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Event-triggered Backstepping Control of 2×2 Hyperbolic PDE-ODE Systems

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Abstract: Motivated by vibration control of a mining cable elevator avoiding frequent actions of the actuator which is a massive hydraulic cylinder at the head sheave, we present an event-triggered backstepping boundary controller for a 2×2 coupled hyperbolic PDE-ODE system. A two-step design is proposed including the design of a low-pass-filter-based backstepping boundary stabilization law and the sequent design of an event-trigger mechanism. The proof of the existence of a nonzero minimal dwell-time between two triggering times, and the exponential stability result of the event-based closed-loop system are given in this paper.

Keywords: Distributed parameter system, Event-trigger, Boundary control, PDE-ODE.

1. INTRODUCTION

A mining cable elevator is a vital device used to transport the mines and miners between thousands of metres underground and ground. In order to suppress vibrations in the long compliant cable to reduce the fatigue damage, a vibration control force is applied at the head sheave, which is designed based on the PDE model of the mining cable elevator Wang et al. (2018b)-Wang et al. (2018d) via backstepping. Two challenges existing in implementation of the designed PDE backstepping controller into practice are 1) the control input signal is changing rapidly so that it is hard for the actuator which is a heavy hydraulic cylinder shown in Fig. 1 to follow; 2) the high-frequency components in the control input may in turn become a vibration source for the cable. It is thus required to reduce the action frequency of the actuator, and meanwhile ensure the suppression of the vibrations in the cable.

Designing sampling schemes applied into the control input is a potential solution. Designs of sampled-data control laws of parabolic and hyperbolic PDEs were presented in Fridman et al. (2012); Karafyllis et al. (2018) and Davo et al. (2018); Karafyllis et al. (2017) respectively. Compared with the periodic sampled-data control where unnecessary movements of the massive actuator may exist, event-triggered control where the massive actuator is only animated at the necessary times which are determined by an event-triggered mechanism of evaluating the operation of the elevator, is more feasible for the mining cable elevator from the point of view of energy saving.

The event-triggered control system consists of two elements, namely, a continuous-time feedback control law, and an ETM that determines triggering times of updating the control law Heemels et al. (2012). The key task in the design of ETM is to make sure the minimal dwell time between two triggering times is nonzero meanwhile the exponential stability of the closed-loop system is ensured. Most of ETM implementations are based on feedback control laws of ODE systems. Tabuada (2007) introduced a static ETM based on the existence of an input to state stable control Lyapunov function (ISS-CLF) and a dynamic triggering mechanism which uses an internal dynamic variable was proposed in Girard (2015). The event-triggered algorithms by the value of the derivative of the Lyapunov function were proposed in Marchand et al. (2013) and Seuret et al. (2014). Recently, Espitia et al. (2016a) and Espitia et al. (2016b) originally developed event-triggered strategies to boundary control of linear hyperbolic PDEs with dissipativity boundary conditions. Event-triggered boundary control of 2×2 coupled transport PDEs and reaction-diffusion PDEs are also addressed in Espitia et al. (2018) and Espitia et al. (2019), respectively.

Compared with Espitia et al. (2018) which designed an event-triggered backstepping controller for a 2×2 hyperbolic PDEs, in addition to an ODE coupled at the uncontrolled boundary of the PDE in our paper, the main contribution here lies in considering the proximal reflection term in the 2×2 hyperbolic PDEs, where ETM design would become more difficult because a higher-order continuous-time boundary stabilization law is required. A two-step design is proposed to solve this problem, where a low-pass-filter-based backstepping boundary stabilization law is designed, based on which the ETM is designed sequently to determine the triggering times, shown in Fig.2.

The rest of the paper is organized as follows. The concerned model and a continuous-time state-feedback controller combining the backstepping method and low-pass filter design are presented in Section 2. The event-trigged mechanism is built in Section 3, and the exponential stability of the event-based closed-loop system is proved via Lyapunov analysis in Section 4. The conclusion and future work are proved in Section 5.

2. PLANT AND CONTINUOUS-TIME CONTROL LAW

The plant considered in this paper is

$$z(0,t) = pw(0,t) + U(t), \tag{1}$$
$$z_t(x,t) = -q_1 z_x(x,t) + c_1 w(x,t) + c_1 z(x,t), \tag{2}$$
$$w_t(x,t) = q_2 w_x(x,t) + c_2 w(x,t) + c_2 z(x,t), \tag{3}$$
$$w(1,t) = qz(1,t) + C_1 X(t), \tag{4}$$
$$\dot{X}(t) = AX(t) + Bz(1,t) \tag{5}$$

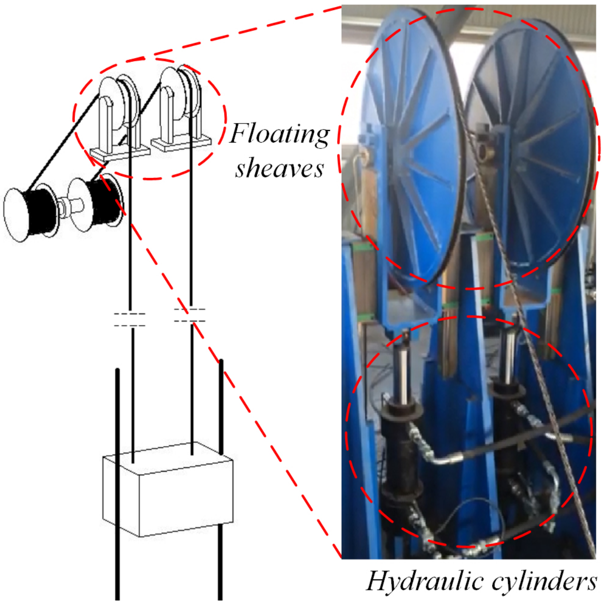

Fig. 1. Motivation from mining cable elevator with hydraulic-driven head sheaves.

