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ICL-Based Adaptive Switching Control Strategy for Aircraft Following Change in System Dynamics ⋆

Ryan A. Licitra * **Scott A. Nivison** ** **Emily A. Doucette** **

* *NRC Postdoctoral Research Associate, Air Force Research Laboratory, Munitions Directorate, 1350 Poquito Rd N, Shalimar, FL 32579 USA (e-mail: rlicitra@ufl.edu).*
** *Air Force Research Laboratory, Munitions Directorate, 101 W. Eglin Blvd., Eglin AFB, FL 32542 USA (e-mail: scott.nivison@us.af.mil, emily.doucette@us.af.mil).*

Abstract: This paper investigates a switching control strategy for a fixed-wing flight vehicle subject to a dramatic change in dynamics. When a system-altering fault occurs, the vehicle switches from a fixed-gain, model-based controller to an adaptive control strategy that uses recorded data to identify the current system online. The adaptive controller employs a data-driven integral concurrent learning scheme to estimate the mass properties, as well as lift and drag coefficients online in an effort to match the faulted system as closely as possible with cataloged flight conditions. Stability is proven to be preserved even through failed attempts to switch from the adaptive controller back to a model-based controller, as long as the developed dwell-time and finite excitation conditions for the adaptive subsystem are satisfied.

Keywords: Adaptive control, concurrent learning, fixed-wing aircraft, neural networks, switched systems.

1. INTRODUCTION

Autonomous control of fixed-wing aircraft systems have been relevant to various applications for many years, including those in commercial and military settings. Now, with modern computing power on the rise, research in this area is more prevalent than ever. Among the many methods used in the stabilization of aircraft systems, various direct and indirect adaptive control methods have been thoroughly investigated and successfully flown on flight vehicles as seen in An et al. (2017); Fiorentini et al. (2009a,b); Gregory et al. (2009); Wise et al. (2008); Xu et al. (2014). Furthermore, with the recent resurgence of neural network (NN) estimation techniques, a number of results that leverage them have emerged as well (Lee and Kim (2001); Li et al. (2019); Nivison and Khargonekar (2017, 2018); Xu et al. (2011, 2013, 2016, 2018)).

A number of results employ an estimation strategy to determine the aerodynamic coefficients needed by the controller. The authors in Wise et al. (2008) employ an $\mathcal{L}_1$ adaptive control-based strategy to demonstrate stability for two common flight control applications. The result in Gregory et al. (2009) uses $\mathcal{L}_1$ adaptive control as well in a NASA flight test study. In Fiorentini et al. (2009a,b); Xu et al. (2014), the authors prove stability using traditional adaptive control to estimate the uncertainties in linearly parameterized form. The result in An et al. (2017) employs a similar strategy using a barrier Lyapunov function-based approach and assuming a small angle approximation on flight path angle.

The authors in Lee and Kim (2001) use a back-stepping strategy with NN estimators to stabilize a 6-DoF (degree of freedom) aircraft model under the assumption that it is held at a constant velocity. In Xu et al. (2011, 2013), the authors utilize NN estimates of the aerodynamics functions in a discrete-time controller to stabilize 3-DoF aircraft systems. Similar to this paper, the results in Xu et al. (2016, 2018) use NNs to estimate faulted system dynamics and prove stability via the small-gain theorem (Xu et al. (2016)) and using a barrier Lyapunov function-based strategy (Xu et al. (2018)). A fuzzy-approximation-based approach is used in Li et al. (2019) to guarantee stability and robustness of an air-breathing aircraft system with input constraints.

Additionally, the authors in Nivison and Khargonekar (2017, 2018) developed a sparse neural network (SNN) framework in adaptive control which focuses on encouraging local learning and reducing computational complexity through intelligent switching and segmentation of the state-space. The sparsification techniques enable local approximations across the segments which efficiently characterize regions of the state-space with significant varying dynamics and creates a global result.

Some results rely on high-gain or high-frequency robust control strategies to deal with the uncertainties in a nonlinear aircraft system, in lieu of estimation methods.

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In Wilcox et al. (2010), the authors utilize high-frequency control to compensate for bounded disturbances, yielding exponential stability of an aircraft system considering aerothermoelastic effects. The authors in MacKunis et al. (2008) employ a a controller that leverages a robust integral of the signum of the error (RISE) term along with adaptive dynamic inversion techniques to show asymptotic stability for the aircraft system.

In this paper, the longitudinal dynamics of a fixed-wing aircraft are assumed to be stabilized initially via a model-based (nominal) controller that relies on look-up tables to find the fixed-gains of an optimal controller for specific flight conditions (see Lavretsky and Wise (2013)). Optimal control is often preferred for flight applications, as many trajectory-following guidance approaches rely on energy efficient and preserving solutions, traits that adaptive methods usually can't offer. It is further assumed that a fault occurs at some point in time, causing the system to become unstable, and prompting the switch to an adaptive controller, which is developed in this paper using neural network estimators to approximate the unknown coefficients online in situations where lookup tables may become unreliable or inaccurate. The weights of the NNs are updated in real time using an integral-based concurrent learning (ICL) (see Parikh et al. (2019)) scheme that is designed to improve parameter estimation and facilitate the switched systems analysis. The ICL strategy only requires a finite excitation (FE) condition to be met to ensure weight parameter convergence, which can be verified online, in contrast with the persistence of excitation (PE) condition often associated with adaptive control, which can be difficult to satisfy and cannot generally be verified.

The main contribution of this paper is in the use of a switching system to help facilitate the stabilization of a faulted aircraft system, as well as in the implementation of the ICL scheme which is shown to improve the performance of the NN estimators. The stability analysis in Section 5 establishes an exponentially decaying bound on the Lyapunov function (a function of squared system error signals) facilitated by the ICL-based weight update laws. This bound is used along with similar bounds that are assumed for the nominally controlled (stable) system and a faulted (unstable) system to determine an overall stable error bound on the switching system.

2. PROBLEM FORMULATION

Consider a fixed-wing aircraft that may experience a physical fault while tracking a prescribed trajectory. This fault could be caused by partial mechanical failures as well as physical changes to the air frame. The now faulted aircraft system, initially stabilized by a given nominal controller (which will not be detailed in this paper), may begin to go unstable.

Once this fault occurs and the system states begin to go unstable, the nominal controller is rendered ineffective [1] and an alternative policy is necessary to re-stabilize the aircraft. The main objective of this paper is to design a simultaneous learning and control strategy that approximates the aerodynamic coefficients online and re-stabilizes the system. Once the system is re-stabilized and the uncertain coefficients are sufficiently learned, the adaptive model may be fed back into the nominal controller to again yield improved performance. This idea motivates the use of a switched systems analysis to ensure overall system stability through physical changes (faults) and policy changes (adaptation to the faults).

[1] If an aircraft controller is designed to utilize look-up tables for aerodynamic data, it will likely fail when the data no longer accurately represents the physical system.

