Platoon Stability Conditions Under Inter-vehicle Additive Noisy Communication Channels

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Abstract: This paper studies the behavior of a platoon control system under the presence of inter-vehicle noisy communication channels. A set of homogeneous vehicles modelled as LTI systems with a predecessor-following topology is analyzed. Our main contribution is to study the stochastic scenario when additive white noise is affecting the communication between agents. We aim to provide conditions for mean square string stability and look over its relationship with the mean and variance of the tracking error. Finally, through computational analysis, we discuss the scalability, convergence and boundedness properties related to string stability in stochastic multi-agent systems.

Keywords: Stochastic string stability, Vehicular platoon control, Time-headway, Additive white noise, Communication channels.

1. INTRODUCTION

A platoon of autonomous vehicles, is a controlled multi-agent system where several vehicles aim to travel as a string/chain at a common velocity, while keeping a safe distance within them. Some benefits of automatized platoon systems include the reduction of traffic congestion and pollution emissions, safe driving, and efficient fuel consumption (Wang et al., 2018). Achieving some of these goals requires each vehicle to follow its predecessor as closely as possible. However, by reducing the inter-vehicle distance it is important to guarantee that disturbances do not amplify along the string. This property, known as string stability (Peppard, 1974), has been extensively studied in recent years; it is of particular interest for platoon control to ensure good tracking performance and avoid collisions between vehicles. In multi-agent systems, string stability is a desirable property that guarantees the scalability and convergence of the whole system, i.e. the number of agents can increase or decrease without compromising the performance and safety of the platoon.

Although string stability for the deterministic case has been deeply studied (Feng et al., 2019; Swaroop and Hedrick, 1996; Stüdli et al., 2017), string stability in a stochastic setting remains poorly explored, except in a few cases (see e.g. (Socha, 2004)). Nevertheless, very

well studied tools from the deterministic case can be useful in systems with stochastic signals. The condition of string stability arises from the need to ensure a bounded system that avoids the amplification of the error signals downstream of the platoon. Hence, for a platoon to be string stable, it is not enough just to satisfy the internal stability of the closed-loop system (Li et al., 2017).

Frequency-domain string stability is the commonly used definition for the analysis of linear platoon systems with predecessor-follower topology (Seiler et al., 2004; Middleton and Braslavsky, 2010; Öncü et al., 2012). In this context, a system is said to be string stable if, for any frequency, the magnitude peak of a relevant sequence of transfer functions can be bounded uniformly with the number of agents. In Section 3, we demonstrate that a similar idea is also useful for a platoon system driven by stochastic signals.

In a platoon, agents exchange information (e.g. speed, inter-vehicle distance, control inputs) with each other and depending on how that information flows, a different communication scheme arises (Li et al., 2017; Wang et al., 2018; Feng et al., 2019). Particularly, in this paper we analyze the predecessor-following topology where each vehicle exchanges information only with the nearest agent. In this topology, the communication flows unidirectionally and each vehicle receives information only from its immediate predecessor. Wireless communications between agents are subject to issues such as communication constraints, noise,
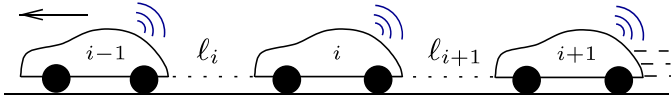
Fig. 1. Platoon configuration with predecessor-follower topology.

Fig. 2. Sequence for N number of agents

Fig. 3. Feedback loop for i-th agent

packet loss or delays. These problems can compromise the performance of the platoon and therefore it is necessary to extend the stability analysis for stochastic models, studying the effects produced by unreliable communications in multi-agent systems. As discussed in a recent survey on platooning and string stability (Feng et al., 2019), sufficient conditions for string stability of stochastic systems are still not available.

The present work aims to extend the analysis of platoon systems with stochastic inputs by considering additive white noise in the communication channel. To the best of our knowledge, this approach has not been considered before whilst most of the investigation is focused on lossy channels (Vargas et al., 2018; Acciani et al., 2019) and communication delays (Xiao et al., 2009; Qin et al., 2015; Di Bernardo et al., 2014; Wang et al., 2018; Qin et al., 2016). In this framework, we propose a notion of mean square string stability and provide necessary and sufficient conditions to achieve it. We also analysis the stationary behavior of the position error mean and variance of each agent.