$\forall(x,t) \in [0,1] \times [0,\infty)$. $X(t) \in \mathbb{R}^{n\times 1}$ are ODE states. $z(x,t) \in \mathbb{R}, w(x,t) \in \mathbb{R}$ are states of the 2×2 coupled hyperbolic PDEs. $U(t)$ is the control input to be designed. q_1 and q_2 are positive transport velocities and $p,\, q \neq 0$ are arbitrary constants. Damping coefficients c_1, c_2 are negative constants. The plant parameters satisfy $e^{c_2/q_2+c_1/q_1}|pq| < 1$, which holds in the elevator application and also meets the condition of existence of delay-robust stabilization proposed in Auriol et al. (2018). (A,B) is assumed as controllable.

Remark 1. That axial vibration dynamics of the mining cable elevator consisting of the mining cable and cage is described by a wave PDE -ODE system, which can be transformed to the 2×2 coupled transport PDE-ODE system considered in this paper via Riemann transformation Wang et al. (2018d).

For the concern plant, a feedback controller would be designed via traditional PDE backstepping as follows. Introduce a PDE backstepping transformation in the following form Meglio et al. (2018); Wang et al. (2018a):

$$\begin{aligned}\alpha(x,t) =& z(x,t) - \int_x^1 M(x,y)z(y,t)dy \\ &- \int_x^1 N(x,y)w(y,t)dy - \gamma(x)X(t), \end{aligned} \tag{6}$$

$$\begin{aligned}\beta(x,t) =& w(x,t) - \int_x^1 D(x,y)z(y,t)dy \\ &- \int_x^1 J(x,y)w(y,t)dy - \lambda(x)X(t)\end{aligned} \tag{7}$$

the inverse transformation of which is

$$\begin{aligned}z(x,t) =& \alpha(x,t) - \int_x^1 \mathscr{M}(x,y)\alpha(y,t)dy \\ &- \int_x^1 \mathscr{N}(x,y)\beta(y,t)dy - \mathscr{G}(x)X(t),\end{aligned} \tag{8}$$

$$\begin{aligned}w(x,t) =& \beta(x,t) - \int_x^1 \mathscr{D}(x,y)\alpha(y,t)dy \\ &- \int_x^1 \mathscr{J}(x,y)\beta(y,t)dy - \mathscr{P}(x)X(t).\end{aligned} \tag{9}$$

Through the above backstepping transformation, (1)-(5) is converted to

$$\begin{aligned}\alpha(0,t) =& p\beta(0,t) + \int_0^1 \bar{K}_1(x)\alpha(x,t)dx \\ &+ \int_0^1 \bar{K}_2(x)\beta(x,t)dx + \bar{K}_3 X(t) + U(t),\end{aligned} \tag{10}$$

$$\alpha_t(x,t) = -q_1\alpha_x(x,t) + c_1\alpha(x,t) \tag{11}$$

$$\beta_t(x,t) = q_2\beta_x(x,t) + c_2\beta(x,t) \tag{12}$$

$$\beta(1,t) = q\alpha(1,t), \tag{13}$$

$$\dot{X}(t) = \hat{A}X(t) + B\alpha(1,t), \tag{14}$$

where $\hat{A}$ is Hurwitz recalling (A,B) is controllable, and $\bar{K}_1(x), \bar{K}_2(x), \bar{K}_3$ satisfy

$$\begin{aligned}\bar{K}_1(x) =& pD(0,x) - M(0,x) + \int_0^x \bar{K}_1(y)M(y,x)dy \\ &+ \int_0^x \bar{K}_2(y)D(y,x)dy,\end{aligned} \tag{15}$$

$$\begin{aligned}\bar{K}_2(x) =& -pJ(0,x) + N(0,x) + \int_0^x \bar{K}_1(y)N(y,x)dy \\ &+ \int_0^x \bar{K}_2(y)J(y,x)dy,\end{aligned} \tag{16}$$

$$\begin{aligned}\bar{K}_3 =& \int_0^1 \bar{K}_2(x)\lambda(x)dx + \int_0^1 \bar{K}_1(x)\gamma(x)dx \\ &+ p\lambda(0) - \gamma(0).\end{aligned} \tag{17}$$

The condition of the kernels $M(x,y)$, $N(x,y)$, $\gamma(x)$, $D(x,y)$, $J(x,y)$, $\lambda(x)$, $\mathscr{M}(x,y)$, $\mathscr{N}(x,y)$, $\mathscr{G}(x)$, $\mathscr{D}(x,y)$, $\mathscr{J}(x,y)$, $\mathscr{P}(x)$ are obtained by matching (10)-(14) and (1)-(5) via (6)-(9). Details and the well-posedness proof of the equations of condition on those kernels are shown in Meglio et al. (2018); Wang et al. (2018a).