2.1 Aircraft Control Model

The longitudinal 3-DoF equations of motion for a fixed-wing flight vehicle are given by Stevens and Lewis (2003); Bolender and Doman (2007); Dickinson et al. (2015) as

$$\dot{V}_T = \frac{1}{m}\left(T\cos\left(\alpha\right) - D\right) - g\sin\left(\theta - \alpha\right) \tag{1}$$

$$\dot{\alpha} = \frac{1}{mV_T}\left(-T\sin\left(\alpha\right) - L\right) + q + \frac{g}{V_T}\cos\left(\theta - \alpha\right) \tag{2}$$

$$\dot{\theta} = q \tag{3}$$

$$\dot{q} = \frac{M}{I_{YY}} \tag{4}$$

$$\dot{h} = V_T\sin\left(\theta - \alpha\right), \tag{5}$$

where m is the mass of the vehicle, I_{YY} is the moment of inertia, T is the thrust, and D, L, and M are the drag force, lift force, and moment that act upon the aircraft. The measurable state vector, $x \in \mathbb{R}^5$, is defined as

$$x \triangleq \left[V_T \;\alpha\; \theta\; q\; h\right],$$

where $V_T \in \mathbb{R}^+$ is the true airspeed, $\alpha \in \mathbb{R}$ is the angle of attack (AOA), $\theta \in \mathbb{R}$ is the pitch angle, $q \in \mathbb{R}$ is the pitch rate, and $h \in \mathbb{R}^+$ is the altitude. The aerodynamic forces and moment in (1)-(5) are approximated in terms of their aerodynamic coefficients as (see Stevens and Lewis (2003); Dickinson et al. (2015))

$$\begin{aligned} T &\approx \bar{q}S_D C_T \\ D &\approx \bar{q}SC_D \\ L &\approx \bar{q}SC_L \\ M &\approx \bar{q}Sc_{ref}C_M, \end{aligned}$$

where $\bar{q} \in \mathbb{R}^+$ is dynamic pressure, defined as $\bar{q} \triangleq \frac{1}{2}\rho V_T^2$, $\rho \in \mathbb{R}^+$ is the measurable air density, $c_{ref} \in \mathbb{R}^+$ is the mean aerodynamic chord, and $S \in \mathbb{R}$ and $S_D \in \mathbb{R}$ are known geometric properties of the aircraft. The coefficients themselves are written in terms of the individual contributions from each force or moment from each state, and are defined as

$$\begin{aligned} C_T &= C_{T_0}\left(\alpha, Ma\right) + C_{T_{V_T}}\left(\alpha, Ma\right) + C_{T_{\delta_T}}\left(\alpha, Ma\right)\delta_T \\ C_D &= C_{D_0}\left(\alpha, Ma\right) + C_{D_\alpha}\left(\alpha, Ma\right) \\ &\quad + C_{D_{V_T}}\left(\alpha, Ma\right) + C_{D_{\delta_e}}\left(\alpha, Ma\right) \\ C_L &= C_{L_0}\left(\alpha, Ma\right) + C_{L_\alpha}\left(\alpha, Ma\right) \\ &\quad + C_{L_{V_T}}\left(\alpha, Ma\right) + C_{L_{\delta_e}}\left(\alpha, Ma\right) \\ C_M &= C_{M_0}\left(\alpha, Ma\right) + C_{M_\alpha}\left(\alpha, Ma\right) \\ &\quad + C_{M_{V_T}}\left(\alpha, Ma\right) + C_{M_{\delta_e}}\left(\alpha, Ma\right)\delta_e, \end{aligned} \tag{6}$$

where $Ma \in \mathbb{R}^+$ is the Mach number, and the control inputs $\delta_T \in \mathbb{R}$ and $\delta_e \in \mathbb{R}$ are assumed to have the linear relationship shown with their respective coefficients. Let

$$\mathcal{F}_C \triangleq \Big\{T_0, T_{V_T}, T_{\delta_T}, D_0, D_\alpha, D_{V_T}, D_{\delta_e}, ... \\ L_0, L_\alpha, L_{V_T}, L_{\delta_e}, M_0, M_\alpha, M_{V_T}, M_{\delta_e}\Big\} \quad (7)$$

define a set containing all of the coefficient subscripts in (6).

Assumption 1. The aircraft system in 1-5 experiences and automatically detects [2] a system change (i.e., a fault) that causes the look-up tables used by the nominal controller to become unreliable.

2.2 Switched Systems Notation

Let $t_0^f \in \mathbb{R}^+$ represent the first time that a fault occurs. Because the proposed control strategy involves the given system switching between stable and unstable subsystems, a switched systems analysis will be carried out to ensure that the subsequent control strategy can be shown to successfully stabilize the system in (1)-(5), even if the system switches multiple times. The key to ensuring overall system stability is to ensure that the switches occur such that the subsequently developed sufficient dwell-time conditions are satisfied. To facilitate this, let $t_k^a \in \mathbb{R}^+$, $t_k^s \in \mathbb{R}^+$, and $t_k^f \in \mathbb{R}^+$ denote the k^{th} instance in time that the system is switched to the adaptive (controller designed in this paper), nominal (assumed stable controller), and faulted (unstable) mode, respectively, where $k \in \mathbb{N}$. The contiguous dwell-times in the k^{th} activation of the system operating in the adaptive, nominal, and faulted mode are denoted by $\Delta t_k^a \in \mathbb{R}^+$, $\Delta t_k^s \in \mathbb{R}^+$, and $\Delta t_k^f \in \mathbb{R}^+$, and defined as $\Delta t_k^a \triangleq t_k^s - t_k^a$, $\Delta t_k^s \triangleq t_k^f - t_k^s$, and $\Delta t_k^f \triangleq t_{k+1}^a - t_k^f$, respectively.

3. CONTROL OBJECTIVE AND FUNCTION APPROXIMATION

The focus of the following development is to design a controller for the system in (1)-(5) to track predetermined [3] velocity and AOA trajectories. The velocity error, $e_V \in \mathbb{R}$, is introduced as

$$e_V \triangleq V_T - V_r, \quad (8)$$

where $V_r \in \mathbb{R}$ is the reference velocity [4]. Similarly, the AOA error signal $e_\alpha \in \mathbb{R}$ is defined as

$$e_\alpha \triangleq \alpha - \alpha_r, \quad (9)$$

where $\alpha_r \in \mathbb{R}$ is the reference AOA signal.

Because the moment term (M) used to control the pitch of the aircraft only appears in the pitch rate dynamics in (4), a back-stepping strategy is necessary to track a desired AOA reference trajectory. To this end, a pitch error signal, $e_q \in \mathbb{R}$, is introduced as

$$e_q \triangleq q - q_d, \quad (10)$$

where $q_d \in \mathbb{R}$ is a desired pitch signal that will be subsequently designed as a virtual control input to the AOA error system in (9).

[2] See Xu et al. (2018); Alwi and Edwards (2008); Hwang et al. (2009) for fault detection methods and analysis.

[3] In practice, these desired trajectories could be generated online.

[4] The reference signals V_r and α_r, and their first derivatives $\dot{V}_r$ and $\dot{\alpha}_r$, are assumed to be known and bounded.

