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Health-aware battery charging via iterative nonlinear optimal control syntheses ⋆

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Abstract: There is an increasing recognition of the critical importance of charging for the safety and life of lithium-ion batteries. This paper proposes a computationally efficient optimal control approach for the problem of real-time charging control. By incorporating specific constraints that must be satisfied during charging, a health-aware operation is promoted. To determine the optimal charging current in the given setup, a recently proposed iterative framework for solving constrained optimal control problems is leveraged. It is found that the resulting optimal charging currents can be expressed in terms of a piecewise-affine time-invariant state feedback law, which results in a high computational efficiency for the optimal control solution.

Keywords: Emerging applications in battery control, computational methods.

1. INTRODUCTION

Lithium-ion batteries represent one of the most important energy storage technologies, with a dominant role in consumer electronics and ever-increasing adoption in the sectors of electrified transportation, smart grid, and energy-efficient buildings. While their popularity is due to their high energy/power density, long cycle life and low self-discharge rate, they are also known to be vulnerable to overcharge, overdischarge and abusive use. This necessitates battery management systems (BMSs) that monitor and control the operation of battery systems to ensure safety and performance. A central function of BMSs is charging control, which regulates the charging process to reduce side reactions and improve battery longevity.

Current popular industrial practices, e.g., constant-current / constant-voltage (CC/CV) charging, usually rely on empirical knowledge or coarse-grained understanding about a battery, and are thus inadequate in reducing damaging effects. Hence, the past years have witnessed an exponentially growing interest in model-based charging control, which focuses on exploiting a physics-based characterization of a battery and its health conditions to determine the best way to do charging. Nonlinear optimal control has provided a significant means to achieve this end. This approach has found a great use in optimizing current profiles for minimum-time charging, while satisfying some constraints on a battery's internal states (Klein et al., 2011), bounding the mechanical stresses inside the battery within a limit (Suthar et al., 2014), or minimizing charging loss (Hu et al., 2013). Recent studies have further revealed its promise in addressing more factors involved in charging, e.g., electro-thermal-aging effects (Perez et al., 2017; Liu et al., 2018), user needs (Fang et al., 2017; Ouyang et al., 2018), and cell equalization (Ouyang et al., 2018).

Model predictive control (MPC) is another methodology, which allows predictive optimization of charging over a future time horizon in a closed control loop. The study in (Klein et al., 2011) proposes to use nonlinear MPC (NMPC) to approximate the optimal charging solution over the full horizon. However, NMPC requires heavy online computation, which can be unaffordable for BMSs. This has motivated a search for computationally efficient NMPC-based charging control methods. One way toward this aim is to use intrinsic properties of a battery model, e.g., the differential flatness of the lithium-ion diffusion as in (Liu et al., 2016, 2017), to reduce the computational cost. One can also introduce model simplification to streamline optimization by using linear or successively linearized models (Xavier and Trimboli, 2015; Zou et al., 2018), or simple data-driven models (Torchio et al., 2015).

In this paper, we propose a novel approach to health-aware charging control via iterative nonlinear optimal control syntheses, cf. (Zeng, 2019a,b) and (Vu and Zeng, 2020). We consider a recently developed equivalent circuit model (ECM) to characterize a battery's dynamics, namely, the nonlinear double capacitor (NDC) model. The NDC model differs from other popular ECMs, e.g., the Thevenin's model, with its capability of simulating the charge diffusion inside an electrode. It thus allows us to perform charging control with an awareness of reducing diffusion-related health degradation. Using the NDC model, we formulate an optimal charging control problem subject to health-related constraints. The computationally efficient iterative scheme proposed by the authors in (Vu and Zeng, 2020) is then adapted to the present setup to synthesize the control input for the optimal charging control problem that is optimal with respect to the actual full time horizon. Due to the computational efficiency of the iterative scheme, we can not only readily compute a solution for one particular initial condition but also easily apply the same procedure

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to a dense grid of several different initial conditions. In doing so, it is found that the optimal solution for different initial states can all be summarized in terms of one underlying rather simple piecewise-affine time-invariant state feedback law, which is particularly convenient for practical implementations.

The remainder of the paper is organized as follows. Section 2 introduces the control theoretic formulation of the optimal charging control problem. Section 3 illustrates the adoption of the aforementioned iterative optimal control synthesis to the considered problem. A detailed case study for the establishment of the global piecewise-affine time-invariant state feedback law is demonstrated in Section 4. Finally, Section 5 gathers concluding remarks.