Taking Laplace transformation into (10)-(14), we have

$$s\alpha(x,s) = -q_1\alpha_x(x,s) + c_1\alpha(x,s), \tag{18}$$

$$s\beta(x,s) = q_2\beta_x(x,s) + c_2\beta(x,s), \tag{19}$$

$$\beta(1,s) = q\alpha(1,s), \tag{20}$$

$$(sI - \hat{A})X(s) = B_1\alpha(1,s). \tag{21}$$

We obtain the following relationship

$$\alpha(x,s) = e^{\frac{(c_1-s)x}{q_1}}\alpha(0,s), \tag{22}$$

$$\beta(1,s) = q\alpha(1,s) = qe^{\frac{(c_1-s)}{q_1}}\alpha(0,s), \tag{23}$$

$$\beta(x,s) = e^{\frac{(c_2-s)(1-x)}{q_2}}\beta(1,s) = e^{\frac{(c_2-s)(1-x)}{q_2}}qe^{\frac{(c_1-s)}{q_1}}\alpha(0,s), \tag{24}$$

$$\beta(0,s) = e^{\frac{(c_2-s)}{q_2}}qe^{\frac{(c_1-s)}{q_1}}\alpha(0,s), \tag{25}$$

$$X(s) = (sI - \hat{A})^{-1}B_1e^{\frac{(c_1-s)}{q_1}}\alpha(0,s). \tag{26}$$

(10) can thus be written in the frequency domain as

$$\begin{aligned}\alpha(0,s) =& U(s) + \Bigg[pe^{\frac{(c_2-s)}{q_2}}qe^{\frac{(c_1-s)}{q_1}} + \int_0^1 \bar{K}_1(x)e^{\frac{(c_1-s)x}{q_1}}dx \\ &+ \bar{K}_3(sI-\hat{A})^{-1}B_1e^{\frac{(c_1-s)}{q_1}} \\ &+ \int_0^1 \bar{K}_2(x)e^{\frac{(c_2-s)(1-x)}{q_2}}qe^{\frac{(c_1-s)}{q_1}}dx\Bigg]\alpha(0,s).\end{aligned} \tag{27}$$

Choosing the controller as

$$\begin{aligned}U(s) =& -\Omega(s)\Bigg[pe^{\frac{(c_1-s)}{q_2}}qe^{\frac{(c_1-s)}{q_1}} + \int_0^1 \bar{K}_1(x)e^{\frac{(c_1-s)x}{q_1}}dx \\ &+ \bar{K}_3(sI-\hat{A})^{-1}B_1e^{\frac{(c_1-s)}{q_1}}\end{aligned}$$

$$+\int_0^1 \bar{K}_2(x)e^{\frac{(c_1-s)(1-x)}{q_2}} q e^{\frac{(c_1-s)}{q_1}} dx\Big]\alpha(0,s)$$
$$=-\Omega(s)\bar{\psi}(s)\alpha(0,s) \quad (28)$$

where $\Omega(s)$ is a first-order low-passing filter as follows

$$\Omega(s)=\frac{a_0}{s+a_0} \quad (29)$$

which can be realized by RC circuits. The constant a_0 is a design parameter making sure

$$0<|1-\Omega(s)|<\frac{1}{|\bar{\psi}(s)|} \quad (30)$$

for $s\in\mathbb{C}$, $\mathbb{R}(s)\geq 0$. Note that $\bar{\psi}(s)$ in (28) is a proper transfer function. Inserting (28) into (27), one obtain

$$[1-(1-\Omega(s))\bar{\psi}(s)]\alpha(0,s)=0 \quad (31)$$

which means $\alpha(0,s)=0$ because $1-(1-\Omega(s))\bar{\psi}(s)$ is nonzero considering (30). Therefore, applying the backtepping transformation and the control input $U(t)$ which is realized from $U(s)$ in (28), then (10) can be regarded as

$$\alpha(0,t)=0. \quad (32)$$

Therefore, the target system is obtained (11)-(14) and (32) and an extend dynamic of the dynamic controller (28), of which the realization is shown as following.

Realization of $U(s)$: $U(t)$ is the output signal of the low-passing filter Ω of which the input signal is $U_b(t)$ derived from previous backstepping design Meglio et al. (2018),

$$U_b(t)=-p\beta(0,t)-\int_0^1\bar{K}_1(x)\alpha(x,t)dx$$
$$-\bar{K}_3X(t)-\int_0^1\bar{K}_2(x)\beta(x,t)dx. \quad (33)$$

