

Carrier-vehicle system for delivery in city environments

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Abstract: In this paper we present an extension of the carrier-vehicle problem for the case of delivery in an urban environment. The small vehicle, namely a drone, performs the delivery of goods at the customer address while the large vehicle is in charge of transporting, launching, recovering and servicing the drone. In this work it is assumed that the take-off and landing points are not at the location of the customer but fixed spots predefined by the city. In this context, the truck is allowed to advance during the drone delivery, providing a landing location closer to the following client and reducing the route completion time. The selection of these spots is restricted by the autonomy of the drone and the velocity of both vehicles. The urban environment is addressed by defining a different distance metric for the aerial and the terrestrial vehicle, respectively. The paper presents a mixed-integer linear programming formulation which allows to solve the given problem of computing the truck routes and selecting the optimal takeoff/landing spots in reasonable time. Illustrative examples of this problem and a computational analysis of the presented solution conclude the paper.

Keywords: Mission planning and decision making, Multi-vehicle systems, Trajectory and Path Planning.

1. INTRODUCTION

The use of autonomous vehicles is incessantly gaining ground in daily life activities. Their technological advancements in terms of performance and reliability have been gathering the attention of several fields (Hengstler et al., 2016; Batalden et al., 2017). In this context, delivery and transportation tasks have raised as one of their main areas of application, where the capabilities of these systems are highly appreciated and demanded (Bagloe et al., 2016).

Transportation and delivery problems have been widely studied by the scientific literature. How to combine a fleet of vehicles and to compute their optimal routes has been established as one of the well known combinatorial optimization problems in the literature. The so-called Vehicle Routing Problem (VRP), introduced by the two seminal papers Dantzig and Ramser (1959); Clarke and Wright (1964), has been subject of several studies and variants all over the years (Kumar and Panneerselvam, 2012; Braekers et al., 2016).

The city environment is often included in this kind of problems. Most cities present complicate street distributions and areas with no access for big transportation vehicles. In this context, the use of groups of heterogeneous vehicles has been proved to provide levels of flexibility and capabilities unable to be found in homogeneous groups (Salhi et al., 2014; Koç et al., 2016). The combination of a slow

big vehicle, e.g. trucks, with smaller and faster vehicles, e.g. drones, is lately coming into focus. Recent works propose a combination of both kind of vehicles, such that the deliveries are divided based on location and accessibility (Murray and Chu, 2015; Agatz et al., 2018; Ha et al., 2018). In these works, still, the role of both vehicles remains similar as the deliveries are performed by any of them.

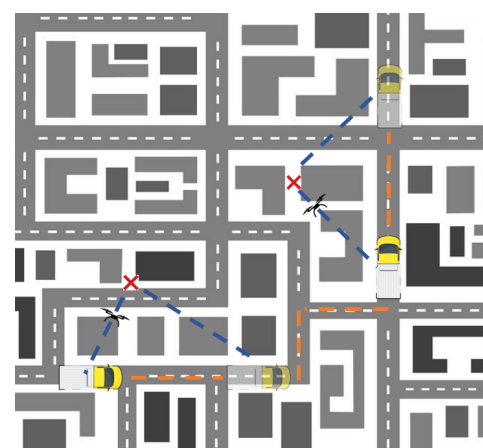


Fig. 1. Schematic of the carrier-vehicle system.

In Garone et al. (2011), the concept of the carrier-vehicle system is firstly introduced for the case of rescue missions. In this case, the small vehicle is in charge of visiting the target points while the big vehicle must transport, service and recover the drone. In Poikonen et al. (2017), a similar

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concept is applied to delivery systems, where the drone is the only agent in charge of the delivery of the goods.

At this point, the majority of works of last-mile delivery assume the launching of these vehicles at the location of the customer. This assumption can become unrealistic in certain city distributions due to physical or legal constraints, and also provide sub-optimal results in systems with drones of large autonomy or slow trucks. Given that, papers like Boysen et al. (2018); Poikonen and Golden (2020) drop the assumption of launching the drone always at the customer location and provide a heuristic approach to select the take-off/landing points based on the routes taken by the truck.

In this paper we follow this recent line of thought, where we define the case where only certain spots in the city allow the takeoff and the recovery of the drones. Moreover, these spots are not linked to any particular customer and must be chosen according to each mission. The main idea is to select the route and the optimal takeoff/landing spots depending on the customer locations such that the route completion time is minimized. This concept can be seen in Fig. 1.

A novelty in this paper is that, extending the idea presented in Garone et al. (2011), the truck is allowed to advance during the drone delivery, providing a different location for the landing and the takeoff. This novel approach in delivery systems permits to improve the mission completion time by adapting the route to the speed of both vehicles and the autonomy of the drone. Additionally, contrary to the cited papers, we assume two different distance metrics for the vehicles, based on the fact that aerial and a terrestrial vehicles are subject to different route constraints.

The main contribution of this paper is to formulate the routing problem as the selection of the optimal take-off and landing locations for each delivery. This approach allows to obtain an optimal solution to this problem based on a mixed-integer linear program that solves the problem in reasonable time.

The remainder of the paper is organized as follows. In Section 2, the problem is stated and defined. In Section 3, the characteristics of the urban environment are detailed and characterized. In Section 4, a mixed-integer formulation for the carrier-vehicle system is presented. Section 5 shows several numerical simulations and computational analysis to support the validity of the presented formulation. Finally, in Section 6, we present some conclusion and future works.