2. THE HEALTH-CONSCIOUS BATTERY CHARGING CONTROL PROBLEM

The battery charging control problem consists of raising the battery's state of charge (SoC) to a desirable level, $\bar{r}$, while satisfying some health-related constraints. The following subsections will describe the setup in more detail.

2.1 Model Description

We consider the NDC model for batteries, which was recently proposed in (Tian et al., 2020) and is shown in Fig. 1. This model includes two parallel RC circuits for charge storage, R_b-C_b and R_s-C_s with $R_b > R_s$ and $C_b \gg C_s$, which are analogous to an electrode's bulk inner part and surface region, respectively. In addition, the model characterizes the terminal voltage as dependent on the voltage source $U = h(V_s)$, where V_s is the voltage across C_s, as well as the internal resistance R_0. With this structure, the NDC model can not only simulate the charge diffusion inside an electrode but also capture the nonlinear voltage behavior. This makes it more accurate than other ECMs while still maintaining a simple structure.

The NDC model's dynamics is governed by the following state-space equations:

$$\begin{bmatrix} \dot{V}_b(t) \\ \dot{V}_s(t) \end{bmatrix} = \tilde{A} \begin{bmatrix} V_b(t) \\ V_s(t) \end{bmatrix} + \tilde{B} I(t), \tag{1a}$$

$$V(t) = h(V_s(t)) + R_0(V_s(t)) I(t), \tag{1b}$$

where $V_b(t)$ is the voltage across C_b at time t, and $I(t)$ is the applied current at time t with $I(t) > 0$ for charging and $I(t) < 0$ for discharging, and $\tilde{A}$ and $\tilde{B}$ are given by

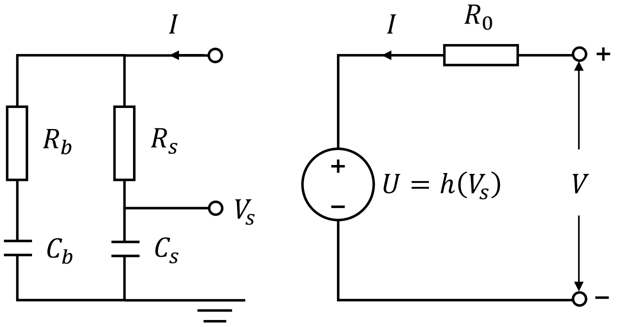


Fig. 1. The nonlinear double-capacitor model (NDC).

$$\tilde{A} = \begin{bmatrix} \frac{-1}{C_b(R_b+R_s)} & \frac{1}{C_b(R_b+R_s)} \\ \frac{1}{C_s(R_b+R_s)} & \frac{-1}{C_s(R_b+R_s)} \end{bmatrix}, \ \tilde{B} = \begin{bmatrix} \frac{R_s}{C_b(R_b+R_s)} \\ \frac{R_b}{C_s(R_b+R_s)} \end{bmatrix}.$$

Without loss of generality, we usually limit $V_b(t)$ and $V_s(t)$ between 0 and 1 V. That is, the battery is fully depleted when $V_b(t) = V_s(t) = 0$, and fully charged when $V_b(t) = V_s(t) = 1$.

In addition, $h(V_s(t))$ can be parameterized as a fifth-order polynomial to offer sufficient accuracy:

$$h(V_s(t)) = \sum_{i=0}^{5} \alpha_i V_s^i(t),$$

where α_i for $i = 0, 1, \ldots, 5$ are coefficients whose values are empirically selected and listed in Table 1. Finally, R_0 is also assumed to monotonically increase with respect to V_s. Such a dependence is described as

$$R_0(V_s(t)) = \beta_1 + \beta_2 e^{-\beta_3(1-V_s(t))},$$

where $\beta_i \geq 0$ for $i = 1, 2, 3$.

2.2 Constraints

To ensure the health-conscious and safe charging, some constraints must be enforced to avoid any abusive charging. A summary of them is as follows.

