)$ where Q_k and R_k are the covariance matrices of the state and observation noise, respectively. Suppose further the initial state $\mathbf{x}_0$ to be a Gaussian random vector $\mathbf{x}_0 \sim \mathcal{N}(\bar{\mathbf{x}}_0, P_0)$ independent of $\mathbf{w}_k$ and $\boldsymbol{\epsilon}_k$, where P_0 denotes its covariance matrix.

The state estimation problem consists in determining the state vector $\mathbf{x}_k$ given the knowledge of the model matrices (including the noise covariances), the input and output sequences up to time k. In the stated hypothesis and under suitable observability conditions Simon (2006), Jazwinski (2007), Anderson and Moore (2012), the optimal (i.e. with least estimate covariance) solution is given by the Kalman filter (KF). Following Anderson and Moore (2012), the equations of the recursive KF are:

$$\hat{\mathbf{x}}_k^- = A_{k-1}\,\hat{\mathbf{x}}_{k-1}^+ + B_{k-1}\,\mathbf{u}_{k-1} \qquad (3)$$

$$P_k^- = A_{k-1}P_{k-1}^+ A_{k-1}^\top + Q_{k-1} \qquad (4)$$

$$K_k = P_k^- C_k^\top (C_k P_k^- C_k^\top + R_k)^{-1} \qquad (5)$$

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + K_k(\mathbf{y}_k - C_k \hat{\mathbf{x}}_k^-) \qquad (6)$$

$$P_k^+ = P_k^- - K_k C_k P_k^-. \qquad (7)$$

where $\hat{\mathbf{x}}_k^-$ denotes the estimate of $\mathbf{x}_k$ before processing the measurements at time k, $\hat{\mathbf{x}}_k^+$ denotes the estimate

of $\mathbf{x}_k$ after processing the measurements at time k, P_k^- denotes the covariance of the estimation error of $\hat{\mathbf{x}}_k^-$ and P_k^+ denotes the covariance of the estimation error of $\hat{\mathbf{x}}_k^+$. It should be noticed that the standard Kalman filter estimates simultaneously both, the state and the error covariance matrix.

The Kalman filter is highly sensitive to outliers and has an asymptotic breakdown point of zero Rousseeuw and Leroy (2003). Indeed, if the assumption of Gaussian measurements noise is violated or the observations are contaminated by outliers, the optimality of the filter fails and its performance can deteriorate significantly. Even a single outlier in the observed data can compromise the result of the estimation. A similar situation may occur in regression and parameter identification scenarios when measurements include outliers. In such cases one may consider robust regression approaches as the Least Median of Squares (LMS) estimator described in Rousseeuw and Leroy (2003). In short, the LMS parameter estimate $\hat{\boldsymbol{\theta}}_{LMS}$ is obtained by minimizing with respect to the unknown parameter $\boldsymbol{\theta}$ the median of the square residual $\mathbf{r}_k = \mathbf{y}_k - \hat{\mathbf{y}}_k(\boldsymbol{\theta})$ norms being $\hat{\mathbf{y}}_k(\boldsymbol{\theta})$ the model estimated measurement. In particular

$$J_{LMS}(\boldsymbol{\theta}) = \text{median}\left\{\|\mathbf{r}_1\|^2, \|\mathbf{r}_2\|^2, \ldots, \|\mathbf{r}_n\|^2\right\} \qquad (8)$$

$$\hat{\boldsymbol{\theta}}_{LMS} = \arg\min_{\boldsymbol{\theta}} J_{LMS}(\boldsymbol{\theta}). \qquad (9)$$

The resulting estimator has the highest possible breakdown point, i.e. 50%. For an intuitive interpretation of the LMS, notice that in case of linear regression of data points on a line in the plane, the LMS line corresponds to the center line of the thinnest stripe of the plane containing half plus one of the data points.

3. LMS BASED ROBUST STATE ESTIMATOR

The proposed estimator consists in performing state estimation by minimizing a least median of square (LMS) cost function on a sliding window of data, i.e. according to a moving horizon strategy.