Furthermore, the various uncertain terms that appear in the nonlinear system in (1)-(5) will be approximated using NNs. In the interest of space, the exact definition of each unknown function will not be listed, however, the majority of these functions follow the form

$$f_j = \frac{C_j}{m_j}, \quad (11)$$

for all $j \in \mathcal{F}_C$, where the numerator is a (lift/drag/moment) coefficient with a subscript from (7) ($C_{L_0}, C_{T_{\delta_T}}$, etc.) and the denominator is the relevant mass property (mass, m, for the lift and drag coefficients, and moment of inertia, I_{YY}, for the moment coefficients). Each of these functions will be estimated by a NN (see Lewis (1999)) as [5]

$$f_j(x) = W_j^T \sigma_j(x) + \epsilon_j(x), \quad (12)$$

where, for all $j \in \mathcal{F}_C$, $f_j : \mathbb{R}^5 \to \mathbb{R}$ are the functions containing the unknown aerodynamic coefficients and mass properties, $\sigma_j : \mathbb{R}^5 \to \mathbb{R}^{L_j}$ are known, bounded, locally Lipschitz, vectors of basis functions, $W_j \in \mathbb{R}^{L_j}$ are vectors of the unknown ideal weights, $L_j \in \mathbb{N}$ is the number of neurons [6] used in the NN in (12), and $\epsilon_j : \mathbb{R}^5 \to \mathbb{R}$ are the function approximation residual errors.

Remark 1. The function approximation residual errors can be upper bounded by positive constants that can be made arbitrarily small based on the Stone-Weierstrass theorem (see Cotter (1990)), i.e., $\bar{\epsilon}_j \triangleq \sup_{x\in\chi, t\in[0,\infty)} \|\epsilon_j(x(t))\|$, $\forall j \in \mathcal{F}_C$. The Stone–Weierstrass theorem requires that the states remain in a compact set (i.e., $x(t) \in \chi$). The stability and switching analyses in Sections 5 and 6 show that if $x(0)$ is bounded, then $x(t) \in \chi$, where χ is a compact simply connected set such that $\chi \subset \mathbb{R}^5$.

Let [7] $\tilde{W}_j(t) \triangleq W_j - \hat{W}_j(t)$ denote the parameter estimation error for the weights associated with the j^{th} element in the function set $\mathcal{F}_C$, where $\hat{W}_j \in \mathbb{R}^{L_j}$ is the estimate of the ideal function approximation weight vector associated with the j^{th} function.

4. CONTROLLER DEVELOPMENT

The following development introduces an adaptive controller that stabilizes the uncertain nonlinear system in (1)-(5).

4.1 Velocity Controller

Taking the time derivative of (8) and using (1) and the notation in (11), the open-loop aircraft velocity error dynamics can be written as

[5] For notational brevity, functional dependence on states and time will be henceforth suppressed, except for when introducing new terms and where necessary for clarity.

[6] With reasonable knowledge of the operating domain, most modern, consumer-grade computers (including the ones commonly mounted to a quadcopters) are capable of meeting the computational requirements for these estimators to function.

[7] For the purposes of notational clarity in later sections, any defined NN weight error $\tilde{W}_j$ has an associated ideal weight W_j and weight estimate $\hat{W}_j$, for all $j \in \mathcal{F}_C$.

$$\dot{e}_V = \bar{q}S_D \cos(\alpha)\left(f_{T_0} + f_{T_{V_T}} + f_{T_{\delta_T}}\delta_T\right) - \bar{q}S\left(f_{D_0} + f_{D_\alpha} + f_{D_{V_T}} + f_{D_{\delta_e}}\right) + F_V, \qquad (13)$$

where the lowercase $f_{(\cdot)}$ functions are defined by the form in (11), and $F_V = -g\sin(\theta-\alpha) - \dot{V}_r$ contains the measurable quantities in the velocity dynamics in (1). Using the NN approximation form in (12), (13) can be rewritten as

$$\dot{e}_V = \bar{q}S_D\cos(\alpha)\Bigg(W_{T_0}^T\sigma_{T_0} + \epsilon_{T_0} + W_{T_{V_T}}^T\sigma_{T_{V_T}} + \epsilon_{T_{V_T}} + \left(W_{T_{\delta_T}}^T\sigma_{T_{\delta_T}} + \epsilon_{T_{\delta_T}}\right)\delta_T\Bigg) - \bar{q}S\Bigg(W_{D_0}^T\sigma_{D_0} + \epsilon_{D_0} + W_{D_\alpha}^T\sigma_{D_\alpha} + \epsilon_{D_\alpha} + W_{D_{V_T}}^T\sigma_{D_{V_T}} + \epsilon_{D_{V_T}} + W_{D_{\delta_e}}^T\sigma_{D_{\delta_e}} + \epsilon_{D_{\delta_e}}\Bigg) + F_V. \qquad (14)$$

Furthermore, by adding and subtracting the term $\bar{q}S_D\cos(\alpha)\hat{W}_{T_{\delta_T}}^T\sigma_{T_{\delta_T}}\delta_T$ and rearranging, (14) becomes [8]

$$\dot{e}_V = \tilde{W}_{T_{\delta_T}}^T Y_{T_{\delta_T}} + W_{T_*}^T Y_{T_*} + W_D^T Y_D + W_{\epsilon\delta_T}^T Y_{\epsilon\delta_T} + E_{T_*} + E_D + E_{\epsilon\delta_T} + F_V + \bar{q}S_D\cos(\alpha)\hat{W}_{T_{\delta_T}}^T\sigma_{T_{\delta_T}}\delta_T. \qquad (15)$$

This enables the design of the thrust controller δ_T as

$$\delta_T \triangleq \frac{1}{\bar{q}S_D\cos(\alpha)}\frac{1}{\hat{W}_{T_{\delta_T}}^T\sigma_{T_{\delta_T}}}\Bigg(-K_V e_V - F_V - \hat{W}_{T_*}^T Y_{T_*} - \hat{W}_D^T Y_D - \hat{W}_{\epsilon\delta_T}^T Y_{\epsilon\delta_T}\Bigg), \qquad (16)$$

where $K_V(V_T) \triangleq k_{V1} + k_{V2}(\bar{q}S_D)^2 + k_{V3}(\bar{q}S)^2$, and $k_{V1}, k_{V2}, k_{V3} \in \mathbb{R}^+$ are positive constant control gains. Substituting (16) into (15) yields the closed-loop velocity error system

$$\dot{e}_V = -K_V e_V + \tilde{W}_1^T Y_1 + E_{T_*} + E_D + E_{\epsilon\delta_T}, \qquad (17)$$

where

$$\tilde{W}_1 \triangleq \left[\tilde{W}_{T_0}^T\ \tilde{W}_{T_{V_T}}^T\ \tilde{W}_{T_{\delta_T}}^T\ \tilde{W}_{D_0}^T\ \tilde{W}_{D_\alpha}^T\ \tilde{W}_{D_{V_T}}^T\ \tilde{W}_{D_{\delta_e}}^T\ \tilde{W}_{\epsilon\delta_T}^T\right]^T = \left[\tilde{W}_T^T\ \tilde{W}_D^T\ \tilde{W}_{\epsilon\delta_T}^T\right]^T$$

and

$$Y_1 \triangleq \left[Y_{T_0}^T\ Y_{T_{V_T}}^T\ Y_{T_{\delta_T}}^T\ Y_{D_0}^T\ Y_{D_\alpha}^T\ Y_{D_{V_T}}^T\ Y_{D_{\delta_e}}^T\ Y_{\epsilon\delta_T}^T\right]^T = \left[Y_T^T\ Y_D^T\ Y_{\epsilon\delta_T}^T\right]^T.$$

