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Switchable PID Controller Tuning Based on Golden Section Reduction Rule

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Abstract: Resulting from the complexity and sensitivity instinct, high-order control system design is always a challenge faced by engineers from process industry. To solve this challenging task, a switchable control scheme consisted of two sub-controllers tuned based on two complementary simplified models respectively is proposed in this paper, and golden section is introduced into process model reduction to simplify parameter tuning. Inspired by adaptive control strategy, the controller switch mechanism depends on the time domain response similarity between the original high-order model and the simplified model. Besides, PID controller structure and corresponding parameter tuning method, such as Ziegler-Nichols method and SIMC method, are remained to guarantee the facility and efficiency of sub-controller design. The proposed method has been tested on several examples (balanced, lag-dominant, and a delay-dominant process) and the comparison with other tuning method based on step-response data resulting in favorable control performance.

Keywords: high-order system, model reduction, multiple models, switching, PID control.

1. INTRODUCTION

In the chemical process, the complexity of the reactant interaction mechanism brings high-order characteristics and uncertainty to the process model. The features of chemical process cause two major contradictions (Luyben and William, 2007),

(1) Traditional controller design generally base on low-order system, such as FOPDT and SOPDT.

(2) Modern control tools derivate from astronomy and aerospace mostly, they are extremely sensitive to model precision.

Therefore, simplifying a high-order system to FOPDT or SOPDT models first then designing proper controller becomes a trend emerges in recent years. Related design methods were designed by Åström (1995) and Hägglund (2019). Commonly used reduction methods include graph method, contained in Ljung (1999)., half rule proposed by Skogestad (2003) and the popular controller structure includes PID, MPC. The typical representative of model simplification plus PID framework is SIMC, and an improvement was presented in Grimholt (2018). A variant of predictive function control can be regarded as model simplification plus MPC, as given in Jian-guo Wu (2010) and Hongxin Wu (2009).

The advantages are obvious, on the one hand, low-order model-based design method avoids generating complicated controller; on the other hand, designing controller for low-order model reduced the design difficulty. More profoundly, this design approach improves controller robustness, since the simplified model has contained several cases of parameter drift.

In view of the advantages described above, model reduction plus PID controller scheme is chosen as the framework to hit the balance of control performance with the facility of design and implementation in this paper. Golden section rate is introduced into model reduction and two reduction rules are presented as Positive Golden Section Rule and Positive Golden Section Rule, these two rules split high-order system into symmetry simplified low-order transfer functions and a fusion mechanism based on bias selection combine two models together. Control strategy draw on the spirit of Multiple Models Adaptive Control (MMAC), as given in Narendra (1994), Han (2012) and Wang (2017), a switchable PID controller is applied to implement the adaptive strategy, and SIMC tuning method is remained to adjust PID parameters. Benefit from the improvement of internal model accuracy, the combination of SIMC tuning method and the model switching controller design proposed in this paper has achieved better control performance than the original design, and is more resistant to model perturbation.

After this introductory section, this paper is organized as follows. The proposed golden section reduction rule is introduced in Section 2. The controller design method and corresponding simulation are given in Section 3 some additional discussions about controller robustness are given in Section 4. The paper is then concluded in Section 5.

2. MODEL REDUCTION

Designing controller directly for high-order system is not economical, turn them into FOPDT, SOPDT model first would be a better solution.

FOPDT:

$$G(s) = \frac{k}{\tau_1 s + 1} e^{-Ls} \tag{1}$$

SOPDT:

$$G(s)=\frac{k}{(\tau_1 s+1)(\tau_2 s+1)}e^{-Ls} \tag{2}$$

In Equation (1) and (2), τ_1 and τ_2 are time constants of plant, assumed that $\tau_1 > \tau_2$, L is the delay time. The dynamic of high-order systems in the process industry are greatly affected by capacity and delay components. From this perspective, high-order systems can be divided into Lag-dominant, Balanced and Delay-dominant System, determined by the ratio of time constant and delay time in simplified model,

$$ratio=\tau_1 / L \tag{3}$$

if the ratio>1, the system is categorized into lag-dominant system, and ratio=1 indicates the balanced situation, other systems are classified as delay-domain system. For the three types of high-order systems, reasonable selection of time constants and delay times can make the low-order systems time-domain responses close to higher-order systems.