First, the SoC must be constrained to prevent any potential overcharging:

$$\mathrm{SoC}_{\min} \leq \mathrm{SoC}(t) \leq \mathrm{SoC}_{\max},$$

where

$$\mathrm{SoC}(t) = \frac{C_b V_b(t) + C_s V_s(t)}{C_b + C_s} \times 100\%.$$

During charging operations, $V_b(t)$, $V_s(t)$, and the charging current $I(t)$ should each be kept within an allowable range. Hence,

$$V_{b,\min} \leq V_b(t) \leq V_{b,\max},$$
$$V_{s,\min} \leq V_s(t) \leq V_{s,\max},$$
$$I_{\min} \leq I(t) \leq I_{\max}.$$

Besides, we introduce an additional constraint associated with the relationship between $V_s(t)$, $V_b(t)$ and $\mathrm{SoC}(t)$, i.e.,

$$V_s(t) - V_b(t) \leq \gamma_1 \mathrm{SoC}(t) + \gamma_2,$$

where $\gamma_1 \leq 0$ and $\gamma_2 \leq 0$ are two coefficients. Here, $V_s(t) - V_b(t)$ represents an analogy to the lithium-ion concentration gradient (Tian et al., 2020), which, if too large, can cause many undesirable side effects and thus needs to be restricted.

Finally, the terminal voltage must also be subject to limitations at all time to circumvent safety issues, which imposes the following constraints

$$V_{\min} \leq V(t) \leq V_{\max}.$$

The above descriptions outline the optimal charging control problem which seeks to increase the SoC to a target under a set of explicit constraints ensuring battery health. In the next section, we will adopt a computational iterative optimal control method into this context.

Table 1. Battery model parameters.

Variable	C_b	C_s	R_b	R_s	α_0	α_1	α_2	α_3	α_4	α_5	β_1	β_2	β_3
Value	9913	887	0.025	0	3.2	3.041	- 11.475	24.457	-23.536	8.513	0.09	0.35	10

3. ITERATIVE OPTIMAL CONTROL SYNTHESIS FOR BATTERY CHARGING

In this section, we consider the problem of synthesizing optimal control inputs for the practical solution of the health-aware battery charging problem. With the definition of states and inputs

$$x(t) := \begin{bmatrix} V_b(t) & V_s(t) \end{bmatrix}^\top \quad \text{and } u(t) := I(t),$$

along with the zero-order hold assumption that

$$u(t) \equiv u_k, \;\; t \in [k\Delta T, \, (k+1)\Delta T],$$

for some $u_k \in \mathbb{R}$ some sampling period $\Delta T > 0$, we obtain the exact discretization of the NDC model (1a) as

$$x_{k+1} = Ax_k + Bu_k, \tag{2}$$

where the matrices are given by the well-known equations

$$A = e^{\tilde{A}\Delta T} \quad \text{and} \quad B = \int_0^{\Delta T} e^{\tilde{A}(\Delta T - \tau)} \tilde{B} d\tau.$$

For the later numerical simulations, the integral for B is evaluated numerically using the trapezoidal rule.

Given an initial state $x_0 \in \mathbb{R}^n$ and a nominal control input $u_0, u_1, \ldots, u_{N-1}$, by iterating (2), we have

$$\underbrace{\begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}}_{=: X_U} = \begin{bmatrix} Ax_0 \\ \vdots \\ A^N x_0 \end{bmatrix} + \underbrace{\begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & & \vdots \\ \vdots & \vdots & & 0 \\ A^N B & A^{N-1}B & \cdots & B \end{bmatrix}}_{=: H} \underbrace{\begin{bmatrix} u_0 \\ \vdots \\ u_{N-1} \end{bmatrix}}_{=: U}.$$

Then, for any perturbation of the nominal control input

$$\Delta U := \begin{bmatrix} \delta u_0 \\ \vdots \\ \delta u_{N-1} \end{bmatrix},$$

the resulting state trajectory of (2) due to the control input $U + \Delta U$ can be conveniently described as

$$X_{U+\Delta U} = X_U + H\Delta U.$$

Moreover, the amount of changes of the system trajectory at each time step due to the control perturbation ΔU can also be directly quantified as follows

$$\Delta X := \begin{bmatrix} \delta x_1 \\ \vdots \\ \delta x_N \end{bmatrix} := \begin{bmatrix} H_1 \Delta U \\ \vdots \\ H_N \Delta U \end{bmatrix} = H\Delta U,$$

where H_k is the k-th horizontal block of the H matrix, i.e.,

$$H_k = \begin{bmatrix} A^k B & \cdots & B & 0 & \cdots & 0 \end{bmatrix}.$$

To begin the establishment for all the constraints of the problem formulation, we first rewrite the constraint on SoC as

$$\mathrm{SoC}_{\min} \leq \underbrace{\begin{bmatrix} \frac{C_b}{C_b + C_s} & \frac{C_s}{C_b + C_s} \end{bmatrix}}_{=: M_{\mathrm{SoC}}} \underbrace{\begin{bmatrix} V_b(k\Delta T) \\ V_s(k\Delta T) \end{bmatrix}}_{= x_k} \leq \mathrm{SoC}_{\max}.$$