4.2 AOA Controller

The open-loop AOA error dynamics are obtained by taking the time derivative of (2) to yield

$$\dot{e}_\alpha = q + f_\alpha - \frac{\bar{q}S}{V_T}\left(f_{L_0} + f_{L_\alpha} + f_{L_{V_T}} + f_{L_{\delta_e}}\right) + F_\alpha, \qquad (18)$$

where $f_\alpha = -\frac{T\sin(\alpha)}{mV_T}$, the remaining lowercase $f_{(\cdot)}$ functions are defined by the form in (11), and $F_\alpha = \frac{g}{V_T}\cos(\theta-\alpha) - \dot{\alpha}_r$ contains the measurable terms in (2). The expression in (18) can be rewritten using the NN-based approximation form in (12) to yield

$$\dot{e}_\alpha = q + W_\alpha^T\sigma_\alpha + \epsilon_\alpha - \frac{\bar{q}S}{V_T}\Bigg(W_{L_0}^T\sigma_{L_0} + \epsilon_{L_0} + W_{L_\alpha}^T\sigma_{L_\alpha} + \epsilon_{L_\alpha} + W_{L_{V_T}}^T\sigma_{L_{V_T}} + \epsilon_{L_{V_T}} + W_{L_{\delta_e}}^T\sigma_{L_{\delta_e}} + \epsilon_{L_{\delta_e}}\Bigg) + F_\alpha. \qquad (19)$$

After adding and subtracting the desired pitch q_d, the expression in (19) can be rearranged and rewritten as

$$\dot{e}_\alpha = q_d + e_q + W_2^T Y_2 + E_\alpha + E_L + F_\alpha, \qquad (20)$$

where

$$\tilde{W}_2 \triangleq \left[\tilde{W}_{L_0}^T\ \tilde{W}_{L_\alpha}^T\ \tilde{W}_{L_{V_T}}^T\ \tilde{W}_{L_{\delta_e}}^T\ \tilde{W}_\alpha^T\right]^T = \left[\tilde{W}_L^T\ \tilde{W}_\alpha^T\right]^T$$

and

$$Y_2 \triangleq \left[Y_{L_0}^T\ Y_{L_\alpha}^T\ Y_{L_{V_T}}^T\ Y_{L_{\delta_e}}^T\ Y_\alpha^T\right]^T = \left[Y_L^T\ Y_\alpha^T\right]^T.$$

In order to regulate the back-stepping error, the virtual control input q_d is designed as

$$q_d = -K_\alpha e_\alpha - \hat{W}_2^T Y_2 - F_\alpha, \qquad (21)$$

where $K_\alpha(V_T) \triangleq k_{\alpha1} + k_{\alpha2} + k_{\alpha3}\left(\frac{1}{2}\rho S V_T\right)^2$, and $k_{\alpha1}, k_{\alpha2}, k_{\alpha3} \in \mathbb{R}^+$ are positive constant control gains. Using (21), (20) is rewritten as

$$\dot{e}_\alpha = -K_\alpha e_\alpha + e_q + \tilde{W}_2^T Y_2 + E_\alpha + E_L. \qquad (22)$$

4.3 Pitch Controller

In order for the overall control system to be stable, the back-stepping error introduced in Section 3 must be regulated as well. Taking the time derivative of (10), the open-loop pitch (back-stepping) dynamics are written as

$$\dot{e}_q = \bar{q}Sc_{ref}\left(f_{M_0} + f_{M_\alpha} + f_{M_{V_T}} + f_{M_{\delta_e}}\delta_e\right) - f_q - F_q, \qquad (23)$$

where F_q and f_q contain the known and unknown terms that arise from taking the time derivative of (21), respectively, and the remaining lowercase $f_{(\cdot)}$ functions are defined by the form in (11). After substituting the NN approximations of the form in (12), (23) can be rewritten as

$$\dot{e}_q = \bar{q}Sc_{ref}\Bigg(W_{M_0}^T\sigma_{M_0} + \epsilon_{M_0} + W_{M_\alpha}^T\sigma_{M_\alpha} + \epsilon_{M_\alpha} + W_{M_{V_T}}^T\sigma_{M_{V_T}} + \epsilon_{M_{V_T}} + \left(W_{M_{\delta_e}}^T\sigma_{M_{\delta_e}} + \epsilon_{M_{\delta_e}}\right)\delta_e\Bigg) - \left(W_q^T\sigma_q + \epsilon_q\right) - F_q. \qquad (24)$$

Using a strategy similar to that in Section 4.1, adding and subtracting the term $\bar{q}Sc_{ref}\hat{W}_{M_{\delta_e}}^T\sigma_{M_{\delta_e}}\delta_e$ enables (24) to be rearranged as [9]

$$\dot{e}_q = \tilde{W}_{M_{\delta_e}}^T Y_{M_{\delta_e}} + W_{M_*}^T Y_{M_*} + W_q^T Y_q + W_{\epsilon\delta_e}^T Y_{\epsilon\delta_e} + E_{M_*} + E_q + E_{\epsilon\delta_e} - F_q + \bar{q}Sc_{ref}\hat{W}_{M_{\delta_e}}^T\sigma_{M_{\delta_e}}\delta_e, \qquad (25)$$

where $Y_{M_{\delta_e}} \triangleq \bar{q}Sc_{ref}\sigma_{M_{\delta_e}}\delta_e$, $Y_{M_*} \triangleq \bar{q}Sc_{ref}\left[\sigma_{M_0}^T\ \sigma_{M_\alpha}^T\ \sigma_{M_{V_T}}^T\right]^T$, $Y_q \triangleq -\sigma_q$, $Y_{\epsilon\delta_e} \triangleq \bar{q}Sc_{ref}\sigma_{\epsilon\delta_e}$,

[8] The term $\epsilon_{T_{\delta_T}}\delta_T$ is estimated by a NN as $f_{\epsilon\delta_T} = \epsilon_{T_{\delta_T}}\delta_T = W_{\epsilon\delta_T}^T\sigma_{\epsilon\delta_T} + \epsilon_{\epsilon\delta_T}$.

[9] The term $\epsilon_{M_{\delta_e}}\delta_e$ is estimated by a NN as $f_{\epsilon\delta_e} = \epsilon_{M_{\delta_e}}\delta_e = W_{\epsilon\delta_e}^T\sigma_{\epsilon\delta_e} + \epsilon_{\epsilon\delta_e}$.