2.1 Review of Classical Method

(1) Graph Method. This method is a non-parametric method, which originates from the system identification. It directly estimates the model parameters of FOPDT and SOPDT from the information on the step response curve, which is suitable for systems with self-balance process. Although this method is relatively rough, it is widely used in engineering practice because of its simple and clear rules. More detailed about so called Graph Method could be found in Ljung (1999).

(2) Least Square Method. This method is similar to the ordinary system identification process, optimizes following cost function.

$$\hat{\theta}=\arg\min_{\theta}\sum_{i=1}^{N}[z(i)-h^T(i)\theta]^2 \tag{4}$$

In Equation (4), $z(i)$ represents the true output value collected from plant in time i, $h(i)$ is data from Hankel matrix and θ is the coefficient of selected model, e.g., ARMAX.

(3) Half Rule. Consider a high-order transfer function of this form. Assume K_j and T_i are sorted in descending order ,in the absence of RHPs and zeros,

$$\begin{aligned}G(s)&=\frac{a_0+a_1 s+\cdots+a_m s^m}{b_0+b_1 s+\cdots+b_{n-1}s^{n-1}+b_n s^n}e^{-Ls}\\&=\frac{\prod_{j=1}^{m}(K_j s+1)}{\prod_{i=1}^{n}(T_i s+1)}e^{-Ls}\end{aligned} \tag{5}$$

FOPDT:

$$\begin{aligned}\hat{G}(s)&=\frac{1}{(T_1+0.5T_2)s+1}e^{-L's}\\L'&=L+0.5T_2+\sum_{i=3}^{n}T_i+\sum_{j=1}^{m}K_j\end{aligned} \tag{6}$$

SOPDT:

$$\begin{aligned}\hat{G}(s)&=\frac{1}{(T_1 s+1)((T_2+0.5T_3)s+1)}e^{-L's}\\L'&=L+0.5T_3+\sum_{i=4}^{n}T_i+\sum_{j=1}^{m}K_j\end{aligned} \tag{7}$$

In Equation (6) and (7), L' is the new delay time of simplified model.

2.2 The Golden Section Reduction Rule

Draw on the spirit of MMAC, which updates the process model continuously by identifier and selects the corresponding controller to cope with model parameters drift during the reaction process. Inspired by this idea, this paper intends to introduce a multi-model framework with a PID controller to implement a simplified adaptive control scheme and eliminate the online identification process by adopting high- order model reduction method. The IMC-PID could obtain a robust controller with good control performance by adjusting only one parameter, the filter constant, so it is very popular in the process industry. The SIMC tuning method is also derived from IMC-PID, and further simplifies the tuning steps. SIMC has been widely used for its outstanding performance, therefore we intend to retain the tuning rule in this paper, and strength SIMC performance from improved internal model accuracy.

Similar to Half Rule, GSR is also an analytical approximation method, and introduces the golden section ratio into simplification. In addition, since GSR includes a dual model fusion mechanism, which combines PGSR and NGSR together, the approximation accuracy is higher than Half Rule. And at each sampling time, there is always a simplified model in GSR with better approximation effect compared to Half Rule. Considered a plant described in Equation (8), the comparison of NGSR and PGSR with Half Rule simulations is shown in the figure 1.

$$G(s)=\frac{1}{(s+1)^4} \tag{8}$$

Although GSR rule can be applied to any high-order model of the form Equation (11) mathematically, the actual simulation results show that if the model does not have self-balance characteristics, the approximation error will be quite large, which is contrary to the requirement of internal model control. Therefore, this paper only considers the self-balance high-order process with no zero points and RHPs.

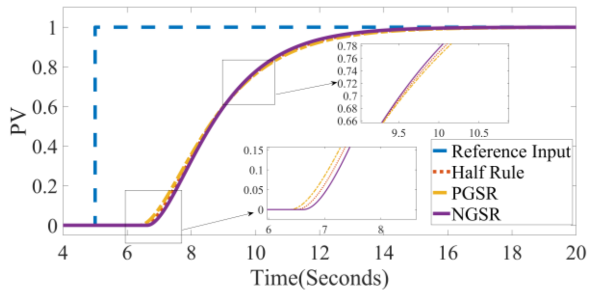


Fig. 1. Comparison of the step response between GSR method and Half Rule to simplify the high-order system to SOPDT model.