The present paper is arranged as follows. Section 2 describes the platoon configuration and defines mean square stability. In Section 3 we define the mean square string stability conditions derived from the mean and variance of the tracking error. Some results obtained by simulation are shown and discussed in Section 4. Finally, in Section 5 we present the conclusions.

2. PROBLEM FORMULATION

2.1 Vehicle Model and Platoon Configuration

We consider a one-dimensional set of $N \in \mathbb{N}$ vehicles moving with a predecessor-following configuration as shown in Fig. 1. Each agent, denoted with the sub-index i where $i = 1, 2, ..., N$, follows its predecessor at a desired distance, except for the leader $(i = 1)$ who follows an independent navigation.

It is reasonable to assume that in a real setting, the communications will be carried out digitally, and since most of the controllers are implemented in digital computers, the interconnected systems can be described in an adequate fashion with discrete time models. Moreover, the treatment of the stochastic aspects is more manageable in the discrete time description of systems (see for instance Qin et al. (2016)). For these reasons, we focus on platoons described by discrete time systems.

At any time instant $k \in \mathbb{N}$, it is expected that the i-th agent is capable of measuring its current position $y_i(k)$ and receives, through a communication channel, the measured position of its predecessor $y_{i-1}(k)$. Platooning control focuses not only on tracking tasks but also on maintaining the inter-vehicle distance $\ell_i(k) \triangleq y_i(k) - y_{i-1}(k)$, as close as possible to a desired reference $r_i(k)$.
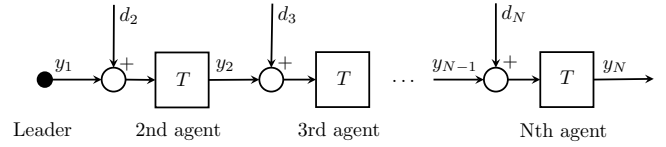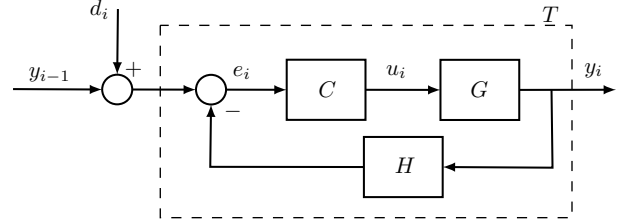
As mentioned, all vehicles are integrated into a network topology. In this scenario, we contemplate the case where the inter-vehicle communication channel between the i-th agent and its predecessor the is affected by an additive noise $d_i(k)$.

Except for the leader, we assume the platoon to be homogeneous, i.e. all the vehicles have the same dynamics. Thus, we represent each agent by a closed loop feedback system whose transfer functions is denoted by $T(z)$. Hence, the platoon setup have the form shown in Fig. 2.

The controller design must ensure that each agent is able to achieve zero steady-state error for ramp references (leader moving at constant speed, and without communication noise). However, in a predecessor follower topology with constant spacing policy, and a closed-loop configuration with one degree of freedom controller, ramp tracking and string stability are not compatible (Wang et al., 2018). Thus, a solution to this issue is a two degrees of freedom closed-loop controller architecture as shown in Fig. 3 where G represents the plant, C is the controller and H adds the extra degree of freedom (dof).

With a 2-dof controller, the filter H can be appropriately chosen to include the effect of the vehicles speed into the spacing policy. Then, we choose $H = (1 + \eta) - \eta/z$, where $\eta > 0$ is the *time headway* constant that weighs the speed at which a vehicle approaches its predecessor. Therefore, the desired inter-vehicle distance between agents can be calculated as

$$r_i(k) = \epsilon_i + \eta \left[y_i(k) - y_i(k-1) \right] \qquad (1)$$

where $\epsilon_i > 0$ represents the minimum desired distance between agents. For simplicity in the explanation we consider the length of each vehicle equal to zero and $\epsilon_i = 0$. Consequently, from (1), the tracking error is given by

$$e_i(k) = y_{i-1}(k) - y_i(k) + d_i(k) - \eta \left[y_i(k) - y_i(k-1) \right]. \quad (2)$$

Clearly, the complementary sensitivity function of the local closed loop system T, and the corresponding sensitivity function S are given by

$$T = \frac{GC}{1 + GCH}, \qquad S = \frac{1}{1 + GCH}.$$

Assumption 1. We consider the following assumptions:

(1) The noises d_i are a sequence of stationary and mutually independent white noises, with mean μ_d and variance P_d.
(2) Each noise d_i is uncorrelated with the initial state of each agent.
(3) $T(z)$ is stable and strictly proper.
(4) The product of $G(z)C(z)$ has, at least, double integral action.