Recalling the structure of the low-pass filter Ω (64), $U(t)$ is the solution of the following ODE driven by $U_b(t)$

$$\dot{U}(t)+a_0U(t)=a_0U_b(t). \quad (34)$$

3. EVENT-TRIGGER MECHANISM

In this section, we introduce an event-triggered control scheme for stabilization of the 2×2 coupled hyperbolic PDE-ODE system (1)-(5). It relies on both the continuous-time control $U(t)$ and a dynamic event-triggered mechanism (ETM) which determines triggering times t_k ($k\geq 0$ and $t_0=0$) when the actuator signal is updated. In other words, the event-triggered form $U_d(t)$ is the value of the continuous-time $U(t)$ at the time instants t_k for $t\in[t_k,t_{k+1})$, i.e.,

$$U_d(t)=U(t_k). \quad (35)$$

Inserting $U_d(t)$ into (1), we have

$$z(0,t)=pw(0,t)+U_d(t). \quad (36)$$

A deviation $d(t)$ between a continuous-time controller and the event-based one is given as

$$d(t)=U(t)-U_d(t). \quad (37)$$

Then (36) can be written as

$$z(0,t)=pw(0,t)+U(t)-d(t). \quad (38)$$

Recalling the backstepping transformations and designs of $U(t)$ in Section 2, the target system becomes (11)-(14) with

$$\alpha(0,t)=-d(t), \quad (39)$$

and an extend dynamics of the dynamic controller (34). The ETM to determine the triggering times of U_d is designed as

$$t_{k+1}=\inf\{t\in R^+|t>t_k|d(t)^2\geq\theta V(t)-m(t)\}, \quad (40)$$

where $m(t)$ satisfies the ordinary differential equation,

$$\dot{m}(t)=-\eta m(t)+\lambda_d d(t)^2-\sigma V(t)-\kappa_1\alpha(1,t)^2-\kappa_2\beta(0,t)^2 \quad (41)$$

with an initial condition $m(0)<0$. Positive constants θ, η, λ_d, σ, κ_1, κ_2 are to be determined later. $V(t)$ is given as

$$V(t)=r_cX(t)^TPX(t)+\frac{1}{2}r_a\int_0^1 e^{\delta_1 x}\beta(x,t)^2dx$$
$$+\frac{1}{2}r_b\int_0^1 e^{-\delta_2 x}\alpha(x,t)^2dx \quad (42)$$

where a positive definite matrix $P=P^T$ is the solution to the Lyapunov equation $\hat{A}^TP+P\hat{A}=-\hat{Q}$, for some $\hat{Q}=\hat{Q}^T>0$. The positive constants $r_a,r_b,r_c,\delta_1,\delta_2$ are to be determined in the next section.

Lemma 1. Considering $d(t)$ defined in (37), there exists a positive constant λ_a such that

$$\dot{d}(t)^2\leq\lambda_a\Big(d(t)^2+\alpha(1,t)^2+\beta(0,t)^2+\|\alpha(\cdot,t)\|^2$$
$$+\|\beta(\cdot,t)\|^2+|X(t)|^2\Big) \quad (43)$$

for $t\in(t_k,t_{k+1})$, where λ_a only depends on the parameters of the plant, the continuous-time control law and the low-pass filter.

Proof. Recalling (36), (8)-(9) and applying Cauchy-Schwarz inequality, we have

$$U_d(t)^2\leq\lambda_{ud}\Big(\alpha(0,t)^2+\beta(0,t)^2+\|\alpha(\cdot,t)\|^2$$
$$+\|\beta(\cdot,t)\|^2+|X(t)|^2\Big) \quad (44)$$

for some positive λ_{ud}.

Taking the time derivative of (37) and recalling (33)-(34), we have

$$\dot{d}(t)^2=\dot{U}(t)^2$$
$$\leq a_0^2U(t)^2+a_0^2U_b(t)^2$$
$$\leq 2a_0^2U_d(t)^2+2a_0^2d(t)^2+a_0^2U_b(t)^2$$
$$\leq\lambda_a\big[d(t)^2+\alpha(1,t)^2+\beta(0,t)^2+\|\alpha(\cdot,t)\|^2$$
$$+\|\beta(\cdot,t)\|^2+|X(t)|^2\big],\ t\in(t_k,t_{k+1}) \quad (45)$$

for some positive λ_a, where (44) and (39) are used. Note that $\dot{U}_d(t)=0$ because it is constant for $t\in(t_k,t_{k+1})$. The proof is completed.

Lemma 2. Considering $m(t)$ defined in (41), it holds that $m(t)<0$.

Proof. According to (40), events are triggered to guarantee,

$$d(t)^2\leq\theta V(t)-m(t). \quad (46)$$

Inserting (46) into (41), one obtain

$$\dot{m}(t)\leq-[\eta+\lambda_d]m(t)+[\lambda_d\theta-\sigma]V(t)$$
$$-\kappa_1\alpha(1,t)^2-\kappa_2\beta(0,t)^2$$
$$\leq-[\eta+\lambda_d]m(t) \quad (47)$$

with choosing

$$\theta\leq\frac{\sigma}{\lambda_d}. \quad (48)$$

Hence, we conclude that $m(t)<0$ recalling the initial condition $m(0)<0$.

The following lemma proves the existence of a nonzero minimal dwell time independent of initial conditions. It contributes to reduction of the actuation frequency and the well-posedness of the resulting closed-loop system.

Lemma 3. There exists a minimal dwell-time $\tau > 0$ between two triggering times, i.e., $t_{k+1} - t_k \geq \tau > 0$ for all $k \geq 0$.