Let be N the size of the sliding window, the proposed approach is built on the derivation of a state estimate at the current time k by using the information given by the measurements and inputs within the window, i.e. $\mathbf{y}_{k-N+1}$, $\mathbf{y}_{k-N+2}, \ldots, \mathbf{y}_k, \mathbf{u}_{k-N+1}, \mathbf{u}_{k-N+2}, \ldots, \mathbf{u}_k$ with the integer $N \geq 1$. The estimates of $\mathbf{x}_{k-N+1}, \mathbf{x}_{k-N+2}, \ldots, \mathbf{x}_k$ are based on such information and on an "estimation" $\hat{\mathbf{x}}_{k-N}^+$ of the state $\mathbf{x}_{k-N}$ at the beginning of the moving window. In the following, the estimates of $\mathbf{x}_{k-N+1}, \mathbf{x}_{k-N+2}, \ldots, \mathbf{x}_k$ at time k are denoted by $\hat{\mathbf{x}}_{k-N+1|k}, \hat{\mathbf{x}}_{k-N+2|k}, \ldots, \hat{\mathbf{x}}_{k|k}$, respectively. This is equivalent to a smoothing problem within the sliding window that can be solved with the standard Rauch-Tung-Striebel (RTS) smoother Rauch et al. (1965). The RTS smoother is implemented by first running the standard Kalman filter equations (3-7) forward in time to the current time k, and then implementing the RTS smoother equations (10-15) backward in time to the initial time $k - N + 1$:

1. Initialize the RTS smoother as follows:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_k^+ \tag{10}$$

$$P_{k|k} = P_k^+ \tag{11}$$

2. For $i = k-1, \ldots, k-N+1$ execute:

$$\mathcal{I}_{i+1}^- = (P_{i+1}^-)^{-1} \tag{12}$$

$$K_i = P_k^+ A_k^\top \mathcal{I}_{i+1}^- \tag{13}$$

$$P_{i|k} = P_i^+ - K_i(P_{i+1}^- - P_{i+1|k})K_i^\top \tag{14}$$

$$\hat{\mathbf{x}}_{i|k} = \hat{\mathbf{x}}_{i|k}^+ + K_i(\hat{\mathbf{x}}_{i+1|k} - \hat{\mathbf{x}}_{i+1}^-). \tag{15}$$

By seeking inspiration from the algorithms for the computation of the LMS estimator Rousseeuw and Leroy (2003), a robust procedure (alogorithm) is proposed by repeatedly drawing subsamples of measurements in a sliding window set $Y_k = \{\mathbf{y}_{k-N+1}, \mathbf{y}_{k-N+2}, \ldots, \mathbf{y}_k\}$. Subsamples are denoted as S_k^j where k is the current time index and j is an index running on all possible m subsamples obtained keeping only L data values out of the N ones in Y_k. In particular m can be computed as the combinations without repetitions of L elements out of N

$$m = C_L^N = \binom{N}{L} = \frac{N!}{L!(N-L)!}.$$

By example if $N = 3$ and $L = 2$ then $m = 3$ and at time, say, $k = 8$ the corresponding subsamples of Y_8 would be $S_8^1 = \{\mathbf{y}_6, \mathbf{y}_7, \cdot\}$, $S_8^2 = \{\mathbf{y}_6, \cdot, \mathbf{y}_8\}$, and $S_8^3 = \{\cdot, \mathbf{y}_7, \mathbf{y}_8\}$. For each subsample S_k^j one can determine the RTS estimates $\hat{\mathbf{x}}_{k-N+1|k}^{S_k^j}$, $\hat{\mathbf{x}}_{k-N+2|k}^{S_k^j}$, $\ldots$, $\hat{\mathbf{x}}_{k|k}^{S_k^j}$.

These estimates leave out the measurements not belonging to the subsample. In correspondence of such points, the standard RTS smoothing equation can still be used by assuming that the measurements have a diverging covariance R_i, namely with R_i^{-1} tending to the zero matrix being i in the range $[k-N+1, k]$.