A proper controller design will naturally yield a stable and strictly proper transfer function $T(z)$. On the other hand, the double integral action in Assumption 1 is required for zero steady-state error for ramp inputs.

2.2 Stability notions

To analyze the convergence properties of the proposed platoon, we first recall the concept of *mean square stability*, commonly adopted to study dynamical systems where stochastic processes are involved, and also the concept of *string stability*, used to analyze the behavior of a platoon of dynamical agents.

The notion of MSS is useful for systems having stochastic stationary processes as inputs. In that case, a linear system is mean square stable (MSS) when the system state converges in a mean-square sense as the time grows unbounded (see, for instance Åström (2012)). In our setup, this notion of stability could be applied to each individual in the platoon, since the convergence is over the time. Thus, a necessary and sufficient condition for $T(z)$ and $S(z)$ to be MSS is that all their poles are inside the unit circle. This guarantees that the mean and variance of the error converge to finite values. Given Assumption 1 it is clear that in our setup, each vehicle in the platoon is MSS by design.

On the other hand, there are several definitions of platoon string stability, most of them for a deterministic setup (Feng et al., 2019). However, all these definitions focus on how disturbances propagate as the number of agents increase. Roughly speaking, we could say that a platoon is string stable if the detrimental effects of the disturbances (measured with some specific metric) do not amplify along the string of vehicles. Otherwise, the platoon is called string unstable.

This notion of string stability cannot be applied for the platoon configuration of this paper using the usual metrics for deterministic signals, due to the stochastic nature of the additive noises affecting the communication channel. For that reason, we propose an alternative definition to study string stability in this stochastic setting.

Definition 2. The platoon described in this section is said to be **mean square string stable** (M3S) if and only if the mean and variance of the tracking errors converge to finite values as the number of agents grows unbounded, when the channel noises are stationary processes.

The definition of mean square string stability corresponds to the notion of mean square convergence applied to a two dimensional stochastic process, where not only the time but also the agent positions within the string are important.

2.3 Problem Definition

In this paper, our main goal is to find necessary and sufficient conditions such that the vehicle platoon under study is mean square string stable.

3. CONDITIONS FOR MEAN SQUARE STRING STABILITY

In this section we present the main results of this paper.

Theorem 3. Consider a platoon satisfying the setup defined in Section 2. The platoon is mean square string stable if and only if the sensitivity functions $T(e^{j\omega})$ and $S(e^{j\omega})$ satisfy

$$|T(e^{j\omega})| \leq 1, \quad \forall \omega, \qquad (3)$$

and

$$|S(e^{j\omega^*})| = 0, \quad \forall \omega^* \text{ such that } |T(e^{j\omega^*})| = 1. \qquad (4)$$

Proof. Since T is assumed to be properly designed, it is clear that, for a given vehicle, its corresponding tracking error is mean square stable when $k \to \infty$, which implies that the stationary mean, variance and spectrum, are well defined. On the other hand, it is not difficult to see that the error of the i-th vehicle can be expressed, in the time domain, as follows

$$e_{i+1}(k) = h_T(k) * e_i(k) + h_S(k) * d_{i+1}(k), \qquad (5)$$

where h_T and h_S denote the impulse response of $T(z)$ and $S(z)$ respectively. The expected value of $e_{i+1}(k)$ can be written as

$$\mu_{e_{i+1}}(k) = h_T(k) * \mu_{e_i}(k) + h_S(k) * \mu_{d_{i+1}}(k).$$

In the frequency domain we can write

$$\mu_{e_{i+1}}(z) = T(z)\mu_{e_i}(z) + S(z)\mu_{d_{i+1}}(z) \qquad (6)$$
$$= T(z)\mu_{e_i}(z) + S(z)\mu_d. \qquad (7)$$