Proof. We know from (46), the events are triggered to guarantee $d(t)^2 \leq \theta V(t) - m(t)$ for all $t \geq 0$. Define a function ψ as

$$\psi(t) = \frac{d(t)^2 + \frac{1}{2}m(t)}{\theta V(t) - \frac{1}{2}m(t)}. \tag{49}$$

Note that $\psi(t_{k+1}) = 1$ because the event is triggered and $\psi(t_k) \leq 0$ because of $m(t) \leq 0$ and $d(t_k) = 0$. Note that $\psi(t)$ is a continuous function on $[t_k, t_{k+1}]$ due to continuity and well-posedness of this class of 2×2 hyperbolic PDE-ODE system according to Meglio et al. (2018). By the intermediate value theorem, there exists $t^* \in [t_k, t_{k+1}]$ to make $\psi(t^*) = 0$. The minimal τ depends on the time it takes for $\psi(t)$ from 0 to 1.

Taking the time derivative of $V(t)$ (42), we obtain,

$$\begin{aligned}
\dot{V}(t) =& - r_c X(t)^T Q X(t) + 2 r_c X^T P B \alpha(1,t) \\
&+ q_2 r_a \int_0^1 e^{\delta_1 x} \beta(x,t) \beta_x(x,t) dx \\
&- q_1 r_b \int_0^1 e^{-\delta_2 x} \alpha(x,t) \alpha_x(x,t) dx \\
&+ r_a c_2 \int_0^1 e^{\delta_1 x} \beta(x,t)^2 dx + r_b c_1 \int_0^1 e^{-\delta_2 x} \alpha(x,t)^2 dx \\
=& - r_c X(t)^T Q X(t) + 2 r_c X^T P B \alpha(1,t) \\
&+ \frac{1}{2} q_2 r_a e^{\delta_1} \beta(1,t)^2 - \frac{1}{2} q_2 r_a \beta(0,t)^2 \\
&- \frac{1}{2} \delta_1 q_2 r_a \int_0^1 e^{\delta_1 x} \beta(x,t)^2 dx \\
&- \frac{1}{2} q_1 r_b e^{-\delta_2} \alpha(1,t)^2 + \frac{1}{2} q_1 r_b d(t)^2 \\
&- \frac{1}{2} \delta_2 q_1 r_b \int_0^1 e^{-\delta_2 x} \alpha(x,t)^2 dx + r_a c_2 \int_0^1 e^{\delta_1 x} \beta(x,t)^2 dx \\
&+ r_b c_1 \int_0^1 e^{-\delta_2 x} \alpha(x,t)^2 dx
\end{aligned} \tag{50}$$

where (39) is used. It is straightforward to have

$$\dot{V}(t) \geq -\mu_0 V - \lambda_\alpha \alpha(1,t)^2 - \lambda_\beta \beta(0,t)^2 + \lambda_{1d} d(t)^2 \tag{51}$$

for some positive $\mu_0, \lambda_\alpha, \lambda_\beta$ and $\lambda_{1d} = \frac{1}{2} q_1 r_b$.

According to (42), defining $\Omega(t) = \|\alpha(\cdot,t)\|^2 + \|\beta(\cdot,t)\|^2 + |X(t)|^2$, the following inequality holds

$$\xi_1 \Omega(t) \leq V(t) \leq \xi_2 \Omega(t) \tag{52}$$

for some positive ξ_1, ξ_2 Taking the derivative of (49) and using (41), (43), (51), (52), we have

$$\begin{aligned}
\dot{\psi}(t) =& \frac{2d(t)\dot{d}(t) + \frac{1}{2}\dot{m}(t)}{\theta V(t) - \frac{1}{2}m(t)} - \frac{\theta \dot{V}(t) - \frac{1}{2}\dot{m}(t)}{\theta V(t) - \frac{1}{2}m(t)} \psi(t) \\
\leq& \frac{1}{\theta V(t) - \frac{1}{2}m(t)} \Big[2\lambda_a \Big(d(t)^2 + \alpha(1,t)^2 + \beta(0,t)^2 \\
&+ \|\alpha(\cdot,t)\|^2 + \|\beta(\cdot,t)\|^2 + |X(t)|^2 \Big) + 4d(t)^2 + \frac{1}{2}\dot{m}(t) \Big] \\
&- \frac{1}{\theta V(t) - \frac{1}{2}m(t)} \Big[\theta \Big(-\mu_0 V(t) - \lambda_\alpha \alpha(1,t)^2 \\
&- \lambda_\beta \beta(0,t)^2 + \lambda_{1d} d(t)^2 \Big) - \frac{1}{2}\dot{m}(t) \Big] \psi(t) \\
\leq& \frac{1}{\theta V(t) - \frac{1}{2}m(t)} \Big[2\lambda_a d(t)^2 + 2\lambda_a \alpha(1,t)^2 + 2\lambda_a \beta(0,t)^2 \\
&+ 2\lambda_a \|\alpha(\cdot,t)\|^2 + 2\lambda_a \|\beta(\cdot,t)\|^2 + 2\lambda_a |X(t)|^2 \\
&+ 4d(t)^2 - \frac{1}{2}\eta m(t) + \frac{1}{2}\lambda_d d(t)^2 \\
&- \frac{1}{2}\sigma V(t) - \frac{1}{2}\kappa_1 \alpha(1,t)^2 - \frac{1}{2}\kappa_2 \beta(0,t)^2 \Big] \\
&- \frac{1}{\theta V(t) - \frac{1}{2}m(t)} \Big[-\theta \mu_0 V(t) - \theta \lambda_\alpha \alpha(1,t)^2 \\
&- \theta \lambda_\beta \beta(0,t)^2 + \theta \lambda_{1d} d(t)^2 + \frac{1}{2}\eta m(t) - \frac{1}{2}\lambda_d d(t)^2 \\
&+ \frac{1}{2}\sigma V(t) + \frac{1}{2}\kappa_1 \alpha(1,t)^2 + \frac{1}{2}\kappa_2 \beta(0,t)^2 \Big] \psi(t) \\
\leq& \frac{1}{\theta V(t) - \frac{1}{2}m(t)} \Big[\Big(2\lambda_a + 4 + \frac{1}{2}\lambda_d \Big) d(t)^2 \\
&+ \Big(2\lambda_a - \frac{1}{2}\kappa_1 \Big) \alpha(1,t)^2 \\
&+ \Big(2\lambda_a - \frac{1}{2}\kappa_2 \Big) \beta(0,t)^2 + \frac{2\lambda_a}{\xi_1} V(t) - \frac{1}{2}\eta m(t) \Big] \\
&+ \frac{1}{\theta V(t) - \frac{1}{2}m(t)} \Big[\theta \mu_0 V(t) + \Big(\theta \lambda_\alpha - \frac{1}{2}\kappa_1 \Big) \alpha(1,t)^2 \\
&+ \Big(\theta \lambda_\beta - \frac{1}{2}\kappa_2 \Big) \beta(0,t)^2 - \Big(\theta \lambda_{1d} - \frac{1}{2}\lambda_d \Big) d(t)^2 \\
&- \frac{1}{2}\eta m(t) - \frac{1}{2}\sigma V(t) \Big] \psi(t).
\end{aligned} \tag{53}$$