Having computed the RTS estimates, for each subsample S_k^j one can evaluate the median of the square residuals cost

$$J_k^{S_k^j}(\hat{\mathbf{x}}_{k-N}^+) = \underset{i=k-N+1,\ldots,k}{\text{median}} \left\| \mathbf{y}_i - C_i \hat{\mathbf{x}}_{i|k}^{S_k^j} \right\|^2 \tag{16}$$

and select the subsample S_k^{j*} with least median of squares (LMS) cost, namely

$$S_k^{j*} = \arg \min_{j=1,\ldots,m} J_k^{S_k^j}(\hat{\mathbf{x}}_{k-N}^+) \tag{17}$$

that allows to identify the current state estimate $\hat{\mathbf{x}}_{k|k}^*$ and its covariance $P_{k|k}^*$ as $\hat{\mathbf{x}}_{k|k}^* := \hat{\mathbf{x}}_{k|k}^{S_k^{j*}}$ and $P_{k|k}^* := P_{k|k}^{S_k^{j*}}$.

The LMS criterion to select the subsample S_k^{j*} aims at excluding outliers from the measurement set used in the RTS smoothing phase. The computational effort of this approach is strongly dependent on the sliding window size N and on the number m of subsample sets where to compute the RTS estimator. If the inliers every N measurements are actually at least L, the proposed approach has the potential (albeit with no formal gauarantee) to exclude all the outliers. Figure 1 reports an example of the described algorithm with subsamples obtained considering $N = 5$ and $L = 3$ (hence $m = 10$).

The pseudo-code for the proposed outlier robust state estimation is reported in algorithm 1.

As a side result the proposed approach allows to eventually rank the measurements according to the number of times they belong to an optimal subset mimizing the median of squares. Indeed when the sliding window of size N is shifted forward $N-1$ measurements of the new window were also included in the previous window. As the window is shifted further, due to the overlap of successive windows, a certain number of measurements are re-evaluated multiple times. One can thus label measurements according to the number of times they are accepted in the (local) Least Median of Squares optimal subsets. Measurements that are more frequently accepted in the local optimal subsets according to the LMS criteria are candidate inliers. Eventually such data can be used to run an estimator over the whole batch of data or over longer windows than the N data points one used locally as previously described. Details of this extension are not included in this paper for the sake of brevity and will be subject to future work.

Algorithm 1 LMS-RTS based estimation algorithm

Require: L, N, $\hat{\mathbf{x}}_{k-N}^+$, P_{k-N}^+, $\mathbf{y}_{k-N+1}$, $\ldots$, $\mathbf{y}_k$, $\mathbf{u}_{k-N+1}$, $\ldots$, $\mathbf{u}_k$

Ensure: $\hat{\mathbf{x}}_{k|k}^*$, $P_{k|k}^*$

1: **for** $j = 1 : m$

2: compute the RTS smoothing estimation on mobile window using (10-15): $\hat{\mathbf{x}}_{k-N+1|k}^{S_k^j}, \hat{\mathbf{x}}_{k-N+2|k}^{S_k^j}, \ldots, \hat{\mathbf{x}}_{k|k}^{S_k^j}$, $P_{k-N+1|k}^{S_k^j}, P_{k-N+2|k}^{S_k^j}, \ldots, P_{k|k}^{S_k^j}$

3: compute the LMS cost $J_k^{S_k^j}(\hat{\mathbf{x}}_{k-N}^+)$ using (16)

4: **end for**

5: $S_k^{j*} \leftarrow \arg \min_{j=1,\ldots,m} J_k^{S_k^j}(\hat{\mathbf{x}}_{k-N}^+)$

6: $\hat{\mathbf{x}}_{k|k}^*, P_{k|k}^* \leftarrow \hat{\mathbf{x}}_{k|k}^{S_k^{j*}}, P_{k|k}^{S_k^{j*}}$

7: **return** $\hat{\mathbf{x}}_{k|k}^*$, $P_{k|k}^*$

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4. EXAMPLE ON MARINE ROBOTICS APPLICATIONS.