In a similar fashion, since $d_{i+1}(k)$ is not correlated with $e_i(k)$, we have

$$R_{e_{i+1}}(s, k) = \sum_{\ell=0}^{\infty} \sum_{j=0}^{\infty} h_T(\ell)h_T(j)R_{e_i}(s - k, k - \ell)$$
$$+ \sum_{\ell=0}^{\infty} \sum_{j=0}^{\infty} h_S(\ell)h_S(j)R_{d_{i+1}}(s - k, k - \ell), \quad (8)$$

which implies that the stationary power spectral density for $e_{i+1}(k)$, say $\phi_{e_{i+1}}(e^{j\omega})$, satisfy

$$\phi_{e_{i+1}}(e^{j\omega}) = |T(e^{j\omega})|^2\phi_{e_i}(e^{j\omega}) + |S(e^{j\omega})|^2\phi_{d_{i+1}}(e^{j\omega}),$$
$$= |T(e^{j\omega})|^2\phi_{e_i}(e^{j\omega}) + |S(e^{j\omega})|^2P_d. \qquad (9)$$

Clearly, both (7) and (9) are linear recursions in terms of the agent position index i. Additionally, $\mu_{d_{i+1}}(z)$ and $\phi_{d_{i+1}}(e^{j\omega})$ do not depend on i nor ω.

If $|T(e^{j\omega})| > 1$ for any ω the sequence generated by (9) is obviously unbounded. For $\omega = 0$, $S(1) = 0$ and $T(1) = 1$, due to the double integral action in the closed loop. Therefore, (9) generates a constant sequence. For any other $\omega > 0$, it is straightforward to conclude that $|T(e^{j\omega})| < 1$ is a sufficient condition for the mean and power spectrum of the error to converge as the number of vehicles grows unbounded. On the other hand,

the existence of any ω^* such that $|S(e^{j\omega^*})| \neq 0$ when $|T(e^{j\omega^*})| = 1$, implies that (9) would yield an increasing sequence. Therefore $|S(e^{j\omega^*})| = 0$ when $|T(e^{j\omega^*})| = 1$ is necessary. The existence of the power spectrum is also necessary for the existence of the corresponding stationary variance, which is guaranteed since T is, by design, a MSS system. ∎

Theorem 3 presents an analytical condition to guarantee string stability in a stochastic setting as defined in Definition 2. It is clear then that, to ensure that the mean and the variance of the tracking error do not amplify along the string, the magnitude of $|T(e^{j\omega})|$ must be less than one for all frequencies.

Remark 4. It is important to note that the necessary and sufficient condition presented in Theorem 3 is almost identical to conditions for string stability in a deterministic and linear setup (see, for instance Seiler et al. (2004)). The main difference is that in some deterministic setups, $|T(e^{j\omega})| = 1$ for any ω may not have a detrimental effect on the string stability property (as neither amplifying nor attenuating disturbances could be considered a favorable outcome). However, when considering additive noises, the *marginal string stability* case may no longer be a favorable one.

Now we focus on the steady state values of a mean square string stable platoon. First, we notice that, for each vehicle, the stationary value of the mean of the error is zero since each controller is designed to achieve perfect tracking in steady state in absence of noise, and since the channel noises are assumed to be zero mean processes. On the other hand, studying the stationary behavior of the variance of the tracking errors is not trivial, which leads us to the following results.

Corollary 5. Consider a platoon satisfying the setup defined in Section 2, and assume that $|T(e^{j\omega})| < 1$ for all $\omega > 0$. The variance of the tracking error of the N-th vehicle when $k \to \infty$, is given by

$$P_{e_N} = \|F_N(z)\|_2^2 P_d, \tag{10}$$

where P_d is the variance of the additive noise and

$$F_N(z) = S(z) \begin{bmatrix} 1 & T(z) & T(z)^2 & \cdots & T(z)^{N-2} \end{bmatrix}. \tag{11}$$

Furthermore, the variance of the tracking error of the N-th vehicle when $k \to \infty$ and $N \to \infty$, is given by

$$\lim_{N \to \infty} P_{e_N} = \left\| \frac{S(z)}{M(z)} \right\|_2^2 P_d, \tag{12}$$

where $M(z)$ is a stable and minimum phase spectral factor such that $1 - T(z)T(z)^{\sim} = M(z)M(z)^{\sim}$.

Proof. To ease notation we omit the arguments z and k during the rest of the proof.

It is easy to see that the dynamics of the platoon is such that the position error for the N-th vehicle (last agent in the string) can be written in terms of the leader position and the channel noises as

$$e_N = S \sum_{i=0}^{N-2} \left(T^i \, d_{N-i} \right) + ST^{N-2} \, y_1 \tag{13}$$

$$= \begin{bmatrix} F & ST^{N-2} \end{bmatrix} \begin{bmatrix} d \\ y_1 \end{bmatrix} \tag{14}$$

for $i \in \mathbb{N}_0$, and where $d = [d_N \; d_{N-1} \; \cdots \; d_2]^T$ and $F(z)$ is as in (11). Since the noises d_i are all i.i.d white processes with variance P_d, and y_1 is a deterministic signal, it follows that the spectrum of the error e_N can be calculated as

$$\phi_{e_N} = F(z) \, P_d \, F(z)^{\sim},$$

where $(\cdot)^{\sim}$ denotes the para-hermitian operator.