PGSR-FOPDT:

$$\begin{aligned}\hat{G}(s) &= \frac{1}{(T_1+0.618T_2)s+1}e^{-L's} \\ L' &= L+0.382T_2+\sum_{i=3}^{n}T_i+\sum_{j=1}^{m}K_j\end{aligned} \tag{9}$$

PGSR-SOPDT:

$$\begin{aligned}\hat{G}(s) &= \frac{1}{(T_1s+1)((T_2+0.618T_3)s+1)}e^{-L's} \\ L' &= L+0.382T_3+\sum_{i=4}^{n}T_i+\sum_{j=1}^{m}K_j\end{aligned} \tag{10}$$

NGSR-FOPDT:

$$\begin{aligned}\hat{G}(s) &= \frac{1}{(T_1+0.382T_2)s+1}e^{-L's} \\ L' &= L+0.618T_2+\sum_{i=3}^{n}T_i+\sum_{j=1}^{m}K_j\end{aligned} \tag{11}$$

NGSR-SOPDT:

$$\begin{aligned}\hat{G}(s) &= \frac{1}{(T_1s+1)((T_2+0.382T_3)s+1)}e^{-L's} \\ L' &= L+0.618T_3+\sum_{i=4}^{n}T_i+\sum_{j=1}^{m}K_j\end{aligned} \tag{12}$$

From the mathematical expression of simplified model, Half Rule model parameters are between the model parameters of PGSR and NGSR, and the model simplification results can be approximated as a compromise between PGSR and NGSR, which explains the phoneme that the step response of Half Rule simplified model is surrounded by the step response of PGSR and NGSR simplified models to some extent. The symmetry between the PGSR and NGSR parameters allows the two models to behave differently at different stages of the step response. Generally speaking, the short delay time of PGSR means faster tracking performance, while the constant time of NGSR simplified model is similar to original model, which means NGSR model better at imitating the dynamic of real plant.

2.3 Evaluation Index

Use the following two indicators to evaluate similarity between simplified model with original process model

(1) Maximum Absolute Error (MAE)

Indicates the maximum error between the simplified model and the original model.

$$MAE = \max(y_s - y) \tag{13}$$

Where y_s and y represent the output response of the same step signal input for the simplified model and the real model, respectively.

(2) Rate of Loss (RoL)

Indicates the weight of the error band in the step response output curve, calculated as follows,

$$RoL = \frac{\int_0^t y(\tau)d\tau}{\int_0^t |y(\tau)-y_s(\tau)|d\tau} \tag{14}$$

2.4 Simulations

Example E1:

$$G_1(s) = \frac{1}{(s+1)^4} \tag{15}$$

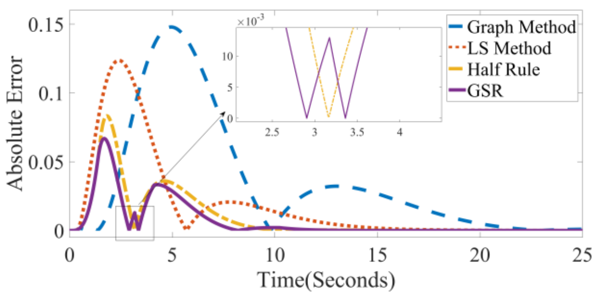
Example E2:

$$G_2(s) = \frac{1}{(s+1)(0.1s+1)(0.01s+1)(0.001s+1)} \tag{16}$$

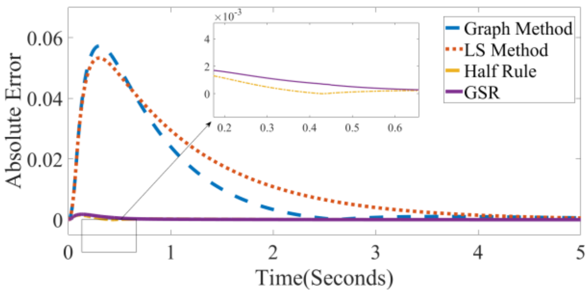
Example E3:

$$G_3(s) = \frac{1}{(s+1)^4}e^{-5s} \tag{17}$$

As the figure 2 shows, Half Rule and GSR are much better than Graph Method and LS Method while ensuring simple operation.