The upper bound and lower bound of $V_b(k\Delta T)$, $V_s(k\Delta T)$, and the charging current can be presented as follows

$$\underbrace{\begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}}_{=: M_{\mathrm{states}}} \underbrace{\begin{bmatrix} V_b(k\Delta T) \\ V_s(k\Delta T) \end{bmatrix}}_{x_k} \leq \underbrace{\begin{bmatrix} -V_{b,\min} \\ -V_{s,\min} \\ V_{b,\max} \\ V_{s,\max} \end{bmatrix}}_{=: v_{\mathrm{bound}}},$$

$$I_{\min} \leq u_k \leq I_{\max}.$$

The relationship between $V_s(k\Delta T)$, $V_b(k\Delta T)$ and the $\mathrm{SoC}(k\Delta T)$ is then characterized as follows

$$\underbrace{\left(\begin{bmatrix} -1 & 1 \end{bmatrix} - \gamma_1 M_{\mathrm{SoC}} \right)}_{M_{\mathrm{differ}}} \underbrace{\begin{bmatrix} V_b(k\Delta T) \\ V_s(k\Delta T) \end{bmatrix}}_{x_k} \leq \gamma_2.$$

Finally, we consider the nonlinear constraints on the terminal voltage defined in (1b) at each time step, i.e.,

$$V_{\min} \leq V(x_k, u_k) \leq V_{\max}.$$

In this paper, we introduce a simple yet effective technique to handle this rather general nonlinear constraint of the form $g(x, u) \leq 0$ by rolling out the overall solution of the nonlinearly constrained optimal control problem as a sequence of linearly constrained quadratic programs.

More specifically, the idea of this quite general principle of iterative optimal control synthesis is to iteratively compute the solution of the optimal control problem as follows. We start with a predetermined input U, which results in a state-evolution given $x_0, x_1, x_2, \ldots, x_N$. This nominal input is then incrementally altered so as to yield an improvement in the solution of the nonlinearly constrained optimal control problem at the next stage. The incremental nature of these iterations allows us to effectively relax the nonlinear constraint into a linear constraint by the consideration of the linear approximation of $V(x_k, u_k)$ at (x_k, u_k), i.e.,

$$V(x_k + \delta x_k, u_k + \delta u_k) \approx V(x_k, u_k) + \begin{bmatrix} \frac{\partial V}{\partial x} & \frac{\partial V}{\partial u} \end{bmatrix} \begin{bmatrix} \delta x_k \\ \delta u_k \end{bmatrix}.$$

As a result, $V_{\min} \leq V_{U+\Delta U}(x_k, u_k) \leq V_{\max}$ becomes

$$V_{\min} \leq V(x_k, u_k) + \underbrace{\begin{bmatrix} \frac{\partial V}{\partial x} & \frac{\partial V}{\partial u} \end{bmatrix}_{(x_k, u_k)}}_{=: \mathbf{J_V}(x_k, u_k)} \begin{bmatrix} \delta x_k \\ \delta u_k \end{bmatrix} \leq V_{\max}.$$

The achieve the aforementioned incremental improvement, at each stage of the iterative process, we solve the following linearly constrained quadratic optimization problem

$$
\begin{aligned}
\underset{\Delta U}{\text{minimize}} \quad & q\|\text{diag}(M_{\text{SoC}})(X + H\Delta U) - \bar{r}\|^2 \\
& \qquad + r\|U + \Delta U\|^2 + \gamma\|\Delta U\|^2, \\
\text{subject to} \quad & \text{SoC}_{\min} \leq M_{\text{SoC}}(x_k + H_k\Delta U) \leq \text{SoC}_{\max}, \\
& M_{\text{states}}(x_k + H_k\Delta U) \leq v_{\text{bound}}, \qquad (4) \\
& M_{\text{differ}}(x_k + H_k\Delta U) \leq \gamma_2, \\
& I_{\min} \leq u_k + \delta u_k \leq I_{\max}, \\
& V_{\min} \leq V(x_k, u_k) + \mathbf{J_V}(x_k, u_k) \begin{bmatrix} \delta x_k \\ \delta u_k \end{bmatrix} \leq V_{\max},
\end{aligned}
$$

