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On Linear Quadratic Regulation of Linear Port-Hamiltonian Systems ⋆

Javier Caballeria * **Francisco Vargas** * **Hector Ramirez** *
Yongxin Wu ** **Yann Le Gorrec** **

* *Universidad Técnica Federico Santa María, Valparaíso, Chile. j.caballeria.magna@gmail.com, francisco.vargasp@usm.cl, hector.ramireze@usm.cl*
** *Department of Automation and Micro-Mechatronic Systems, FEMTO-ST UMR CNRS 6174, UBFC, 26 chemin de l'épitaphe, F-25030 Besançon, France. yongxin.wu@femto-st.fr, yann.le.gorrec@ens2m.fr*

Abstract:
The linear quadratic regulator is a widely used and studied optimal control technique for the control of linear dynamical systems. It consists in minimizing a quadratic cost functional of the states and the control inputs by the means of solving a linear Riccati equation. The effectiveness of the linear quadratic regulator relies on the cost function parameters hence, an appropriate selection of these parameters is of mayor importance in the control design. Port-Hamiltonian system modelling arise from balance equations, interconnection laws and the conservation of energy. These systems encode the physical properties in their structure matrices, energy function and definition of input and output ports. This paper establishes a relation between two classical passivity based control tools for port-Hamiltonian systems, namely control by interconnection and damping injection, with the linear quadratic regulator. These relations allow then to select the weights of the quadratic cost functional on the base of physical considerations. A simple RLC circuit has been used to illustrate the approach.

Keywords: Port-Hamiltonian systems, Linear Quadratic Regulator, Passivity based control, Control by interconnection, Damping injection

1. INTRODUCTION

In classical linear control theory there is an important control technique known as linear quadratic regulation (LQR) (Kwakernaak and Sivan, 1972; Anderson and Moore, 2007), which gives rise to a set of other optimal control and estimation techniques in different setups. In LQR, the goal is to minimize a quadratic cost function that depends on the system state and also on the control signal, and two weighting matrices. An advantage of this technique is that the solution of the problem is well defined (Goodwin et al., 2001) and consists in a state feedback. The weighting matrices are design parameters, which allow to adjust the importance of every state and input of the system. However, the choice of these matrices is not necessarily based on some criteria which permits to interpret the resultant cost function with a physical meaning.

On the other hand, the port-Hamiltonian framework allows to represent a large class of physical systems with respect to a set of structure matrices and Hamiltonian energy function. While the energy function is defined by the parameters of the energy storing elements, the structure matrices represents Kirchoff's interconnection laws and dissipation relations. Furthermore, port-Hamiltonian systems (PHS) are passive systems (van der Shaft, 2017; van der Shaft and Jeltsema, 2014), hence there are many Lyapunov based properties and results which are available for the modeling and control of this class of systems. (Duindam et al., 2009). This system representation allows to design controllers based on closed-loop energy requirements. For instance, a type of control in this context aims to change the rate of convergence of the systems state to the natural equilibrium point, this procedure is denominated *damping injection* (Ortega et al., 2001). Another well-known control technique is the denominated *control by interconnection* (Ortega et al., 2001; van der Shaft, 2017), which aims to change the shape of the closed-loop energy of the system and modify the equilibrium point to a desire one. Unlike LQR control, the design of these controllers is not based on the solution of an optimal control problem but is heavily related with the physical properties of the system.

In this paper some relationships and equivalences between the aforementioned passivity based control (PBC) techniques and the standard LQR are established. This allows to design the weighting matrices of the optimal control problem in the LQR setup in terms of a desired closed-loop energy function. This will be useful because with that election for the weighting matrices, the cost function in the

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linear quadratic regulator setup can be interpreted and designed in terms of the physical properties of the system. On the other hand, the controllers designed by PBC can be interpreted in terms of optimal control and of associated cost function.

In this paper, we first present in Section 2 the class of linear PHS (LPHS) and its essential properties. In Section 3 we introduce two passivity based control approaches for LPHS: damping injection and control by interconnection. In Section 4, the linear quadratic regulator is introduced for linear systems. In Section 5, we establish two propositions that allow to find explicit relations which render damping injection and control by interconnection equivalent to LQR. Section 6 presents a simple RLC circuit as illustrative example. Finally Section 7 presents some conclusions.