A specific example relative to the navigation of an underwater robot is reported. The experiment is performed employing an Underwater Vehicle Manipulator System (UVMS) developed within the ROBUST project ROBUST (2015-2020). The UVMS is composed by three Autonomous Underwater Vehicles (AUVs), each one equipped with 4 tunnel thrusters (2 vertical and 2 lateral at the bow and stern of the AUVs rispectively), and 1 main rear thruster. During the experiment, the UVMS is controlled to obtain a motion along the surge axes at constant speed, and its velocity is acquired using a DVL sensor together with the thrusters commands. The velocity starts from zero and increases gradually until reaching a steady state value. The following dynamic equation for surge velocity is considered:

$$m\dot{v} + D_l v + D_q v|v| = \tau \tag{18}$$

where v denotes the surge velocity, m the mass of the vehicle (including added mass), D_l and D_q the linear and quadratic surge drag coefficients, τ the surge actuation term. The latter can be expressed as:

$$Y_k = \{\mathbf{y}_{k-4}, \mathbf{y}_{k-3}, \mathbf{y}_{k-2}, \mathbf{y}_{k-1}, \mathbf{y}_k\}$$

$$
\left.
\begin{array}{l}
S_k^1 = \{\mathbf{y}_{k-4}, \mathbf{y}_{k-3}, \mathbf{y}_{k-2} \quad\quad\quad\} \rightarrow J_k^{S_k^1} = \underset{i=k-4,\dots,k}{\mathrm{median}} \left\| \mathbf{y}_i - C_i \hat{\mathbf{x}}_{i|k}^{S_k^1} \right\|^2 \\[2mm]
S_k^2 = \{\mathbf{y}_{k-4}, \mathbf{y}_{k-3}, \quad\quad \mathbf{y}_{k-1} \quad\quad\} \rightarrow J_k^{S_k^2} = \quad\quad\quad \vdots \\[2mm]
S_k^3 = \{\mathbf{y}_{k-4}, \mathbf{y}_{k-3}, \quad\quad\quad\quad \mathbf{y}_k\} \rightarrow J_k^{S_k^3} = \quad\quad\quad \vdots \\[2mm]
S_k^4 = \{\mathbf{y}_{k-4}, \quad\quad \mathbf{y}_{k-2}, \mathbf{y}_{k-1} \quad\quad\} \rightarrow J_k^{S_k^4} = \quad\quad\quad \vdots \\[2mm]
S_k^5 = \{\mathbf{y}_{k-4}, \quad\quad \mathbf{y}_{k-2}, \quad\quad \mathbf{y}_k\} \rightarrow J_k^{S_k^5} = \quad\quad\quad \vdots \\[2mm]
S_k^6 = \{\mathbf{y}_{k-4}, \quad\quad\quad\quad \mathbf{y}_{k-1}, \mathbf{y}_k\} \rightarrow J_k^{S_k^6} = \quad\quad\quad \vdots \\[2mm]
S_k^7 = \{\quad\quad \mathbf{y}_{k-3}, \mathbf{y}_{k-2}, \mathbf{y}_{k-1} \quad\quad\} \rightarrow J_k^{S_k^7} = \quad\quad\quad \vdots \\[2mm]
S_k^8 = \{\quad\quad \mathbf{y}_{k-3}, \mathbf{y}_{k-2}, \quad\quad \mathbf{y}_k\} \rightarrow J_k^{S_k^8} = \quad\quad\quad \vdots \\[2mm]
S_k^9 = \{\quad\quad \mathbf{y}_{k-3}, \quad\quad \mathbf{y}_{k-1}, \mathbf{y}_k\} \rightarrow J_k^{S_k^9} = \quad\quad\quad \vdots \\[2mm]
S_k^{10} = \{\quad\quad\quad\quad \mathbf{y}_{k-2}, \mathbf{y}_{k-1}, \mathbf{y}_k\} \rightarrow J_k^{S_k^{10}} = \underset{i=k-4,\dots,k}{\mathrm{median}} \left\| \mathbf{y}_i - C_i \hat{\mathbf{x}}_{i|k}^{S_k^{10}} \right\|^2
\end{array}
\right\}
\underset{j=1,\dots,10}{\min} J_k^{S_k^j} \rightarrow \hat{\mathbf{x}}_{k|k}^{S_k^{j\,*}}
$$

Fig. 1. Example of subsamples with $N = 5$ and $L = 3$ ($m = 10$).