where $\text{diag}(M_{\text{SoC}})$ is a $2N \times 2N$ block diagonal matrix whose diagonal elements are M_{SoC}. The two parameters q and r represent the (tunable) balance ratio between the charging time and the current's magnitude, and γ is a regularization parameter that enforces a penalty on the magnitude of ΔU in order to ensure that the aforementioned incremental change of the system trajectory at each trajectory iteration is valid.

Notably, the formulation (4) is a linearly constrained quadratic program, which can be solved very effectively. Also, to achieve the most efficiency and effectiveness of the iterative optimal control synthesis, one should only apply a small initial control input U and let the system naturally follow its own dynamics to achieve the desired cost functional.

In summary, the trajectory-based iterative procedure for synthesizing the optimal control input for the health-conscious battery control problem is as follows.

Algorithm 1 Charging under health-related constraints

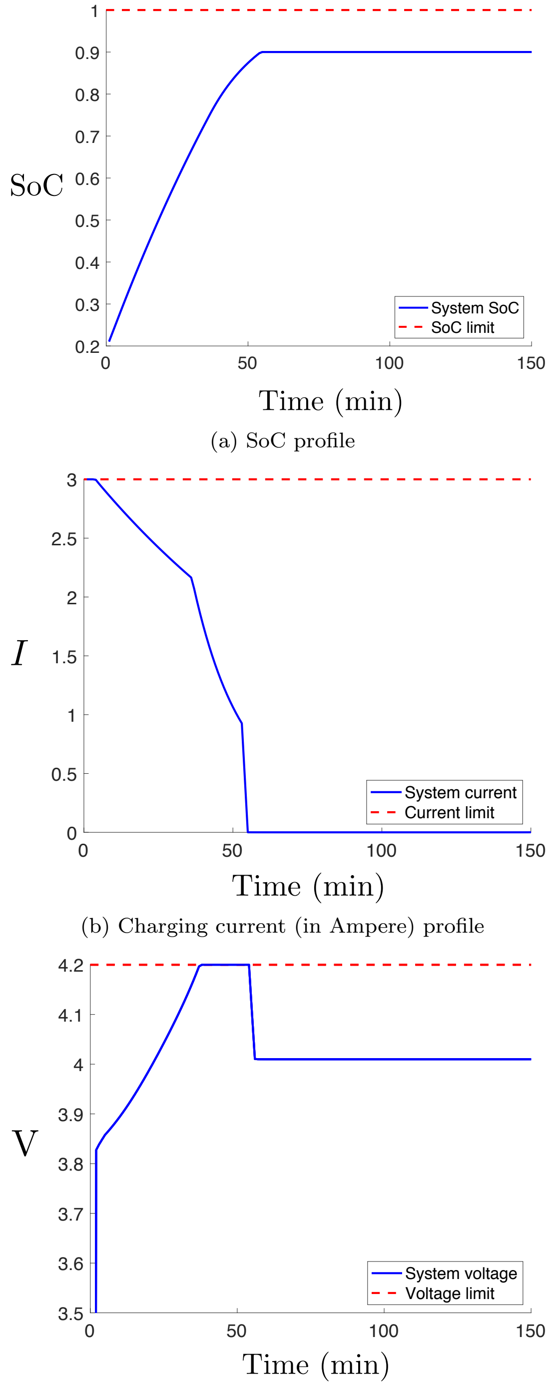
Require: Desired terminal SoC, and an initial input U.
1: Apply the input U to the system and store the resulting state trajectory $x_0, x_1, \ldots, x_N$.
2: Compute H.
3: Solve for ΔU^* of the optimization problem (4).
4: Update the control input via $U = U + \Delta U^*$.
5: Repeat until improvement in cost is too incremental.

4. A DETAILED CASE STUDY

In this section, we present an elaborate case study to validate the proposed optimal control method as well as to investigate in more detail the resulting trade-off between the energy consumption quantified by $\|U\|$ and the charging time (in minutes), as a consequence of different balance ratios of the weights q and r.