2. PORT-HAMILTONIAN SYSTEMS

Port-Hamiltonian systems are defined in, for instance, van der Shaft (2017); van der Shaft and Jeltsema (2014). In this paper we will focus only on the linear case, that is, linear port-Hamiltonian systems (LPHS), described by the equations:

$$\dot{x} = [J - R]Qx + gu \tag{1}$$

$$y = g^T Qx, \tag{2}$$

where $x \in \mathbb{R}^n$ is the state variable, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^n$ are the input and output of the system respectively, $g \in \mathbb{R}^{n\times m}$ is the input mapping to the system, $Q \in \mathbb{R}^{n\times n}$ is related with the energy of the system, $J \in \mathbb{R}^{n\times n}$ represent the interconnection structure matrix and $R \in \mathbb{R}^{n\times n}$ corresponds to the resistive structure of the system.

For LPHS, the input and output are conjugated, hence for physical systems the product $y^T u$ has the unit of power. Additionally, the matrices J, R and Q satisfy

$$J = -J^T \tag{3}$$

$$R = R^T \geq 0 \tag{4}$$

$$Q = Q^T > 0 \tag{5}$$

Note that J is power-conserving and R is responsible for the internal dissipation of energy.

The so called *Hamiltonian* of the system is a function $H(x)$ that represents the total energy of the system such that

$$H : \mathbb{R}^n \to \mathbb{R}^+{}_0, \tag{6}$$

that is, it is a non negative scalar-valued function of the n-dimensional state space of the system. The Hamiltonian for the system defined above can be written as

$$H(x) = \frac{1}{2} x^T Qx. \tag{7}$$

Now, taking the derivative of H with respect to time, we have

$$\begin{aligned} \dot{H} &= x^T Q^T (J - R)Qx + x^T Q^T gu \\ &\leq y^T u, \end{aligned} \tag{8}$$

where it is concluded that the LPHS is a passive system (see e.g. Ortega et al. (1999))

3. CONTROL OF PORT HAMILTONIAN-SYSTEMS

In this section, two control methods based on the energy of the port-Hamiltonian system are shown. These are *Damping Injection* and *Control by Interconnection* (see Ortega et al. (2001) and van der Shaft (2017)).

3.1 Damping Injection

Damping injection control aims to modify the rate of convergence of the system state to a natural equilibrium point by designing the dissipation matrix of the closed loop system. For this, assume that the input u is given by

$$u = -K_d y, \tag{9}$$

where $K_d \geq 0$ is a design parameter. For this case, $\dot{H}$ is given by

$$\dot{H} = -x^T Q^T RQx + y^T u.$$

Then, given (9) and (2) we obtain

$$\begin{aligned} \dot{H} &= -x^T Q^T RQx - x^T Q^T gK_d g^T Qx \\ &= -x^T Q^T (R + gK_d g^T)Qx. \end{aligned} \tag{10}$$

Hence, we can define a desired dissipation matrix of the closed loop system, $R_d = R_d^T \geq 0$, as follows

$$R_d = R + gK_d g^T, \tag{11}$$

where K_d is the design parameter.

3.2 Control By Interconnection

The second control technique studied here is the *Control by interconnection*, that allows us to drive the system to a new desired equilibrium point, different from the natural one(s) by the design of a Lyapunov function that will represent the desired energy of the system.

Consider that the LPHS defined in (1), (2), with Hamiltonian (7), is controlled by the system Σ_c, which is defined by the following equations:

$$\Sigma_c : \begin{cases} \dot{x}_c = (J_c - R_c)(Q_c x_c + Q_p) + gu_c \\ y_c = g_c^T (Q_c x_c + Q_p) \\ H_c = \frac{1}{2} x_c^T Q_c x_c + \frac{1}{2} x_c^T Q_p + \frac{1}{2} Q_p^T x_c + Q_0 \end{cases} \tag{12}$$

where n_c is the dimension of the state vector x_c, g_c is the map from the state to the controller output y_c, the matrices J_c, R_c and Q_c are matrices in $\mathbb{R}^{n_c\times n_c}$, satisfying the properties in (3) and (5) respectively (considering the corresponding sub index c), $Q_p \in \mathbb{R}^{n_c\times 1}$ and Q_0 a scalar.