$E_{M_*} \triangleq \bar{q}Sc_{ref}\left(\epsilon_{M_0}+\epsilon_{M_\alpha}+\epsilon_{M_{V_T}}\right)$, $E_q \triangleq -\epsilon_q$, and $E_{\epsilon\delta_e} \triangleq \bar{q}Sc_{ref}\epsilon_{\epsilon\delta_e}$. The elevator controller δ_e is then designed as

$$\delta_e \triangleq \frac{1}{\bar{q}Sc_{ref}}\frac{1}{\hat{W}^T_{M_{\delta_e}}\sigma_{M_{\delta_e}}}\left(-K_q e_q - e_\alpha + F_q - \hat{W}^T_{M_*}Y_{M_*} - \hat{W}^T_q Y_q - \hat{W}^T_{\epsilon\delta_e}Y_{\epsilon\delta_e}\right), \quad (26)$$

where $K_q(V_T) \triangleq k_{q1}+k_{q2}+k_{q3}\left(\bar{q}Sc_{ref}\right)^2$, and $k_{q1}, k_{q2}, k_{q3} \in \mathbb{R}^+$ are positive constant control gains. Using (26), the expression in (25) can be rewritten in closed-loop form as

$$\dot{e}_q = -K_q e_q - e_\alpha + \tilde{W}_3^T Y_3 + E_{M_*} + E_q + E_{\epsilon\delta_e}, \quad (27)$$

where

$$\tilde{W}_3 \triangleq \left[\tilde{W}^T_{M_0}\ \tilde{W}^T_{M_\alpha}\ \tilde{W}^T_{M_{V_T}}\ \tilde{W}^T_{M_{\delta_e}}\ \tilde{W}^T_q\ \tilde{W}^T_{\epsilon\delta_e}\right]^T = \left[\tilde{W}^T_M\ \tilde{W}^T_q\ \tilde{W}^T_{\epsilon\delta_e}\right]^T$$

and

$$Y_3 \triangleq \left[Y^T_{M_0}\ Y^T_{M_\alpha}\ Y^T_{M_{V_T}}\ Y^T_{M_{\delta_e}}\ Y^T_q\ Y^T_{\epsilon\delta_e}\right]^T = \left[Y^T_M\ Y^T_q\ Y^T_{\epsilon\delta_e}\right]^T.$$

4.4 ICL and Adaptive Update Laws

This section details the ICL function approximation scheme necessary for the implementation of the controllers designed above in Sections 4.1, 4.2, and 4.3. The following steps facilitate the development of the FE condition associated with the ICL strategy that is used below.

Rearranging (15) and integrating both sides yields

$$\int_{t-\Delta t}^{t}\dot{e}_V(\varsigma)\,d\varsigma = \int_{t-\Delta t}^{t}Y_1^T(\varsigma)\,d\varsigma W_1 + \int_{t-\Delta t}^{t}E_1(\varsigma)\,d\varsigma + \int_{t-\Delta t}^{t}F_1(\varsigma)\,d\varsigma, \quad (28)$$

where $E_1 \triangleq E_{T_*} + E_{\epsilon\delta_T} + E_D$, $F_1 \triangleq F_V$, and $\Delta t \in \mathbb{R}^+$ is a positive constant denoting the size of the window of integration. Using the Fundamental Theorem of Calculus, (28) is rewritten as

$$e_V(t) - e_V(t-\Delta t) = \mathcal{Y}_1(t)W_1 + \mathcal{E}_1(t) + \mathcal{F}_1(t), \quad (29)$$

$\forall t \in [\Delta t, \infty)$, where $\mathcal{Y}_1(t) = \int_{t-\Delta t}^{t}Y_1^T(\varsigma)\,d\varsigma$, $\mathcal{E}_1(t) = \int_{t-\Delta t}^{t}E_1(\varsigma)\,d\varsigma$, and $\mathcal{F}_1(t) = \int_{t-\Delta t}^{t}F_1(\varsigma)\,d\varsigma$. In similar fashion, (19) is rearranged and integrated to give

$$\int_{t-\Delta t}^{t}\dot{e}_\alpha(\varsigma)\,d\varsigma = \int_{t-\Delta t}^{t}Y_2^T(\varsigma)\,d\varsigma W_2 + \int_{t-\Delta t}^{t}E_2(\varsigma)\,d\varsigma + \int_{t-\Delta t}^{t}F_2(\varsigma)\,d\varsigma, \quad (30)$$

where $E_2 \triangleq E_\alpha + E_L$ and $F_2 \triangleq q + F_\alpha$. The Fundamental Theorem of Calculus enables (30) to be rewritten as

$$e_\alpha(t) - e_\alpha(t-\Delta t) = \mathcal{Y}_2(t)W_2 + \mathcal{E}_2(t) + \mathcal{F}_2(t), \quad (31)$$

where, $\forall t \in [\Delta t, \infty)$, $\mathcal{Y}_2(t) = \int_{t-\Delta t}^{t}Y_2^T(\varsigma)\,d\varsigma$, $\mathcal{E}_2(t) = \int_{t-\Delta t}^{t}E_2(\varsigma)\,d\varsigma$, and $\mathcal{F}_2(t) = \int_{t-\Delta t}^{t}F_2(\varsigma)\,d\varsigma$. Lastly, the expression in (25) is rearranged and integrated to yield

$$\int_{t-\Delta t}^{t}\dot{e}_q(\varsigma)\,d\varsigma = \int_{t-\Delta t}^{t}Y_3^T(\varsigma)\,d\varsigma W_3 + \int_{t-\Delta t}^{t}E_3(\varsigma)\,d\varsigma + \int_{t-\Delta t}^{t}F_3(\varsigma)\,d\varsigma, \quad (32)$$

where $E_3 \triangleq E_{M_*} + E_{\epsilon\delta_e} + E_q$ and $F_3 \triangleq -F_q$. Finally, applying the Fundamental Theorem of Calculus to (32) yields

$$e_q(t) - e_q(t-\Delta t) = \mathcal{Y}_3(t)W_3 + \mathcal{E}_3(t) + \mathcal{F}_3(t), \quad (33)$$

where, $\forall t \in [\Delta t, \infty)$, $\mathcal{Y}_3(t) = \int_{t-\Delta t}^{t}Y_3^T(\varsigma)\,d\varsigma$, $\mathcal{E}_3(t) = \int_{t-\Delta t}^{t}E_3(\varsigma)\,d\varsigma$, and $\mathcal{F}_3(t) = \int_{t-\Delta t}^{t}F_3(\varsigma)\,d\varsigma$.