$$
\tau = G \begin{bmatrix} \alpha_1 n_1 |n_1| \\ \vdots \\ \alpha_n n_n |n_n| \end{bmatrix}, \tag{19}
$$

where $n_i, i = 1, \dots, 15$ represent the thruster revolutions per minute (RPM) values, while the matrix $G \in \mathbb{R}^{1 \times 15}$ depends on the relative position of each thruster with respect to the body frame, and on the thruster characteristics. A quadratic relationship has been assumed between the thruster's RPM values and its exerted force, neglecting other hydrodynamic terms in the thruster modelling depending, by example, on the water flow velocity through the propeller blades. With regards to the knowledge of the added mass, hydrodynamic parameters, and coefficients in (18, 19), we resort to the values estimated and experimentally identified exploiting the method in Ingrosso et al. (2019). Note that for the sake of simplicity, in order to approximate the non linear model (18) into a linear one, the hydrodynamic drag has been modelled with a linear term only leading to the following equation:

$$m\dot{v} + Dv = \tau \tag{20}$$

with D chosen as $D = D_l + D_q \bar{v}$, being $\bar{v}$ the mean value of the vehicle velocity during the experiment. Assuming to explicitly account for noise, and discretizing the system with a sampling time T_s, the equation (20) can be reformulated in a linear time invariant (LTI) state space setting:

$$x_k = A x_{k-1} + B u_{k-1} + w_{k-1}, \tag{21}$$

$$y_k = C x_k + \epsilon_k, \tag{22}$$

having defined the state as $x = v$, the input as $u = \tau$, and

$$A = 1 - \frac{D}{m} T_s, \quad B = \frac{T_s}{m}, \quad C = 1. \tag{23}$$

The covariances of the state and the observations are $Q = (10^{-3}\, m/s)^2$ and $R = (5 \cdot 10^{-3}\, m/s)^2$, respectively. In order to validate the performance of the proposed algorithm both in absence and in presence of outliers, we firstly considered an experimental data set free of outliers. A sliding window of size $N = 9$ with $L = 5$ and $m = 126$

is considered and the proposed LMS-RTS algorithm is applied. The resulting estimates are shown in figure 2. If compared with a standard KF, the LMS-RTS algorithm does not exhibit degraded performances. Then, a data
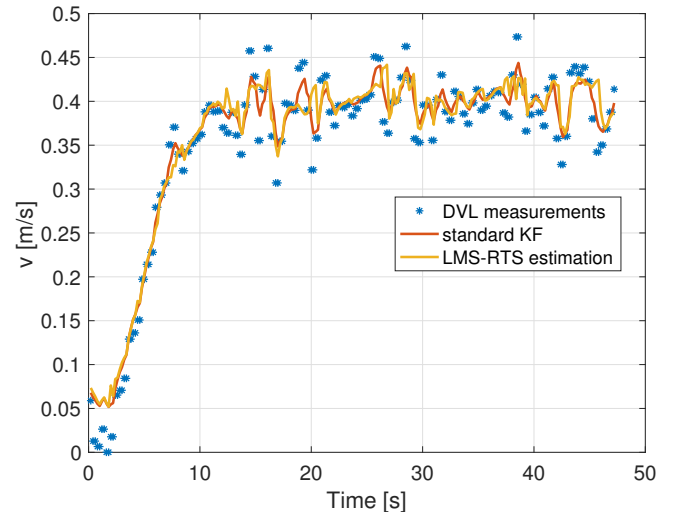

Fig. 2. UVMS velocity measured through a DVL and velocity estimated through the KF and the LMS-RTS algorithm in absence of outliers.

set corrupted by outlier has been considered. Outliers can occur, for example, when the vehicle is very close to the sea bottom due to multi-path phenomena. Figure 3 shows an example of real data set of DVL measurements acquired from the UVMS, it should be noted that the measurement outliers occurred when the depth of the vehicle was maximum, i.e. when the vehicle was close to the sea bottom.