The power preserving interconnection between the system and the controller yields into another passive and port-Hamiltonian system for the closed loop. This is carried out by setting

$$\begin{aligned} u &= -y_c \\ y &= u_c \end{aligned} \tag{13}$$

which yields the following closed loop system:

$$\begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} = \left(\begin{bmatrix} J & -gg_c^T \\ g_c g & J_c \end{bmatrix} - \begin{bmatrix} R & 0 \\ 0 & R_c \end{bmatrix} \right) \begin{bmatrix} Q & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x \\ Q_c x_c + Q_p \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} y \\ y_c \end{bmatrix} = \begin{bmatrix} g & 0 \\ 0 & g_c \end{bmatrix}^T \begin{bmatrix} Q & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x \\ Q_c x_c + Q_p \end{bmatrix} \tag{15}$$

The Hamiltonian of the closed loop system is given by:

$$H_d(x, x_c) = H(x) + H_c(x_c) \tag{16}$$

Now the aim is to write the energy function only in terms of the states of the system, that is $H_d(x)$. Thus, we can assign the minimum of the energy at a new desired point and characterize it in terms of x, the state variable of the system.

In order to achieve this, we have to restrict the dynamics (x, x_c) to a submanifold parameterized only by x. This means that we are looking for Casimir functions (van der Shaft, 2017; Ortega et al., 2001), denoted by $C(x, x_c)$, that can relate each state of the controller with the states of the plant. These functions have the form

$$C(x, x_c) = F(x) - x_c. \tag{17}$$

We can choose the function $F(x)$ as follows:

$$F(x) = Kx = \begin{bmatrix} k_1^1 & \cdots & k_n^1 \\ \vdots & \ddots & \vdots \\ k_1^{n_c} & \cdots & k_n^{n_c} \end{bmatrix} x, \tag{18}$$

where $C(x, x_c) \in \mathbb{R}^{n_c \times 1}$, $K \in \mathbb{R}^{n_c \times n}$ and $x_c \in \mathbb{R}^{n_c \times 1}$. The Casimirs function should be invariant with respect to time, therefore from $\dot{C}(x, x_c) = 0$ we can derive the following matching conditions:

$$[K \;\; -I] \begin{bmatrix} J - R & -g g_c^T \\ g_c g^T & J_c - R_c \end{bmatrix} = 0 \tag{19}$$

Thus, the following four equations can be derived from (19) and are denominated *Matching Equations* :

$$KJK^T = J_c \tag{20}$$
$$RK^T = 0 \tag{21}$$
$$R_c = 0 \tag{22}$$
$$KJ = g_c g^T \tag{23}$$

Finally, with this equations we can design the Hamiltonian of the closed loop as:

$$H_d(x) = H(x) + H_c(Kx - C) \tag{24}$$

So, the closed loop system can be re-written as:

$$\dot{x} = (J - R)\frac{\partial H_d}{\partial x}(x). \tag{25}$$

With this expression we can design $H_d(x)$ and therefore its gradient to obtain a desirable closed loop dynamics, so we choose the following desire Hamiltonian for the closed loop system:

$$H_d(x) = \frac{1}{2}(x - x^*)^T Q_d (x - x^*) \tag{26}$$

where $Q_d = Q_d^T \in \mathbb{R}^{n \times n}$ is a symmetric, definite positive energy matrix and $x^* \in \mathbb{R}^{n \times 1}$ a vector of the desired equilibrium point. This implies that the dynamics are given by:

$$\dot{x} = (J - R)Q_d(x - x^*) \tag{27}$$

It is easy to prove that from equation (24) and using the matching equations (20) to (23), Q_d satisfies the following equation:

$$Q_d = Q + K^T Q_c K \tag{28}$$

Finally, with these design, the input u is given by

$$u = -g_c^T (Q_c x_c + Q_p) \tag{29}$$
$$= -g_c^T (Q_c Kx - Q_c C + Q_p). \tag{30}$$

4. LINEAR QUADRATIC REGULATOR

In this section, a brief review of the *Linear Quadratic Regulator* design is recalled to define the considerate notation and the kind of problem to solve. First, consider a linear time-invariant system with state-space representation:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{31}$$
$$y(t) = Cx(t) + Du(t) \tag{32}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ input and $y \in \mathbb{R}^m$ the output of the system, $x_0 \in \mathbb{R}^n$ the initial state at $t = t_0$ and A, B, C, D matrix of appropriate dimensions. Note that the input and output are assumed to have the same dimensions as the ones of corresponding signals in the port-Hamiltonian framework.