The parameter estimates for each of the unknown weight vectors W_1, W_2, and W_3 are generated from an ICL-based adaptive update law (see Parikh et al. (2019); Bell et al. (2016); Licitra et al. (2019)). The update laws associated with the velocity error, AOA error, and pitch error are given by

$$\dot{\hat{W}}_1 \triangleq \text{proj}\left\{\Gamma_1 Y_1 e_V + k_{CL,1}\Gamma_1 \times \sum_{i=1}^{N_1}\mathcal{Y}^T_{1,i}\left(\Delta e_{V,i} - \mathcal{F}_{1,i} - \mathcal{Y}_{1,i}\hat{W}_1\right)\right\}, \quad (34)$$

$$\dot{\hat{W}}_2 \triangleq \text{proj}\left\{\Gamma_2 Y_2 e_\alpha + k_{CL,2}\Gamma_2 \times \sum_{i=1}^{N_2}\mathcal{Y}^T_{2,i}\left(\Delta e_{\alpha,i} - \mathcal{F}_{2,i} - \mathcal{Y}_{2,i}\hat{W}_2\right)\right\}, \quad (35)$$

and

$$\dot{\hat{W}}_3 \triangleq \text{proj}\left\{\Gamma_3 Y_3 e_q + k_{CL,3}\Gamma_3 \times \sum_{i=1}^{N_3}\mathcal{Y}^T_{3,i}\left(\Delta e_{q,i} - \mathcal{F}_{3,i} - \mathcal{Y}_{3,i}\hat{W}_3\right)\right\}, \quad (36)$$

respectively, where proj$\{\cdot\}$ is a smooth projection operator [10], $\Gamma_1 \in \mathbb{R}^{L_1\times L_1}, \Gamma_2 \in \mathbb{R}^{L_2\times L_2}, \Gamma_3 \in \mathbb{R}^{L_3\times L_3}$ and $k_{CL,1}, k_{CL,2}, k_{CL,3} \in \mathbb{R}^+$ are constant, positive definite control gains, $N_1, N_2, N_3 \in \mathbb{Z}$ are constants that represents the number of saved data points for the associated data stack, $\Delta e_{V,i} \triangleq e_V(t_i) - e_V(t_i - \Delta t)$, $\Delta e_{\alpha,i} \triangleq e_\alpha(t_i) - e_\alpha(t_i - \Delta t)$, $\Delta e_{q,i} \triangleq e_q(t_i) - e_q(t_i - \Delta t)$, and $t_i \in [\Delta t, t]$ represents times when data measurements are available.

The philosophy for using a concurrent learning (see Chowdhary and Johnson (2011b); Parikh et al. (2019)) scheme is based on the idea of utilizing online recorded input and output data to better identify the ideal weights in order to improve function approximation. Data points are selected in real time to be saved in a way that

[10] The limits used in the projection algorithm are based on the known bounds on the unknown functions in (1)-(5). See Section 4.4 in Dixon et al. (2003) for details of the projection operator.

maximizes the minimum eigenvalues of $\sum_{i=1}^{N_1}\left(\mathcal{Y}_{1,i}^T\mathcal{Y}_{1,i}\right)$, $\sum_{i=1}^{N_2}\left(\mathcal{Y}_{2,i}^T\mathcal{Y}_{2,i}\right)$, and $\sum_{i=1}^{N_3}\left(\mathcal{Y}_{3,i}^T\mathcal{Y}_{3,i}\right)$ [11].

For use in the stability analysis that follows in Section 5, the update laws in (34), (35), and (36) can each be rewritten in an equivalent, but non-implementable [12], form as

$$\dot{\hat{W}}_1 = \text{proj}\left\{\Gamma_1 Y_1 e_V + k_{CL,1}\Gamma_1\sum_{i=1}^{N_1}\mathcal{Y}_{1,i}^T\left(\mathcal{Y}_{1,i}\tilde{W}_1+\mathcal{E}_{1,i}\right)\right\}, \tag{37}$$

$$\dot{\hat{W}}_2 = \text{proj}\left\{\Gamma_2 Y_2 e_\alpha + k_{CL,2}\Gamma_2\sum_{i=1}^{N_2}\mathcal{Y}_{2,i}^T\left(\mathcal{Y}_{2,i}\tilde{W}_2+\mathcal{E}_{2,i}\right)\right\}, \tag{38}$$

and

$$\dot{\hat{W}}_3 = \text{proj}\left\{\Gamma_3 Y_3 e_q + k_{CL,3}\Gamma_3\sum_{i=1}^{N_3}\mathcal{Y}_{3,i}^T\left(\mathcal{Y}_{3,i}\tilde{W}_3+\mathcal{E}_{3,i}\right)\right\}, \tag{39}$$

respectively, for all $t > \Delta t$, where $\mathcal{E}_{1,i} \triangleq \mathcal{E}_1(t_i)$, $\mathcal{E}_{2,i} \triangleq \mathcal{E}_2(t_i)$, and $\mathcal{E}_{3,i} \triangleq \mathcal{E}_3(t_i)$.

5. CONTROLLER STABILITY ANALYSIS

This section details the analysis of the overall stability of the system in (1)-(5) when the controllers and update laws designed in Section 4 are implemented. The following analysis is carried out under the assumption that the following FE [13] conditions are satisfied at some point (see Parikh et al. (2019); Bell et al. (2016); Licitra et al. (2019)). The FE conditions associated with the velocity, AOA, and pitch errors are given by

$$\exists\underline{\lambda}_1 > 0, \tau_1 > \Delta t : \forall t \geq \tau_1,\ \lambda_{\min}\left\{\sum_{i=1}^{N_1}\mathcal{Y}_{1,i}^T\mathcal{Y}_{1,i}\right\} \geq \underline{\lambda}_1, \tag{40}$$

$$\exists\underline{\lambda}_2 > 0, \tau_2 > \Delta t : \forall t \geq \tau_2,\ \lambda_{\min}\left\{\sum_{i=1}^{N_2}\mathcal{Y}_{2,i}^T\mathcal{Y}_{2,i}\right\} \geq \underline{\lambda}_2, \tag{41}$$

and

$$\exists\underline{\lambda}_3 > 0, \tau_3 > \Delta t : \forall t \geq \tau_3,\ \lambda_{\min}\left\{\sum_{i=1}^{N_3}\mathcal{Y}_{3,i}^T\mathcal{Y}_{3,i}\right\} \geq \underline{\lambda}_3, \tag{42}$$

respectively, where $\lambda_{\min}\{\cdot\}$ refers to the minimum eigenvalue of $\{\cdot\}$. Because overall system stability is desired, the time at which all three of the above FE conditions have been satisfied will be denoted by $\bar{\tau} \triangleq \max\{\tau_1, \tau_2, \tau_3\}$.