(a)



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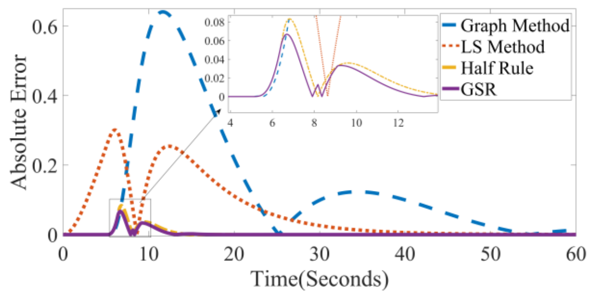


(c)

Fig. 2. Comparison of the absolute error between GSR method and traditional method to simplify the high-order system to SOPDT model.

As the figure 2(b) presents, half-rule and GSR behave almost the same in lag-dominant process, in other cases, GSR performs better than Half Rule.

The MAE and RL of approximation models are given in Table 1.

Table 1. MAE and RoL of simplified models.

Example	Method	MAE	RoL
E1	Graph Method	0.1479	0.029
	LS Method	0.1236	0.0149
	Half Rule	0.0833	0.0073
	GSR	0.0668	0.0055
E2	Graph Method	0.0573	0.0023
	LS Method	0.0533	0.0029
	Half Rule	0.0015	2.76×10-5
	GSR	0.0018	3.47×10-5
E3	Graph Method	0.6414	0.2080
	LS Method	0.3011	0.0998
	Half Rule	0.0833	0.0055
	GSR	0.0668	0.0041

According to Table 1, MAE and RL of GSR are reduced appreciably, verify the effectiveness of the GSR. Considering the two aspects of calculation and fitting accuracy, the comprehensive performance of GSR is the best.

3. CONTROLLER DESIGN

The controller's tuning uses SIMC as the basis, the meanings of symbols appeared in Table 2 are as follows, critical parameters k, τ_1, τ_2, L come from simplified SOPDT model, and k_c, τ_I are instrument variables derived from IMC-PID.

$$k_c = 0.5\tau_1 / kL \tag{18}$$

$$\tau_I = \min\{\tau_1, 8L\} \tag{19}$$

$$u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{de(t)}{dt} \tag{20}$$

Equation (18) and (19) illustrate the computation process of instrument variables. In this paper, standard PID controller structure is used and the mathematical form is described as Equation (20), in which $u(t)$ is manipulate variable, $e(t)$ is the deviation between setpoint and plant variable, K_P, K_I, K_D are controller gains.

SIMC Tuning Rule is listed briefly in Table 2,

Table 2. SIMC Tuning Rule.

Parameter	P	I	D
PI	k_c	k_c/τ_I	-
PID	$k_c(\tau_1+\tau_2)/\tau_I$	k_c/τ_I	$0.5\tau_1\tau_2/kL$

3.1 Simulation

The models in Section 2.4 are used for verification, controller parameters are given in Table 3-5.

Table 3. PID parameters tuned for E1.

Parameters	K	Ki	Kd
Z-N	1.2	0.3	1.2
SIMC	0.83	0.33	0.5
PGSR	0.947	0.362	0.585
NGSR	0.736	0.309	0.427

Table 4. PID parameters tuned for E2.

Parameters	K	Ki	Kd
Z-N	12.687	21.471	1.638
SIMC	4.72	10.579	0.527
PGSR	389	2690	11.015
NGSR	195	1212	7.229

Table 5. PID parameters tuned for E3.

Parameters	K	Ki	Kd
Z-N	0.343	0.024	1.2
SIMC	0.19	0.077	0.12
PGSR	0.205	0.078	0.127
NGSR	0.179	0.076	0.104

The simulation results compared with the traditional design methods are shown in figure 3.