The control problem consist to find a linear time-invariant controller that minimizes the cost function

$$J_u = \int_0^\infty [x^T(t)\Phi x(t) + u^T(t)\Lambda u(t)]dt \tag{33}$$

where $\Phi \in \mathbb{R}^{n \times n}$ is a symmetric and positive semi-definite matrix that penalizes the state, and $\Lambda \in \mathbb{R}^{m \times m}$ is a symmetric and positive definite matrix that penalizes the magnitude of the input. Both Φ and Λ are design parameters that allow to define a comparative importance among each component of the state and also of the control signals. Certainly, for J_u to be finite, the state and the control signal must converge to zero.

The optimal solution to this problem is widely known (see e.g. Anderson and Moore (2007); Goodwin et al. (2001)), yielding the optimal control signal

$$u^o = -K_{lqr} x, \tag{34}$$

where x is the system state and

$$K_{lqr} = \Lambda^{-1} B^T P \tag{35}$$

is a constant matrix such that P is the solution to the following Algebraic Ricatti Equation

$$0 = \Phi - PB\Lambda^{-1}B^T P + PA + A^T P. \tag{36}$$

If we want to drive the state to an equilibrium point x^*, different from the origin, we can modify the cost function and write

$$J_u = \int_0^\infty \left[(x(t) - x^*)^T \, \Phi \, (x(t) - x^*) + (u(t) - u^*)^T \, \Lambda \, (u(t) - u^*) \right] dt, \tag{37}$$

where u^* is the input associated to the steady-state that $x = x^*$. Defining

$$\tilde{x}(t) = x(t) - x^*$$
$$\tilde{u}(t) = u(t) - u^*,$$

the cost function can be re-written as

$$J_u = \int_0^\infty [\tilde{x}^T(t)\Phi\tilde{x}(t) + \tilde{u}^T(t)\Lambda\tilde{u}(t)]dt. \tag{38}$$

Hence, the optimal solution is exactly the same from equation (34) but in terms of the new variables, this means that the optimal input signal is given by:

$$\tilde{u}(t) = -K_{lqr}\tilde{x}(t). \tag{39}$$

Finally, in terms of the original variables

$$u^o(t) = -K_{lqr}(x(t) - x^*) + u^*. \tag{40}$$

5. LINEAR QUADRATIC REGULATOR OF LINEAR PORT-HAMILTONIAN SYSTEMS

In this section, we connect the results in the Linear Quadratic Regulator setup with the damping injection and control by interconnection for the LPHS defined in section 3. In order to do this, in this section we set $A = (J - R)Q$ and $B = g$, which equals the state of the LPHS in (1) with the state of the LTI system in (31).

5.1 Damping Injection as a LQR problem.

Proposition 1. Consider a LPHS described by equations (1), (2), (7) and a Damping injection controller defined by an invertible gain K_d. Then, such controller design corresponds to the optimal solution of the standard LQR problem applied to the LPHS, when the weighting matrices of (33) are chosen as

$$\Lambda = K_d^{-1}, \tag{41}$$

and

$$\Phi = Q^T \left[2R + gK_d g^T\right] Q. \tag{42}$$

Proof. We start this proof with a Lyapunov analysis of the Linear Quadratic Regulator. The following equation will be considered as Lyapunov candidate function (see Khalil (2002))

$$V(x) = \frac{1}{2} x^T P x \tag{43}$$

where $P = P^T \geq 0$ is the solution of the algebraic Ricatti equation (36)

$$\Phi - Pg\Lambda^{-1} gP + P(J - R)Q + Q^T (J - R)^T P = 0, \tag{44}$$

where we have used the fact that $A = (J-R)Q$ and $B = g$.