To facilitate the following Lyapunov-based stability analysis, let $V : \mathbb{R}^{3+L_1+L_2+L_3} \to \mathbb{R}$ be a positive definite, continuously differentiable candidate Lyapunov function, defined as

$$V(\zeta(t)) \triangleq \frac{1}{2}e_V^2 + \frac{1}{2}e_\alpha^2 + \frac{1}{2}e_q^2 + \frac{1}{2}\text{tr}\left(\tilde{W}_1^T\Gamma_1^{-1}\tilde{W}_1\right) + \frac{1}{2}\text{tr}\left(\tilde{W}_2^T\Gamma_2^{-1}\tilde{W}_2\right) + \frac{1}{2}\text{tr}\left(\tilde{W}_3^T\Gamma_3^{-1}\tilde{W}_3\right), \tag{43}$$

which can be upper and lower bounded as

$$\beta_1\|\zeta(t)\|^2 \leq V(\zeta(t)) \leq \beta_2\|\zeta(t)\|^2, \tag{44}$$

where $\zeta(t) \in \mathbb{R}^{3+L_1+L_2+L_3}$ is the stacked state vector, defined as

$$\zeta(t) \triangleq \left[z^T\ \tilde{W}_1^T\ \tilde{W}_2^T\ \tilde{W}_3^T\right]^T,$$

$z(t) \triangleq [e_V\ e_\alpha\ e_q]^T$, $\text{tr}(\cdot)$ denotes the matrix trace operator, and $\beta_1, \beta_2 \in \mathbb{R}^+$ are known positive bounding constants.

The following Theorem proves the stability of the overall aircraft system in (1)-(5) under influence of the control strategy detailed in Section 4, once enough data has been collected to satisfy each of the FE conditions in (40)-(42) (i.e., for all $t \geq \bar{\tau}$) [14].

Theorem 1. The controllers given in (16), (21), and (26), implemented in conjunction with the adaptive update laws in (34), (35), and (36), ensure that all system signals are bounded under closed-loop operation and that, $\forall t \geq \bar{\tau}$,

$$\|\zeta(t)\|^2 \leq \frac{\beta_2}{\beta_1}\|\zeta(t_0)\|^2\exp\left(-\lambda_a(t-t_0)\right) + \kappa_a, \tag{45}$$

where $\lambda_a, \kappa_a \in \mathbb{R}^+$ are subsequently defined positive constants that act as a measure of stability.

Proof. Using (17), (22), and (27), as well as (37)-(39), the time derivative of (43) can be upper bounded as

$$\begin{aligned}\dot{V} \leq & -k_{V1}e_V^2 - k_{\alpha 1}e_\alpha^2 - k_{q1}e_q^2 - k_{CL,1}\underline{\lambda}_1\left\|\tilde{W}_1\right\|^2 \\ & - k_{CL,2}\underline{\lambda}_2\left\|\tilde{W}_2\right\|^2 - k_{CL,3}\underline{\lambda}_3\left\|\tilde{W}_3\right\|^2 + c_{CL} \\ & + \left[\left(\bar{\epsilon}_{T_*} + \bar{\epsilon}_{\epsilon\delta_T}\right)\left(\bar{q}S_D|e_V|\right) - k_{V2}\left(\bar{q}S_De_V\right)^2\right] \\ & + \left[\bar{\epsilon}_D\left(\bar{q}S|e_V|\right) - k_{V3}\left(\bar{q}Se_V\right)^2\right] + \left[\bar{\epsilon}_\alpha|e_\alpha| - k_{\alpha 2}e_\alpha^2\right] \\ & + \left[\bar{\epsilon}_L\left(\frac{1}{2}\rho SV_T|e_\alpha|\right) - k_{\alpha 3}\left(\frac{1}{2}\rho SV_Te_\alpha\right)^2\right] \\ & + \left[\bar{\epsilon}_q|e_q| - k_{q2}e_q^2\right] \\ & + \left[\left(\bar{\epsilon}_{M_*} + \bar{\epsilon}_{\epsilon\delta_e}\right)\left(\bar{q}Sc_{ref}|e_q|\right) - k_{q3}\left(\bar{q}Sc_{ref}e_q\right)^2\right],\end{aligned} \tag{46}$$

where $\underline{\lambda}_j \leq \lambda_{\min}\left\{\sum_{i=1}^{N_j}\mathcal{Y}_{j,i}^T\mathcal{Y}_{j,i}\right\}$, $\forall j \in \{1,2,3\}$, $c_{CL} \in \mathbb{R}^+$ is a positive constant that upper bounds the integrated reconstruction error terms in (37)-(39), and the $\bar{\epsilon}_{(\cdot)} \in \mathbb{R}^+$ terms are known positive upper bounds of the corresponding reconstruction error $\epsilon_{(\cdot)}$. Then, using the completing-the-square technique, (46) can be rewritten as

[11] See Chowdhary and Johnson (2011a) for details on methods of selecting data.

[12] The update laws in (37)-(39) contain $\tilde{W}_{(\cdot)}$ terms, which are unmeasurable.

[13] The FE conditions in (40)-(42) require that the system be sufficiently excited, which is a milder (can be satisfied in finite time) condition than the typical PE condition. For more information about the FE condition and how likely a given system is to satisfy it, see Chowdhary and Johnson (2011b); Chowdhary et al. (2013).

[14] The system can be shown to be stable in a similar fashion for all $t < \bar{\tau}$ as well, but because the aerodynamic function estimates can be shown to be greatly improved following all FE conditions being satisfied, the presented strategy only considers the possibility of the switch from the *adaptive* to *nominal* subsystem after this time. For this reason, the stability analysis for the time period $t < \bar{\tau}$ is omitted.

$$\dot{V} \leq -k_{V1}e_V^2 - k_{\alpha1}e_\alpha^2 - k_{q1}e_q^2 - k_{CL,1}\underline{\lambda}_1 \left\|\tilde{W}_1\right\|^2 - k_{CL,2}\underline{\lambda}_2 \left\|\tilde{W}_2\right\|^2 - k_{CL,3}\underline{\lambda}_3 \left\|\tilde{W}_3\right\|^2 + \frac{\left(\bar{\epsilon}_{T_*} + \bar{\epsilon}_{\epsilon\delta_T}\right)^2}{4k_{V2}} + \frac{\bar{\epsilon}_D^2}{4k_{V3}} + \frac{\bar{\epsilon}_\alpha^2}{4k_{\alpha2}} + \frac{\bar{\epsilon}_L^2}{4k_{\alpha3}} + \frac{\bar{\epsilon}_q^2}{4k_{q2}} + \frac{\left(\bar{\epsilon}_{M_*} + \bar{\epsilon}_{\epsilon\delta_e}\right)^2}{4k_{q3}} + c_{CL}. \tag{47}$$

In (47), the gains k_{V2}, k_{V3}, $k_{\alpha2}$, $k_{\alpha3}$, k_{q2}, and k_{q3} can be made arbitrarily large (based on actuator limits) in order to make each of the extra additive terms as small as possible. The expression in (47) is upper bounded by

$$\dot{V} \leq -\lambda_a V + c_a, \tag{48}$$

where $\lambda_a \triangleq \frac{1}{\beta_2}\min\{k_{V1}, k_{\alpha1}, k_{q1}, k_{CL,1}\underline{\lambda}_1, k_{CL,2}\underline{\lambda}_2, k_{CL,3}\underline{\lambda}_3\}$ is the minimum error decay rate for the closed-loop adaptive system, and $c_a \triangleq \frac{\left(\bar{\epsilon}_{T_*}+\bar{\epsilon}_{\epsilon\delta_T}\right)^2}{4k_{V2}} + \frac{\bar{\epsilon}_D^2}{4k_{V3}} + \frac{\bar{\epsilon}_\alpha^2}{4k_{\alpha2}} + \frac{\bar{\epsilon}_L^2}{4k_{\alpha3}} + \frac{\bar{\epsilon}_q^2}{4k_{q2}} + \frac{\left(\bar{\epsilon}_{M_*}+\bar{\epsilon}_{\epsilon\delta_e}\right)^2}{4k_{q3}} + c_{CL}$. Applying the Comparison Lemma (see Khalil (2002, Lemma 3.4)) to (48) yields

$$V \leq V(t_0)\exp\left(-\lambda_a(t-t_0)\right) + \kappa_a, \tag{49}$$

where $\kappa_a \triangleq \frac{c_a}{\lambda_a}$. Then, (49) along with (44) yields (45).