In order to compare the behaviour of the proposed filter with the KF estimate in the presence of measurement outliers, some artificial ones are added to the same data

set trying to replicate the typical frequency and entity of outliers in DVL measurements.
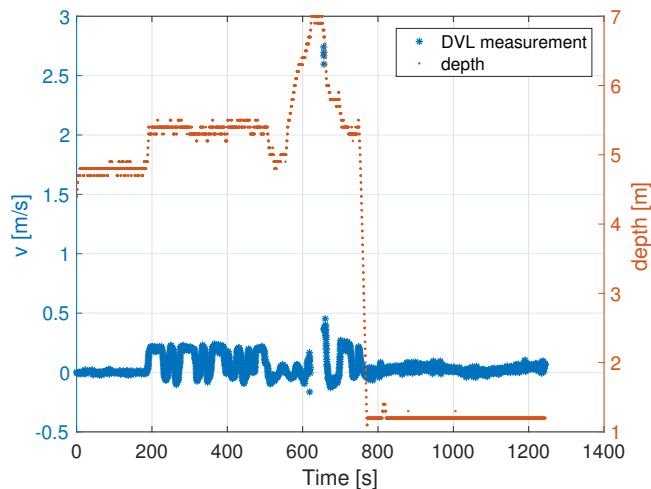

Fig. 3. Example of outliers in a real data set of DVL measurements acquired from the UVMS and corresponding depth of the UVMS.

The results of this experimental example are plotted in 4. The comparison with the KF estimate confirms the enhanced ability to cope with measurement outliers at the cost of an additional computational effort arising from the implementation of the smoother and the LMS minimization. From a computational point of view, Table 1 provides the amount of time required to compute the Kalman and RTS-LMS estimates (pseudo-code in Algorithm 1), respectively. The algorithms have been coded in MATLAB® (version R2017a) on an Apple Laptop with a 3.1 GHz Intel Core i7 processor, 16 GB RAM, running the MAC OS X Version 10.11.6 operating system. Also notice that our MATLAB implementations, for both filters, were not specifically optimized for execution time. Of course, by using dedicated hardware and a suitably optimized code, the execution time can be significantly less.

	Time to process 326 time steps [s]	Average time per step [s]
Kalman Filter	0.02	$6 \cdot 10^{-5}$
LMS-RTS Filter	8.6	$2640 \cdot 10^{-5}$

Table 1. Computational cost of Kalman and LMS-RTS algorithm.

5. CONCLUSION

A sliding window state estimation filter is designed for a linear state space model aiming to achieve enhanced robustness to measurement outliers as compared to conventional Luenberger or Kalman filters. The novel contribution is relative to the use of a Least Median of Squares cost function on a moving horizon and its evaluation on the residuals arising from performing smoothing on the same window rather than estimation only. This allows to better highlight outlying data as due to smoothing on the moving window an outlier eventually entering at the current time step in the window will affect the *past* smoothed
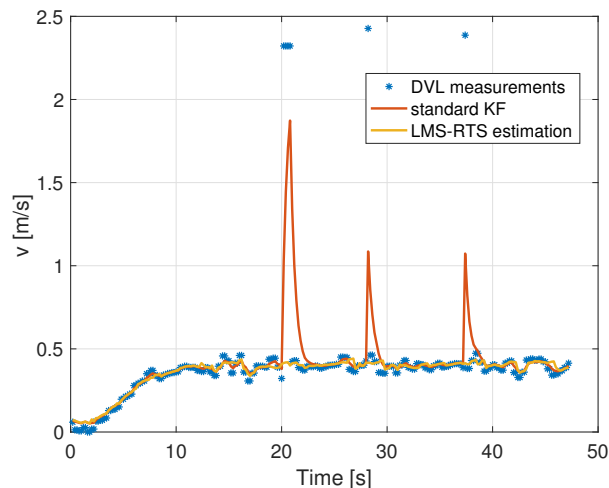

Fig. 4. UVMS velocity measured through a DVL and velocity estimated through the KF and the LMS-RTS algorithm in presence of outliers.

state estimates in the window and not only the future ones. Outlier robustness stems from the fact that not all measurements are processed on the moving window, but only the ones such that the median of squared residuals is minimized on the window. The identification of the specific subsample corresponding to the least median of squares cost on the window is performed through an exhaustive search. This can be quite demanding from a computational point of view as the number of subsamples to evaluate rapidly grows with the window size. Window size and the least number of candidate inliers in the window are tuning parameters. If the true number of inliers is actually larger or equal than the expected one, the proposed solution has the potential to reject all the outliers, otherwise not. The study is motivated by the need to cope with measurement outliers frequently arising in marine robotics navigation and localization scenarios. Numerical results exploiting experimental DVL measurements are illustrated.