Note that the following inequalities

$$\begin{aligned}
&- \frac{\frac{1}{2}\eta m(t)}{\theta V(t) - \frac{1}{2}m(t)} \leq - \frac{\frac{1}{2}\eta m(t)}{-\frac{1}{2}m(t)} = \eta, \\
&\frac{V(t)}{\theta V(t) - \frac{1}{2}m(t)} \leq \frac{V(t)}{\theta V(t)} = \frac{1}{\theta}, \\
&\frac{d(t)^2}{\theta V(t) - \frac{1}{2}m(t)} = \frac{d(t)^2 + \frac{1}{2}m(t) - \frac{1}{2}m(t)}{\theta V(t) - \frac{1}{2}m(t)} \leq \psi(t) + 1
\end{aligned}$$

hold because of $m(t) < 0$. Choose

$$\kappa_1 \geq \max\{2\lambda_a, 2\theta\lambda_\alpha\}, \kappa_2 \geq \max\{2\lambda_a, 2\theta\lambda_\beta\}, \tag{54}$$

we thus obtain

$$\begin{aligned}
\dot{\psi}(t) \leq& \frac{(2\lambda_a + 4 + \frac{1}{2}\lambda_d) d(t)^2 + \frac{2\lambda_a}{\xi_1} V(t) - \frac{1}{2}\eta m(t)}{\theta V(t) - \frac{1}{2}m(t)} + \eta \psi(t) \\
&+ \mu_0 \psi(t) + \Big(\frac{1}{2}\lambda_d - \theta \lambda_{1d} \Big) (\psi(t)^2 + \psi(t)) \\
\leq& \Big(2\lambda_a + 4 + \frac{1}{2}\lambda_d \Big) \psi(t) + 2\lambda_a + 4 + \frac{1}{2}\lambda_d \\
&+ \frac{2\lambda_a V(t)}{\xi_1 \theta V(t) - \frac{1}{2}m(t)} \\
&+ \frac{-\frac{1}{2}\eta m(t)}{\theta V(t) - \frac{1}{2}m(t)} + \mu_0 \psi(t) + \Big(\frac{1}{2}\lambda_d - \theta \lambda_{1d} \Big) \psi(t)^2 \\
&+ \Big(\frac{1}{2}\lambda_d - \theta \lambda_{1d} + \eta \Big) \psi(t)
\end{aligned}$$

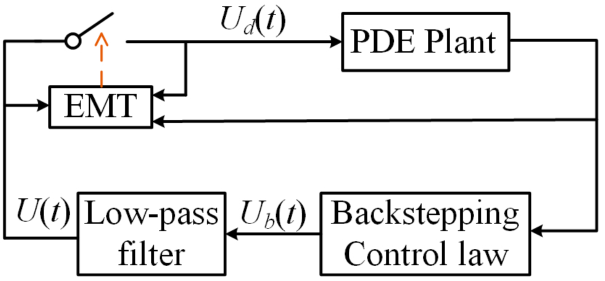

Fig. 2. Event-based closed-loop system.

$$\leq\left(\frac{1}{2}\lambda_d-\theta\lambda_{1d}\right)\psi(t)^2+4+\frac{1}{2}\lambda_d+2\lambda_a+\frac{2\lambda_a}{\xi_1\theta}+\eta$$
$$+(2\lambda_a+4+\lambda_d-\theta\lambda_{1d}+\eta+\mu_0)\psi(t). \quad (55)$$