Finally, the stationary variance of the position error can be obtained from its power spectrum, leading to (10)

$$P_{e_N} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \phi_{e_N}(j\omega) d\omega$$

$$= \frac{1}{2\pi} \left(\int_{-\pi}^{\pi} F(e^{j\omega}) F(e^{j\omega})^* d\omega \right) P_d$$

$$= \|F\|_2^2 \, P_d, \tag{15}$$

where P_d is the variance of the additive noise.

For the case when $N \to \infty$, we notice from (9) that, in such stationary case, the spectrum ϕ_{e_N} satisfies

$$\phi_{e_N} = T \, T^{\sim} \phi_{e_N} + S \, S^{\sim} P_d, \tag{16}$$

where we obtain

$$\phi_{e_N} = (1 - T \, T^{\sim})^{-1} S \, S^{\sim} P_d \tag{17}$$

$$= \frac{S \, S^{\sim}}{M \, M^{\sim}} P_d. \tag{18}$$

The expression in (12) is directly obtained from (17), completing the proof. ∎

Corollary 5 presents the stationary value of the variance of the error of each vehicle, when $k \to \infty$. The existence of such values is guaranteed since $T(z)$ is designed to be a mean square stable system, ensuring convergence in time. Also, since mean square string stability is assumed, when the number of vehicles $N \to \infty$, the variance of the error also converges to a specific value. This imposes a bound for the stationary variance of each vehicle in the platoon.

4. SIMULATION RESULTS

In this section, given a system $T(z)$, we show some results that can be derived from the mean square string stability analysis performed in Section 3.

Consider the following platoon set up

$$G(z) = \frac{1}{z-1}, C(z) = \frac{(1/(1+\eta))z}{(z-1)(z+0.7)}, H(z) = (1+\eta) - \frac{\eta}{z}.$$

To visualize the stable or unstable behavior of the platoon, we chose two different time headway constants; $\eta = 4$ that corresponds to a string stable case, and $\eta = 3$ that corresponds to a string unstable case. The communication channel within each vehicle is considered to be affected by an additive white noise of zero mean an variance P_d. In the simulation, we consider that the leading vehicle moves with constant speed and all the agents at time instant $k = 0$ start from rest.