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REFERENCES

A. Alcocer, P. Oliveira, A.P. (2006). Underwater acoustic positioning systems based on buoys with GPS. In S.M. Jesus and O.C. Rodríguez (eds.), *Proceedings of the Eighth European Conference on Underwater Acoustics, 8th ECUA*, volume 8, 1–8. Carvoeiro, Portugal.

Alessandri, A. and Awawdeh, M. (2016). Moving-horizon estimation with guaranteed robustness for discrete-time linear systems and measurements subject to outliers. *Automatica*, 67, 85 – 93. doi: https://doi.org/10.1016/j.automatica.2016.01.015. URL http://www.sciencedirect.com/science/article/pii/S0005109816000169.

Alessandri, A. and Zaccarian, L. (2018). Stubborn state observers for linear time-invariant systems. *Automatica*,

88, 1 – 9. doi:https://doi.org/10.1016/j.automatica.2017.10.022. URL http://www.sciencedirect.com/science/article/pii/S0005109817305319.

Anderson, B.D. and Moore, J.B. (2012). *Optimal filtering*. Courier Dover Publications.

Bahr, A., Leonard, J.J., and Fallon, M.F. (2009). Cooperative localization for autonomous underwater vehicles. *The International Journal of Robotics Research*, 28(6), 714–728. doi:10.1177/0278364908100561. URL http://dx.doi.org/10.1177/0278364908100561.

Bingham, B. and Seering, W. (2006). Hypothesis grids: improving long baseline navigation for autonomous underwater vehicles. *IEEE Journal of Oceanic Engineering*, 31(1), 209–218. doi:10.1109/JOE.2006.872220.

Caccia, M., Bono, R., Bruzzone, G., and Veruggio, G. (2003). Bottom-following for remotely operated vehicles. *Control Engineering Practice*, 11, 461–470. doi:10.1016/S0967-0661(01)00142-3.

De Palma, D. and Indiveri, G. (2017). Output outlier robust state estimation. *International Journal of Adaptive Control and Signal Processing*, 31(4), 581–607. doi:10.1002/acs.2673. URL http://dx.doi.org/10.1002/acs.2673. ISSN: 08906327.

Distante, C. and Indiveri, G. (2011). RANSAC-LEL: An optimized version with least entropy like estimators. In *2011 18th IEEE International Conference on Image Processing*, 1425–1428. doi:10.1109/ICIP.2011.6115709.

Hawkins, D.M. (1980). *Identification of Outliers*. Springer, Dordrecht. doi:10.1007/978-94-015-3994-4.

Horgan, J. and Toal, D. (2006). Review of machine vision applications in unmanned underwater vehicles. In *2006 9th International Conference on Control, Automation, Robotics and Vision*. IEEE, Singapore, Singapore. doi:10.1109/ICARCV.2006.345250.

Huber, P.J. and Ronchetti, E.M. (2009). *Robust statistics - second edition*. Wiley, New York.

Ingrosso, R., De Palma, D., Avanzini, G., and Indiveri, G. (2019). Dynamic modeling of underwater multi-hull vehicles. *Robotica*, 1–21. doi:10.1017/S0263574719001693.

Jazwinski, A.H. (2007). *Stochastic processes and filtering theory*. Courier Dover Publications.

Lekkas, A.M., Candeloro, M., and Schjølberg, I. (2015). Outlier rejection in underwater acoustic position measurements based on prediction errors. In D.R. Pere Ridao Marc Carreras (ed.), *4th IFAC Workshop on Navigation, Guidance and Control of Underwater Vehicles NGCUV 2015, Dedicated to the memory of Professor Geoff Roberts*, volume 48, 82 – 87. doi:https://doi.org/10.1016/j.ifacol.2015.06.014. URL http://www.sciencedirect.com/science/article/pii/S2405896315002530.

Leonard, J.J. and Bahr, A. (2016). *Springer Handbook of Ocean Engineering.*, chapter Autonomous Underwater Vehicle Navigation, 341–358. Springer, Cham. doi:10.1007/978-3-319-16649-0_14.