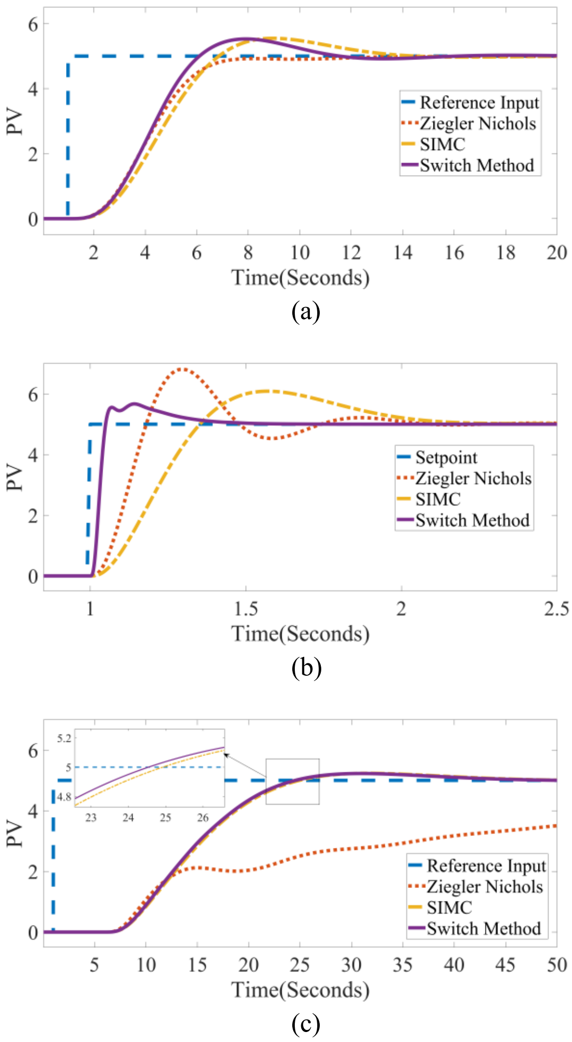


Fig. 3. Performance comparison of different design methods for high-order systems.

It can be seen from figure 3 that the Switch Method has achieved good performance in all three experiments. The accurate statistic of overshoot and raising time comparisons are given in Table 6.

Table 6. Controller dynamic performance comparison.

Process	Overshoot(%)			Raising Time(s)		
	ZN	SIMC	SM	ZN	SIMC	SM
E1	0	10.9	10.5	8.0	6.9	6.1
E2	36.2	21.7	13.3	1.2	1.3	1.1
E3	-	4.5	4.5	-	25.3	24.5

The data shows that the overshoot of Z-N changes drastically and the controller tuning via Z-N cannot guarantee controllability; rise time of SIMC and SM is much shorter, and S-M has a relatively small overshoot.

3.2 *Analysis*

For the delay-dominated system and the balanced system, the design described above is effective, and there are certain advantages in overshoot and adjustment time. For the lag-dominated system, the effect is very significant, but the gains are too large. For this model, Simplify the original model to FOPDT model according to NGSR, then tune PI controller parameters directly. The modified parameters are given in Table 7.

Table 7. PI controller tuning based on PGSR and NGSR.

Parameters	K	Ki	Kd
PGSR	10.731	27.42	-
NGSR	7.13	12.24	-

Simulation result shows in fig.4.

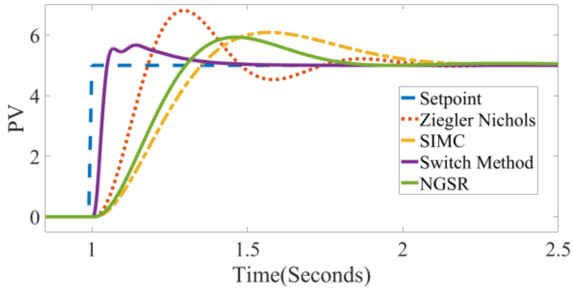


Fig. 4. Performance comparison of PI with others.