If we use the Linear Quadratic Regulator, the optimal control law given by (34) leads to the closed loop dynamics given by

$$\dot{x} = [(J - R)Q - gK_{lqr}]x \tag{45}$$

Then, the derivative of $V(x)$ respect to time becomes

$$\dot{V}(x) = \frac{1}{2} x^T [Q^T (J - R)^T P + P(J - R)Q - 2Pg\Lambda^{-1} g^T P]x \tag{46}$$

This expression can be reduced using the algebraic Ricatti equation (44) as:

$$\dot{V}(x) = -\frac{1}{2} x^T [\Phi + Pg\Lambda^{-1} g^T P]x \tag{47}$$

If we impose that the solution of (44) is $P = Q$, we will have that

$$V(x) = H(x),$$

and thus we have from (10) and (47) that.

$$-\frac{1}{2} x^T [\Phi + Q^T g\Lambda^{-1} g^T Q]x = -x^T Q^T R_d Q x \tag{48}$$

Since such equality is valid for every x, we conclude that

$$\frac{\Phi}{2} = Q^T \left[R_d - \frac{g\Lambda^{-1} g^T}{2}\right] Q \tag{49}$$

On the other hand, comparing the control laws in the LQR and Damping Injection setups, we have that

$$u_{lqr} = -\Lambda^{-1} g^T Q x \tag{50}$$

$$u_d = -K_d g^T Q x. \tag{51}$$

We have the same control input if we choose:

$$K_d = \Lambda^{-1} \tag{52}$$

Given (52), and recalling that $R_d = R + gK_d g^T$, we rewrite (49) to obtain (42). □

The result of *Propisition 1* can be interpreted as if we have a Damping Injection with known gain K_d, an equivalent LQR control problem with this energy-based design can be obtained, where the matrices Φ, Λ, are chosen like in (41) and (42) respectively.

5.2 Control by Interconnection as a LQR problem.

Proposition 2. Consider a LPHS described by equations (1), (2), (7) and a Control by Interconnection design controller defined by some g_c, Q_c, K. Then, if there exist matrices $X = X^T > 0$ and $Y = Y^T \geq 0$ satisfying

$$g_c^T Q_c K = X g^T Y, \tag{53}$$

then such controller design correspond to the optimal solution of the LQR problem, with non-zero equilibrium point, applied to the LPHS, when the weighting matrices of (37) are chosen such that

$$\Lambda = X^{-1}, \tag{54}$$

$$\Phi = YgXg^T Y - Q(J - R)^T Y - Y^T (J - R)Q. \tag{55}$$

Proof. To prove this proposition we will compare the input u given by the implementation of a closed loop with control by Interconnection and the Linear Quadratic Regulator, to the LPHS described by equations (1), (2) and (7). This control inputs are known from the previous sections, having the form

$$u_{cbi} = -g_c^T Q_c K x + g_c^T Q_c C - g_c^T Q_p \tag{56}$$

$$u_{lqr} = -\Lambda^{-1} g^T P x + \Lambda^{-1} g^T P x^* + u^*. \tag{57}$$

First, note that u_{cbi} is the input derived by control by interconnection and u_{lqr} is the one given by the linear quadratic regulator for non zero equilibrium point. Thus, if we want the same control law, the gains that multiply the vector state x have to be the same. This leads to

$$g_c^T Q_c K = \Lambda^{-1} g^T P, \tag{58}$$

which is (53) with $\Lambda^{-1} = X$ and $P = Y$. Hence, from the algebraic Ricatti equation (44) we conclude that

$$\Phi = YgXg^T Y - Q(J - R)^T Y - Y^T (J - R)Q.$$

On the other hand, we notice that the output of the controller Σ_c in steady state is equal to the input in steady state of the plant. Such stationary value also satisfy (31) with $\dot{x} = 0$. This implies that the constant parts in u_{ci} and u_{lqr} are equal, by design. That is

$$g_c^T Q_c C - g_c^T Q_p = \Lambda^{-1} g^T P x^* + u^*,$$

which ensures $u_{cbi} = u_{lqr}$, completiong the proof. □

Proposition 2 allows to design a closed loop according to control by interconnection and then obtain an equivalent LQR controller that satisfies (53) and therefore the requirements of the energy-based control. Nevertheless, the existence of X and Y that satisfy (53) is not guaranteed for every LPHS and control by interconnection design. Unlike Proposition 1, in this case a direct relation between design parameters as the one in (41) is not revealed.