Leone, A., Distante, C., Mastrolia, A., and Indiveri, G. (2006). A fully automated approach for underwater mosaicking. In *OCEANS 2006*, 1–6. doi:10.1109/OCEANS.2006.307006.

Martin, S.C. and Whitcomb, L.L. (2014). Experimental identification of six-degree-of-freedom coupled dynamic plant models for underwater robot vehicles. *IEEE Journal of Oceanic Engineering*, 39(4), 662–671. doi:

10.1109/JOE.2013.2280492.

Mišković, N., Nađ, Đ., and Rendulić, I. (2015). Tracking divers: An autonomous marine surface vehicle to increase diver safety. *IEEE Robotics Automation Magazine*, 22(3), 72–84. doi:10.1109/MRA.2015.2448851.

Morgado, M., Oliveira, P., and Silvestre, C. (2015). Robust outliers detection and classification for usbl underwater positioning systems. In A. Moreira, A. Matos, and G. Veiga (eds.), *CONTROLO '2014 - Proceedings of the 11th Portuguese Conference on Automatic Control*, volume 321 of *Lecture Notes in Electrical Engineering*, 555–565. Springer, Cham. doi:10.1007/978-3-319-10380-8_53.

Rauch, H.E., Striebel, C.T., and Tung, F. (1965). Maximum likelihood estimates of linear dynamic systems. *American Institute of Aeronautics and Astronautics Journal*, 3(8), 1445–1450. doi:10.2514/3.3166. URL https://doi.org/10.2514/3.3166.

ROBUST (2015-2020). ROBUST: Robotic subsea exploration technologies. EU H2020 Project grant agreement ID: 690416 https://cordis.europa.eu/project/id/690416. (last visited on May 9th, 2020).

Rousseeuw, P.J. and Leroy, A.M. (2003). *Robust regression and outlier detection*. John Wiley & Sons.

Ruckdeschel, P., Spangl, B., and Pupashenko, D. (2014). Robust Kalman tracking and smoothing with propagating and non-propagating outliers. *Statistical Papers*, 55(1), 93–123. doi:10.1007/s00362-012-0496-4. URL http://dx.doi.org/10.1007/s00362-012-0496-4.

Simon, D. (2006). *Optimal state estimation: Kalman, H infinity, and nonlinear approaches*. John Wiley & Sons.

Soares, J.M., Aguiar, A.P., Pascoal, A.M., and Martinoli, A. (2013). Joint ASV/AUV range-based formation control: Theory and experimental results. In *2013 IEEE International Conference on Robotics and Automation (ICRA 2013)*, 5579–5585. Karlsruhe, Germany. doi:10.1109/ICRA.2013.6631378. URL http://dx.doi.org/10.1109/ICRA.2013.6631378.

Vaganay, J., Leonard, J., and Bellingham, J. (1996). Outlier rejection for autonomous acoustic navigation. In *Proceedings of the 1996 IEEE International Conference on Robotics and Automation, ICRA 1996*, volume 3, 2174–2181. IEEE. doi:10.1109/ROBOT.1996.506191. URL http://dx.doi.org/10.1109/ROBOT.1996.506191.

van de Ven, P., Johansen, T., Sørensen, A., Flanagan, C., and Toal, D. (2007). Neural network augmented identification of underwater vehicle models. *Control Engineering Practice*, 15(6), 715–725. doi:10.1016/j.conengprac.2005.11.004. URL http://doi.org/10.1016/j.conengprac.2005.11.004.

Vasilijevic, A., Borovic, B., and Vukic, Z. (2012). Underwater vehicle localization with complementary filter: Performance analysis in the shallow water environment. *Journal of Intelligent & Robotic Systems*, 68(3-4), 373–386. doi:10.1007/s10846-012-9766-6. URL http://dx.doi.org/10.1007/s10846-012-9766-6.

Vike, S. and Jouffroy, J. (2005). Diffusion-based outlier rejection for underwater navigation. In *Proceedings of MTS/IEEE OCEANS, 2005*, volume 2, 1037–1042. IEEE. doi:10.1109/OCEANS.2005.1639890. URL http://dx.doi.org/10.1109/OCEANS.2005.1639890.