6. SWITCHED SYSTEMS ANALYSIS

In this section, a brief switched systems analysis is detailed to show that the system states remain bounded, even through multiple subsystem changes. The NN-based adaptive controller detailed in this paper is to be used in the event of a system fault that causes the nominal controller to destabilize the system. In other words, the adaptive controller will be activated when a fault is detected. The aircraft control system will switch between subsystems in the order: *nominal-faulted-adaptive-...*, where the switches are triggered by the following: The system switches from *nominal*-to-*faulted* at the time the fault physically occurs; the switch from *faulted*-to-*adaptive* happens once the fault is detected; and the *adaptive*-to-*nominal* switch is attempted (see Assumption 2) once enough data has been collected to satisfy the FE condition (i.e., $t \geq \bar{\tau}$). To facilitate the switched systems analysis, the following assumptions about the subsystems are made.

Assumption 2. Once the FE conditions in (40)-(42) are satisfied, the NN estimates of the aerodynamic coefficients are sufficiently accurate to enable the retraining [15] of the nominal controller. If this retraining step is unsuccessful, the switch to the nominal mode is skipped, and the switching cycle restarts at the faulted stage.

Assumption 3. The aircraft system in (1)-(5) is initially controlled by a nominal controller that ensures exponential stability of the error states. When a Lyapunov-based analysis is carried out using the candidate Lyapunov function (43), the Comparison Lemma (see Khalil (2002, Lemma 3.4)) can be applied to the resulting differential inequality to yield

$$V\left(t_k^f\right) \leq V\left(t_k^s\right)\exp\left(-\lambda_s \Delta t_k^s\right), \tag{50}$$

[15] This retraining step is a primary matter of future work.

for all $t \in \left[t_k^s, t_k^f\right)$, $k \in \mathbb{N}$, where $\lambda_s \in \mathbb{R}^+$ is a known positive constant that represents the error decay rate when the stable (nominal) subsystem is active.

Assumption 4. The system faults considered in this paper will cause the states will grow unbounded in a way that can be upper bounded by a growing exponential. Carrying out the analysis using the same candidate Lyapunov function (43), the Comparison Lemma (see Khalil (2002, Lemma 3.4)) can be applied to the resulting differential inequality during the time periods $t \in \left[t_k^f, t_{k+1}^a\right)$, $k \in \mathbb{N}$, yielding

$$V\left(t_{k+1}^a\right) \leq V\left(t_k^f\right)\exp\left(\lambda_f \Delta t_k^f\right), \tag{51}$$

where $\lambda_f \in \mathbb{R}^+$ is a known positive constant that represents the error growth rate when the faulted (unstable) subsystem is active. The amount of time that the system spends in the faulted subsystem is upper bounded by a positive constant as $\Delta t_k^f \leq \tau_D$, $\forall k \in \mathbb{N}$, where $\tau_D \in \mathbb{R}^+$ is the worst-case fault detection time.

Furthermore, during time periods spent in the adaptive subsystem (i.e., during the time periods $t \in [t_k^a, t_k^s)$, $k \in \mathbb{N}$), the above result (49) can be rewritten using the switched systems time-keeping notation from Section 2.2 as

$$V\left(t_k^s\right) \leq V\left(t_k^a\right)\exp\left(-\lambda_a \Delta t_k^a\right) + \kappa_a. \tag{52}$$

Theorem 2. The controllers in (16), (21), and (26), and the adaptive update laws in (34)-(36) ensure that all system signals remain bounded under closed loop operation for all time $t \in [0, \bar{\tau})$ and

$$\limsup_t \|\zeta(t)\|^2 \leq \frac{\nu_2}{\beta_1(1-\nu_1)}, \tag{53}$$

where $\nu_1, \nu_2 \in \mathbb{R}^+$ are positive constants, provided there exist sequences $\{\Delta t_k^a\}_{k=0}^\infty$, $\{\Delta t_k^s\}_{k=0}^\infty$, and $\left\{\Delta t_k^f\right\}_{k=0}^\infty$ such that $\forall k \in \mathbb{N}$, the sufficient condition

$$\frac{\lambda_s}{\lambda_f}\Delta t_k^s + \frac{\lambda_a}{\lambda_f}\Delta t_k^a \geq \tau_D \tag{54}$$

is satisfied.

Proof. Considering a single cycle of switching to *adaptive* mode, followed by *nominal* mode, *faulted* mode, and back to *adaptive* mode again, (51) can be written using (50) and (52) as

$$V\left(t_{k+1}^a\right) \leq \nu_1 V\left(t_k^a\right) + \nu_2,$$

where $\nu_1 \triangleq \exp\left(-\lambda_a \Delta t_k^a - \lambda_s \Delta t_k^s + \lambda_f \Delta t_k^f\right)$ and $\nu_2 \triangleq \kappa_a \exp\left(-\lambda_s \Delta t_k^s + \lambda_f \Delta t_k^f\right)$. Let $\{s_k\}_{k=0}^\infty$ be a sequence defined by the recurrence relation $s_{k+1} = Q(s_k)$, with initial condition $s_0 = V(\zeta(t_0^a))$, where $Q : \mathbb{R} \to \mathbb{R}$ is defined as $Q(s) \triangleq \nu_1 s + \nu_2$. Provided that (54) is satisfied, $\nu_1 < 1$, and therefore Q is a contraction (Rudin (1976, Definition 9.22)), and thus all initial conditions, s_0, approach the fixed point $s = \frac{\nu_2}{1-\nu_1}$ (Rudin (1976, Theorem 9.23)). Since the sequence $\{s_k\}$ upper bounds V, in the sense that $V(\zeta(t_k^a)) \leq s_k$, V is ultimately bounded by (53).

7. CONCLUSION

This paper presents a control and estimation scheme for a fixed wing aircraft that experiences a fault which introduces severe uncertainties that begin to dominate the system dynamics. An ICL scheme is employed to help ensure that aerodynamic coefficients are approximated online and yield ultimately bounded system errors. Future work will include a more in-depth study of the stable (nominal) subsystem and how the subsystems interact with one another, the impacts of faults and switching control schemes on flight performance, and the consideration of more complicated dynamics (6-DoF models, input constraints, etc.).

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