From figure 4, the PI controller based on the NGSR simplified model achieves a balance of absolute parameters value and dynamic performance.
When using this scheme for controller design, thanks to the simple rules and light calculations, we can pay more attention on the characteristics of the simplified system, it is significant to obtain a PID controller with good performance.
The steps can be summarized as follows,
Step l: Model Reduction
(1) Simplify original model to SOPDT system
(2) Modify simplified model, if FOPDT also works
Step 2: Controller Design
(1) Controller structure is determined by the simplified model structure, the SOPDT corresponds to the PID controller and the FOPDT corresponds to the PI controller
(2) Adjust controller parameters according to tuning rule

4. DISCUSSION

4.1 *Special case*

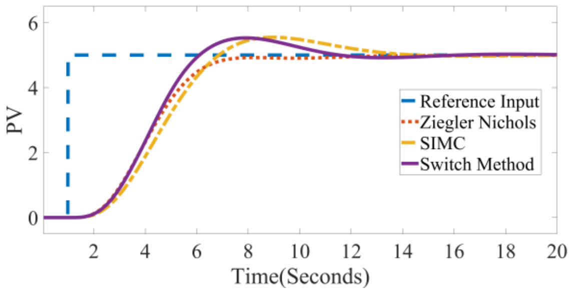
This section discusses the ability of control strategies to resist model parameter drift. Consider the following extremes firstly,

$$G(s)=\frac{1}{(5s+1)(3s+1)(2s+1)(s+1)}e^{-s} \quad (21)$$

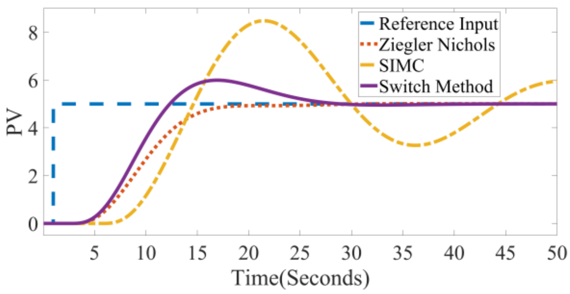
Drift Parameter Model

$$\tilde{G}(s)=\frac{1}{(5s+1)(3s+1)(2s+1)(0.5s+1)}e^{-1.5s} \quad (22)$$

Figure 5 presents the comparison of control performance before and after the parameter drift occurs. Figure 5(a) shows the normal occasion, model parameters are matched with real plant well. Figure 5(b) shows that using the GSR proposed in this paper, the model drift occurred in special case has no effect on simplified model while SIMC method results in unsatisfactory oscillation. The models come from a real chemical system and has been moderately simplified for convenience of explanation.



(a)



(b)

Fig. 5. Influence of parameter drift. Comparison of control performance of different controller design methods under the normal (a) and special parameter drift situations (b).

4.2 General case

Consider a more general case, when one of the parameters is 10% floating, for example,

$$G(s)=\frac{1}{(s+1)^4}\Rightarrow \tilde{G}(s)=\frac{1}{(1.1s+1)(s+1)^3} \quad (23)$$

Simulation result is as follows:

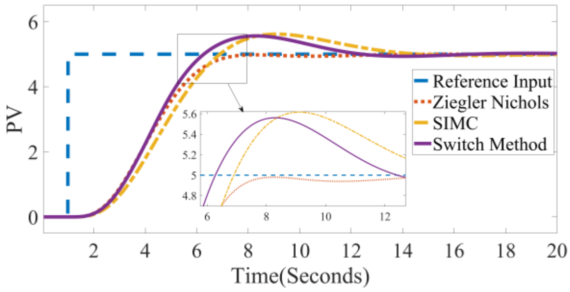


Fig. 6. Comparison under a random chosen condition.

The experiments result shows that Switch Method proposed in this paper can cope with the problem of delay-time and lag-time drift to a certain extent, the robustness of controller is improved, and the performance is acceptable. The static gain drift is not included in, due to the severe impact on system dynamic, and in this case, the gain of the controller should be scaled accordingly to achieve a relatively ideal effect.

5. CONCLUSIONS

Model reduction is a practical and simple method to solve high-order systems control problem, it has been reflected in SIMC, PFC and other design methods. Following this idea, we propose an adaptive model reduction method called GSR (Golden Section Reduction Rule) combined with the golden section ratio, less precision loss could be achieved compared with previous method. Then PID-based controller with switching control strategy could implement on the simplified model.

In simulation section, the design method is verified by several representative examples. It is worth noting that the golden section ratio is not the optimal option, a trade-off between accuracy and computational complexity is explored with adaptive technique in this work. The optimal and universal model reduction scheme and corresponding controller design method will be the direction of our future research.

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