Let us consider a 3 Ah lithium-ion battery cell governed by the NDC model with the parameters shown in Table 1. The charging objective is to raise the SoC from 20% to 90% under the following constraints:

$$
\begin{gathered}
0 \leq \text{SoC} \leq 1, \quad 0 \leq V_b \leq 0.95, \quad 0 \leq V_s \leq 0.95, \\
V_s - V_b \leq -0.04\text{SoC} + 0.08, \\
0 \leq I \leq 3, \quad 0 \leq V \leq 4.2.
\end{gathered}
$$



(a) SoC profile

(b) Charging current (in Ampere) profile

(c) Voltage (in Volts) profile

Fig. 2. Simulation results for the case study.

To this end, with the sampling period of $\Delta T = 60\,\text{s}$ for the model discretization, we can directly apply the overall iterative scheme to synthesize the optimal control input for the lithium-ion battery charging problem whose solution is presented in Fig. 2.

Fig. 2a demonstrates the resultant SoC profile which increases smoothly from 20% to 90% as desired. Its rate of increase gradually declines to prevent any overshooting, which is reflected by the decreasing charging current as shown in Fig. 2b.

Note that the current profile particularly includes four distinct stages. The first stage is brief and shows constant-current charging. Following it are the second to fourth stages. For each of them, the current reduces almost linearly. A quick drop is observed in the fourth stage when the SoC is about to reach 90%.

Both Figs. 2a-2b indicate a slowing pace of charging due to the imposed constraints, which provides a stronger health protection for a lithium-ion battery as the SoC grows. However, because of the accuracy and efficiency of the proposed method, the entire charging process only takes place in less than one hour. This beneficial situation is due to the control law achieving the optimal charging efficiency and constraint satisfaction simultaneously by construction.

Finally, Fig. 2c displays the terminal voltage whose value drops at the end due to the RC transients after the charging has finished.

4.1 Analyzing the choice of weights q and r

Our method to quickly synthesize the optimal solution also allows us to efficiently investigate the trade-off between the energy consumption and the charging time. To this end, we fix $q = 0.5$ while slowly varying the value of r from 10^{-6} to 10^{-2}. As a result, a small gain in energy saving comes with a great cost of a dramatic increase in battery charging duration, as illustrated in Fig. 3. Therefore, the tuning of the balance ratio between q and r does not yield a favorable result, so it is reasonable to only focus on tracking the desired SoC (i.e. $q = 0.5$ and $r = 0$ whose results are demonstrated in Fig. 2).

Due to the result of the above analysis, we choose to only consider the desired SoC tracking problem (i.e. to raise the SoC to 90% with $q = 0.5$ and $r = 0$), which leads us to an ultimately elegant time-invariant state feedback solution to the lithium-ion battery charging control problem, as presented in the next subsection.

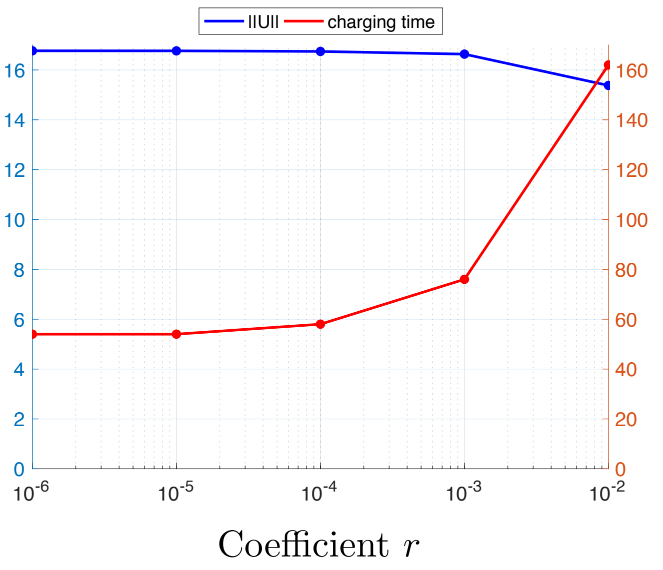


Fig. 3. Trade-off between energy of the optimal control input and charging duration in minutes for varying values of r (with $q = 0.5$ held fixed).