This differential inequality has the form

$$\dot{\psi}\leq n_1\psi^2+n_2\psi+n_3 \quad (56)$$

where

$$n_1=\frac{1}{2}\lambda_d-\theta\lambda_{1d}, \quad (57)$$
$$n_2=2\lambda_a+4+\lambda_d-\theta\lambda_{1d}+\eta+\mu_0, \quad (58)$$
$$n_3=4+\frac{1}{2}\lambda_d+2\lambda_a+\frac{2\lambda_a}{\xi_1\theta}+\eta \quad (59)$$

are positive constants by choosing

$$\theta\leq\min\left\{\frac{\lambda_d}{2\lambda_{1d}},\frac{\sigma}{\lambda_d}\right\} \quad (60)$$

considering (48). The estimate of the minimal time taken by $\psi(t)$ from 0 to 1 is at least Espitia et al. (2018):

$$\tau=\int_0^1\frac{1}{n_1+n_2\bar{s}+n_3\bar{s}^2}d\bar{s}>0. \quad (61)$$

The proof of this lemma is completed.

4. STABILITY ANALYSIS OF THE EVENT-BASED CLOSED-LOOP SYSTEM

The event-based closed-loop system is built as Fig. 2, where a backstepping control law $U_b(t)$ (33) going through a low-pass filter to generate a "smooth" continuous-time stabilization law $U(t)$ (34) which is updated at time instants t_k determined by ET-M (40)-(41), producing an event-triggered control input $U_d(t)$ to regulate the PDE plant (1)-(5). Because $w(0,t)$ associated with H_1 norm exists in the control law, initial conditions of the plant belonging to H_1 is required to ensure the control law being well-defined.

Theorem 1. For any initial data $(z(x,0),w(x,0))\in H^1(0,1)\times H^1(0,1)$, exponential stability of the system (1)-(5) under the event-based controller $U_d(t)$ holds in the sense that exists positive constants Υ_1 and λ_1 such that

$$\left(\|z(\cdot,t)\|^2+\|w(\cdot,t)\|^2+|X(t)|^2+|m(t)|^2+U(t)^2\right)^{\frac{1}{2}}$$
$$\leq\Upsilon_1\left(\|z(\cdot,0)\|^2+\|w(\cdot,0)\|^2+|X(0)|^2+|m(0)|^2+U(0)^2\right)^{\frac{1}{2}}e^{-\lambda_1 t},$$

where $\|\cdot\|$ denotes L_2 norm and $|\cdot|$ is Euclidean norm.

Proof. Define a Lyapunov function as

$$V_a(t)=V(t)-m(t)+\frac{1}{2}U(t)^2 \quad (62)$$

where $m(t)<0$ is defined in (41) and $V(t)$ is given in (42). $U(t)$ is the state of the extend dynamics, i.e., the output of the low-pass filter. There exist positive constants $\bar{\xi}_1,\bar{\xi}_2$ such that

$$\bar{\xi}_1\bar{\Omega}(t)\leq V_a(t)\leq\bar{\xi}_2\bar{\Omega}(t), \quad (63)$$
$$\bar{\Omega}(t)=|X|^2+\|\alpha\|^2+\|\beta\|^2+|m(t)|^2+U(t)^2. \quad (64)$$

Taking the derivative of (62) along (11)-(14), (34), (39), recalling (41), (50), one obtain

$$\begin{aligned}\dot{V}_a(t)=&\dot{V}-\dot{m}(t)+U(t)\dot{U}(t)\\
=&-r_cX(t)^TQX(t)+2r_cX^TPB\alpha(1,t)\\
&+\frac{1}{2}q_2r_ae^{\delta_1}\beta(1,t)^2-\frac{1}{2}q_2r_a\beta(0,t)^2\\
&-\frac{1}{2}\delta_1q_2r_a\int_0^1e^{\delta_1x}\beta(x,t)^2dx\\
&-\frac{1}{2}q_1r_be^{-\delta_2}\alpha(1,t)^2+\frac{1}{2}q_1r_bd(t)^2\\
&-\frac{1}{2}\delta_2q_1r_b\int_0^1e^{-\delta_2x}\alpha(x,t)^2dx+\eta m(t)\\
&-\lambda_dd(t)^2+\sigma V(t)+\kappa_1\alpha(1,t)^2+\kappa_2\beta(0,t)^2\\
&+a_0U(t)(U_b(t)-U(t))\\
\leq&-\left(\frac{3r_c}{4}\lambda_{\min}(Q)-2a_0|\bar{K}_3|^2\right)|X(t)|^2-\left[\frac{1}{2}q_1r_be^{-\delta_2}\right.\\
&\left.-\frac{|PB|^2r_c}{\lambda_{min}(Q)}-\frac{1}{2}q_2r_ae^{\delta_1}q^2-\kappa_1\right]\alpha(1,t)^2\\
&-\left(\frac{1}{2}q_2r_a-\kappa_2-2p^2a_0\right)\beta(0,t)^2\\
&-\left(\frac{1}{2}\delta_1q_2r_a-2a_0\|\bar{K}_2\|^2\right)\int_0^1e^{\delta_1x}\beta(x,t)^2dx\\
&+\left(\frac{1}{2}q_1r_b-\lambda_d\right)d(t)^2\\
&-\left(\frac{1}{2}\delta_2q_1r_b-2a_0\|\bar{K}_1\|^2e^{\delta_2}\right)\int_0^1e^{-\delta_2x}\alpha(x,t)^2dx\\
&+\eta m(t)+\sigma V(t)-\frac{a_0}{2}U(t)^2,\end{aligned}$$

where

$$\begin{aligned}U_b(t)^2\leq&4p^2\beta(0,t)^2+4\|\bar{K}_1\|^2\int_0^1\alpha(x,t)^2dx\\
&+4|\bar{K}_3|^2|X(t)|^2+4\|\bar{K}_2\|^2\int_0^1\beta(x,t)^2dx \quad (65)\end{aligned}$$

which holds by recalling (33), is used.