In Theorem 3 it was shown that a necessary condition for mean square string stability is that for all frequencies ($\omega > 0$), the magnitude of the system transfer function must be less than one. Then, the frequency response of the system for the two cases under analysis is shown in Fig. 5, where the M3S condition is clearly not met for $\eta = 3$ due to the peak gain that exceeds the bounded magnitude of one.
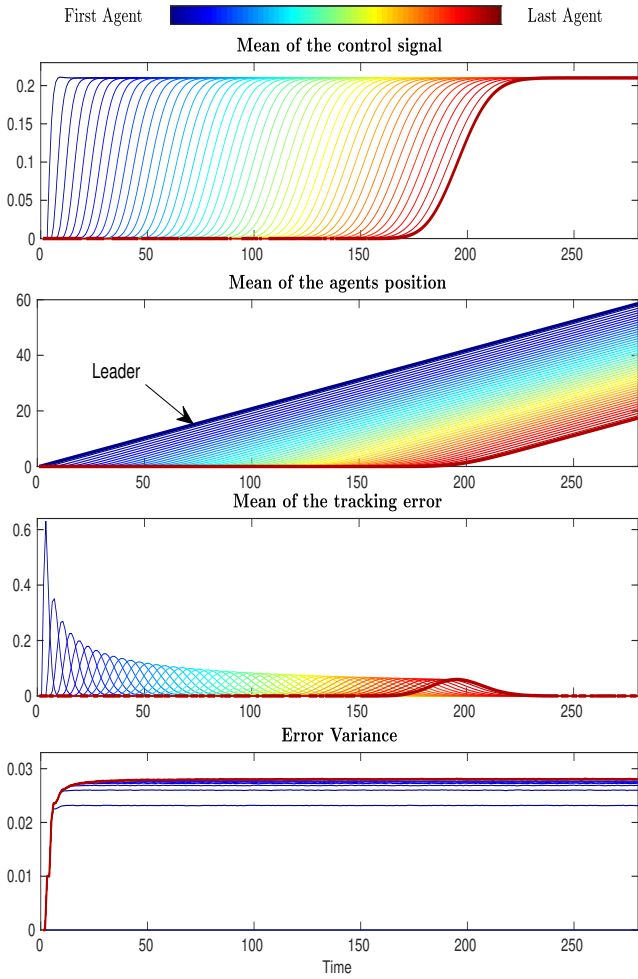
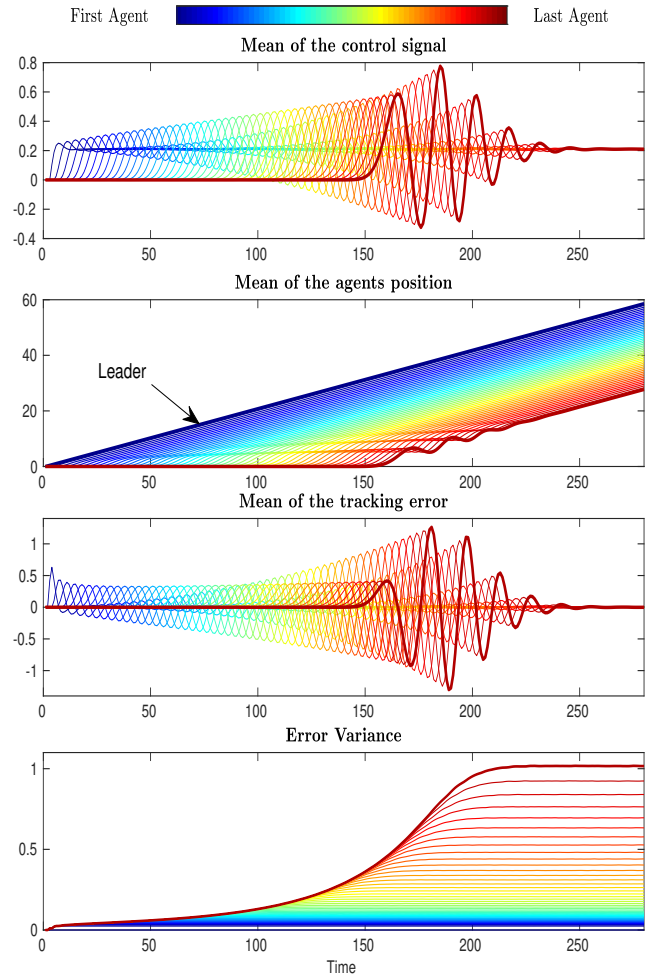Fig. 4. Mean square string stable platoon behavior.

Fig. 5. Frequency response of $T(z)$ for two values of η.

Fig. 6. Mean square string unstable platoon behavior.

found that the stationary variance converges to the value of 0.02804. On the other hand, when the system is string unstable the tracking error variance increases its value while more vehicles are added to the platoon. The unstable behavior can also be seen in the mean of the tracking error where although it converges to zero (expected result due to the integral action of the system), in the transition, oscillations are observed that affect the position of the vehicles causing an unacceptable performance that can lead to vehicle collisions.

5. CONCLUSION

In this work we consider a discrete-time LTI platoon with noisy communication between agents. Two contributions were presented in this paper. First, we extended the analysis of platoons with predecessor follower topology by considering the realistic scenario where the inter-vehicle communication channel is affected by additive white noise. For the mentioned platoon configuration, we propose a notion of mean square string stability and derived necessary and sufficient conditions to achieve it. This implies that if a platoon is string stable in a deterministic setup, then it is also mean square string stable and hence the variance of the tracking error converges to a maximum value regardless of the number of agents added to the platoon.

For a set of 50 agents we simulate the evolution of the system. In Fig. 4 we present the string stable case and in Fig. 6 the string unstable behavior is shown. In both scenarios, four plots are presented. From top to bottom, the mean of the control signal $u(k)$, the mean of the agent's position $y(k)$, the mean of the tracking error $e(k)$ and the variance of the tracking error $P_e(k)$ are shown. The means of all the parameters were obtained from a Monte Carlo simulation with 1×10^6 realizations. Therefore, when the system is string stable, the tracking error variance converges to a maximum constant value. From (12) we

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