6. NUMERICAL EXAMPLE

In this section, we consider the following port-Hamiltonian system:

$$\begin{bmatrix} \dot{q} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -R \end{bmatrix} \begin{bmatrix} \frac{1}{C} & 0 \\ 0 & \frac{1}{L} \end{bmatrix} \begin{bmatrix} q \\ \phi \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_f \tag{59}$$

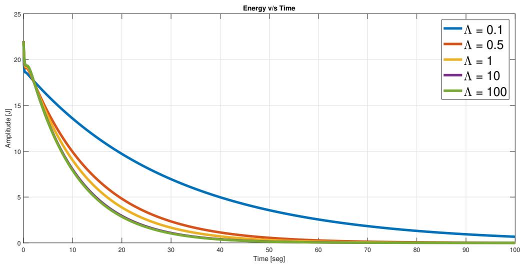

Fig. 1. Time evolution of the Hamiltonian for different LQR controllers

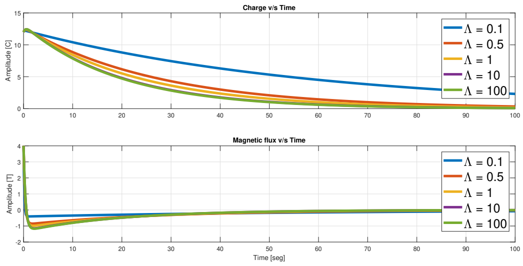

Fig. 2. Time evolution of the states for different LQR controllers

$$y = [0\ 1] \begin{bmatrix} \frac{1}{C} & 0 \\ 0 & \frac{1}{L} \end{bmatrix} \begin{bmatrix} q \\ \phi \end{bmatrix} \tag{60}$$

which correspond to a series RLC circuit with voltage source (Ortega et al., 2001), where the values of the physical parameters are the resistance $R = 4$, the capacitance $C = 6$ and the inductance $L = 2$. Also the initial conditions are $q(0) = 12$ and $\phi(0) = 4$.

First, we use *Proposition 1* to study how the value of Λ affects the energy of the system, more specifically the rate of convergence to its minimum and thus its relation with damping injection. We use the following set of $\Lambda = \{0.1, 0.5, 1, 10, 100\}$ for the simulation, the results are shown in Fig. 1 and 2.

From Fig. 1 it can be seen that for smaller values of Λ the energy tends slower to zero. This can be explained from Fig. 2. where the state ϕ tends faster to zero as the value of Λ is smaller. Thus, the inductor can be interpreted like an open circuit and in consequence the discharge of the capacitor is slower. Also, recalling that $R_d = R + gK_dg^T$ this value increases as Λ decreases (41), hence the relationship between the election of Λ and the increase of the apparent damping R_d depends inversely.

Secondly, we design a controller using control by interconnection and try to find an equivalent LQR controller using *Proposition 2*. From the matching equations (20) to (23) we obtain

$$K = [k_1\ k_2] \tag{61}$$
$$J_c = 0 \tag{62}$$
$$k_2 = 0 \tag{63}$$
$$k_1 = g_c = 1 \tag{64}$$

The desire Hamiltonian function is:

$$H_d(q, \phi) = \frac{1}{2}\frac{(q - q^*)^2}{C + C_a} - \frac{1}{2}\frac{\phi^2}{L} \tag{65}$$

where C_a is a new design parameter that allows to change the capacitance in closed loop of the system. From equation (28) Q_c is

$$Q_c = \frac{1}{C + C_a} - \frac{1}{C}. \tag{66}$$

To find when LQR is equivalent to control by interconnection with the previous design we set Y as

$$Y = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \tag{67}$$

and from (53) from *Proposition 2*, we have the following equality

$$[Q_c\ 0] = \left[X^{-1}b\ X^{-1}c\right] \tag{68}$$

which implies $c = 0$ and $b \neq 0$. This implies

$$Y = \begin{bmatrix} a & b \\ b & 0 \end{bmatrix} \tag{69}$$

and this matrix is non-definite positive. This contradicts one of the properties of Y, in particular $Y \geq 0$, in *Proposition 2* and therefore there is no equivalent LQR controller for this particular system.

7. CONCLUSION

The relation between two classical passivity based control (PBC) schemes, namely damping injection and control by interconnection, with the linear quadratic regulator (LQR) has been established for a class of linear port-Hamiltonian systems (LPHS). The weights of the LQR cost function have been related with the parameters of the Hamiltonian function, the structure matrices and the PBC control gains. Moreover, for the case of damping injection the weights of the LQR cost functional have also been related with the injected dissipation and in the case of control by interconnection with the Casimir functions and shaped Hamiltonian energy. These relations allow to interpret the LQR either as the injection of damping with respect to the physical energy of a system, or as the energy provided to a system in order to shape it. A simple RLC circuit has been used to illustrate the approach.

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