4.2 Uncovering an underlying time-invariant feedback law

Because the computational effort to synthesize the optimal control signal for one initial condition is quite minimal (around 10 seconds on a 2.2 GHz Intel Core i7 computer), it becomes feasible to compute the optimal solution to the battery control problem for a rather dense grid of different initial conditions in the comparatively small state-space of (V_b, V_s). The idea is to compute these solutions offline and investigate the possibility of uncovering a time-invariant state feedback law, which, when applied to the battery system in closed-loop, results in the exact same input as generated by the algorithm. In this way, would not have to solve the optimal control problem online for some given initial state, but its solution is implicitly encoded in the time-invariant state feedback law which has been computed offline. The ability to do so would be significant in view of practical real-time solutions.

To further investigate this, we first apply a mesh (grid) refinement to the relatively small state space of the system, then, for each grid point, we compute the optimal control signal to track the 90% SoC, yet only store the very first control input which will later be used to compute the general feedback law. These first control inputs can then be plotted as a function of the states, as illustrated in Fig. 4. The result reveals the presence of an underlying time-invariant state feedback law $k(V_b, V_s)$, for which $u(t) = k(V_b(t), V_s(t))$ completely coincides with the input computed by the algorithm.

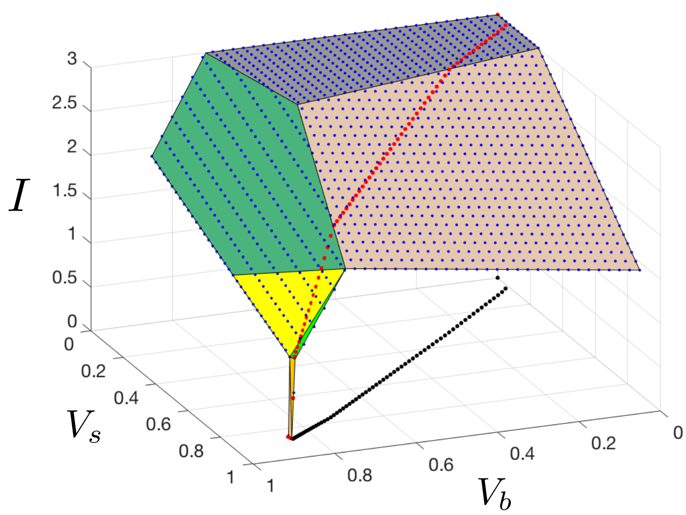


Fig. 4. Given grid points $(V_b^{(i)}, V_s^{(i)})$, we computed the solution $U^{(i)}$ of the overall nonlinear optimal control problem and plotted $(V_b^{(i)}, V_s^{(i)}, U_1^{(i)})$, where $U_1^{(i)}$ is the first entry of $U^{(i)}$. It can be seen that these blue points lie on a surface that can be described using a piecewise affine function (with 7 distinct regions) very well. Moreover, to reinforce the connection of this feedback law $u = k(V_b, V_s)$ to the overall problem, we computed the solution U of the optimal battery control problem for a typical initial state $(V_b(0), V_s(0))$ and then plotted the trajectory $(V_b(k), V_s(k), U_k)$ in red. We observe that this trajectory is fully trapped in the piecewise affine surface, which highlights the validity of the time-invariant feedback law.

Furthermore, the time-invariant state feedback law is evidently a piecewise affine one with fairly few regions of different affine functions. Besides the conceptual relevance of this result, the approach also provides a promising opportunity to derive real-time optimal and highly practical solutions to the lithium-ion battery control problem.

5. CONCLUSIONS

Lying in the center of the clean energy revolution, lithium-ion batteries are rising as an indispensable means of energy storage in many sectors. The charging process is known to play a critical role in their health, longevity, and safety, but how to carry it out remains an open challenge to date. In this study, we considered a nonlinear double capacitor model and formulated a nonlinearly constrained optimal control problem that promotes health consciousness charging. The resulting optimal control problems were solved using a recently developed iterative optimal control methodology. Moreover, we uncovered a piecewise-affine time-invariant state feedback law that ties together the solutions to the optimal charging control problem obtained from different initial states. Its tremendous computational efficiency makes it very desirable for real-time execution on battery management systems. Simulation results showed the effectiveness of our approach in ensuring the health-aware fast charging. Our future work will include an experimental validation of this result.

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