$\kappa_1,\kappa_2,r_a,r_b,r_c,\lambda_d$ are chosen as

$$r_c>\frac{16a_0|\bar{K}_3|^2}{3\lambda_{\min}(Q)}, \quad (66)$$
$$r_a>\frac{1}{q_2}\max\left\{8\max\{\lambda_a,\theta\lambda_\beta\},8p^2a_0,\frac{8a_0\|\bar{K}_2\|^2}{\delta_1}\right\}, \quad (67)$$
$$\begin{aligned}r_b>&\frac{1}{q_1}\max\left\{8\max\{\lambda_a,\theta\lambda_\alpha\}e^{\delta_2},\frac{4r_c|PB|^2}{\lambda_{\min}(Q)}e^{\delta_2}\right.\\
&\left.+2q_2r_ae^{\delta_1}q^2e^{\delta_2},\frac{8a_0}{\delta_2}\|\bar{K}_1\|^2e^{\delta_2}\right\}, \quad (68)\end{aligned}$$

$$2\max\{\lambda_a,\theta\lambda_\alpha\}\leq\kappa_1\leq\frac{1}{2}q_1r_be^{-\delta_2}-\frac{r_c|PB|^2}{\lambda_{\min}(Q)}$$

$$-\frac{1}{2}q_2r_ae^{\delta_1}q^2, \tag{69}$$

$$2\max\{\lambda_a,\theta\lambda_\beta\}\le\kappa_2\le\frac{1}{2}q_2r_a-2p^2a_0, \tag{70}$$

$$\lambda_d=\frac{1}{2}q_1r_b, \tag{71}$$

and δ_1,δ_2 can be arbitrary positive constants. Note that (67) ensures $\frac{1}{2}q_2r_a-2p^2a_0>\frac{1}{4}q_2r_a>2\max\{\lambda_a,\theta\lambda_\beta\}$ and $\frac{1}{2}\delta_1q_2r_a-2a_0\|\bar{K}_2\|^2>\frac{1}{4}\delta_1q_2r_a$, and (68) has the same purpose. The left terms in (69)-(70) are from (54).

We thus arrive

$$\begin{aligned}\dot{V}_a(t)&\le-(\nu_a-\sigma)V+\eta m(t)-\frac{a_0}{2}U(t)^2\\&\le-\min\{(\nu_a-\sigma),\eta,a_0\}V_a(t)\end{aligned} \tag{72}$$

where

$$\nu_a=\frac{1}{\xi_1}\min\left\{\frac{3r_c}{8}\lambda_{min}(Q),\frac{1}{4}\delta_1q_2r_a,\frac{1}{4}\delta_2q_1r_be^{-\delta_2}\right\} \tag{73}$$

and ξ_1 is in (52). By choosing

$$\sigma<\nu_a, \tag{74}$$

(72) then becomes

$$\dot{V}_a\le-\bar{\lambda}_1V_a(t) \tag{75}$$

where $\bar{\lambda}_1=\min\{(\nu_a-\sigma),\eta,a_0\}>0$ and $\eta>0$ is a free constant. Recalling (63)-(64), we obtain

$$\begin{aligned}&|X|^2+\|\alpha\|^2+\|\beta\|^2+|m(t)|^2+U(t)^2\le\frac{\bar{\xi}_2}{\bar{\xi}_1}\bigg(|X(0)|^2\\&+\|\alpha(\cdot,0)\|^2+\|\beta(\cdot,0)\|^2+|m(0)|^2+U(0)^2\bigg)e^{-\bar{\lambda}_1t}.\end{aligned} \tag{76}$$

Recalling the backstepping transformation and its inverse in (6)-(9) which guarantee the equivalence between the target system-(α,β,X) and the original system-(z,w,X). The proof of Theorem 1 is completed.

Remark 2. Adding an additional condition $\theta<\min\{\frac{1}{\lambda_\alpha},\frac{1}{\lambda_\beta}\}$ can decouple the sufficient conditions (60), (66)-(71), (74) of the nonzero dwell time and exponential stability of the event-based closed-loop system, and make the solution easier to get in practice.

5. CONCLUSION

In this paper, an event-triggered backstepping boundary controller for a 2×2 hyperbolic PDE-ODE system is proposed via a two-step design, including the design of a low-pass-filter-based backstepping boundary stabilization law and the design of an ETM which determines triggering times of updating the continuous-time control law. The nonzero minimal dwell time between two triggering times and the exponential stability of the event-based closed-loop system are proved. This design can be applied into vibration control of a mining cable elevator driven by hydraulic cylinders at the head sheaves, where the frequent actions of the massive cylinder can be avoided. The design of an output-feedback form of the event-triggered backstepping boundary controller and the simulation test would be conducted in the future work.

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