2. PROBLEM STATEMENT

In this paper we deal with the problem of transport and delivery of packages by employing a two vehicles system in an urban environment. The aim of the mission is to deliver N packages to a set $P = \{p_1, \dots, p_N\} \in \mathbb{R}^2$ of assigned delivery locations in the shortest time possible.

The system considered for such a task is composed by a big and slow vehicle carrier and a small and fast carried vehicle. The carrier is assumed to be a terrestrial vehicle which must follow the predefined routes given by the city

distribution, e.g. a truck. On the other hand, the carried vehicle is assumed to be an aerial vehicle, namely a drone, which can move freely in the space.

As a result of the physical constraints of the city, the drone is considered as the only agent able to perform the delivery of the goods. The role of the carrier is the one of transporting, launching and recovering the drone such that the delivery is performed in the shortest time possible.

Due to the high speed of the small vehicle and the low altitudes considered, both vehicles are considered as points belonging to the Euclidean space \mathbb{R}^2 . The position of both vehicles is described as $p_c(t) = [x_c(t), y_c(t)]^T$ and $p_v(t) = [x_v(t), y_v(t)]^T$ for the carrier and the drone respectively.

In terms of kinematics, both vehicles are considered as a single integrator with a maximal speed $V_c > 0$ in the case of the carrier and $V_v > 0$ for the vehicle, being $V_v > V_c$. The drone is considered to have a limited autonomy (in time) $a > 0$ but that can be restored by the carrier in a negligible time.

The truck routes must follow the street distribution and the launch and recovery of the drone is restricted to a set $S = \{s_1, \dots, s_{N_p}\} \in \mathbb{R}^2$ of specified spots spread over the city, whose number is $N_p > N$. The customer locations can belong to any point of \mathbb{R}^2 while the takeoff spots must be part of the set formed by the network of streets. An example of this kind of distributions is detailed in Fig. 2.

The launch/recovery spots are not previously assigned to any customer but depend on the chosen route for the truck. Moreover, in order to optimize the delivery route, the drone can be launched and recovered at different spots as far as the reachability of another spot is ensured by the autonomy range of the drone and the speed of the truck.

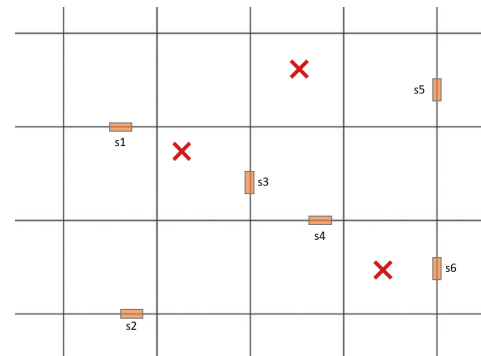


Fig. 2. Schematic of the available stops deployment where the crosses represent the customer locations and the rectangles the truck stops.

In the definition of the problem the following assumptions are considered:

- The order of delivery is given a priori.
- The drone is instantaneously charged and loaded.
- The drone can only visit one single location per flight.

This system, in the case of n customers, presents two kind of time intervals. The time when the vehicle is on board of the truck, denoted by $t_i^{to,t}$, $i = 1, \dots, n+1$ and when the vehicle is airborne denoted by $t_i^{to,d}$, $i = 1, \dots, n$. Knowing

that the drone flight time is limited by the endurance a , the following constraints must be satisfied:

$$0 \leq t_i^{to,l} \leq a \quad i = 1, \dots, n, \quad (1)$$

$$0 \leq t_i^{l,to} \quad i = 1, \dots, n+1. \quad (2)$$

The optimal route for both vehicles is the route which minimizes the sum of all these time intervals. In this framework, based on Garone et al. (2011), this route can be defined and computed by selecting the launch and recovery spots for the drone and the shortest path between these points.

3. CITY ENVIRONMENT CONSTRAINTS

This section presents the adaptation of the carrier-vehicle problem to the case of a city environment. This scenario implies several constraints related to allowed routes and the available take-off and landing spots.

3.1 Distance metrics

Due to the different nature of the vehicles, two different distance metrics are considered. The drone, as an aerial vehicle, is not constrained by the distribution of the streets. Accordingly, the distance travelled by the vehicle between two points $p_1 = \langle x_1, y_1 \rangle$ and $p_2 = \langle x_2, y_2 \rangle$ is given by the Euclidean norm defined as $\|p_1 - p_2\|_2 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

On the other hand, the distance travelled by the carrier is defined by the distribution and length of the grid created by the streets of the city. In this context, the possible paths taken by the carrier are given by the shortest distances in between the available launching spots.

Being the city considered as a Manhattan-like urban area, the Manhattan distance between two points $p_1 = \langle x_1, y_1 \rangle$ and $p_2 = \langle x_2, y_2 \rangle$ is given by $\|p_1 - p_2\|_1 = |x_1 - x_2| + |y_1 - y_2|$, which can be seen illustrated in Fig. 3. However, the distance travelled between two points following the grid is not always equivalent to the the Manhattan distances when these points are not part of the joints of the grid, as it is shown in Fig. 4.

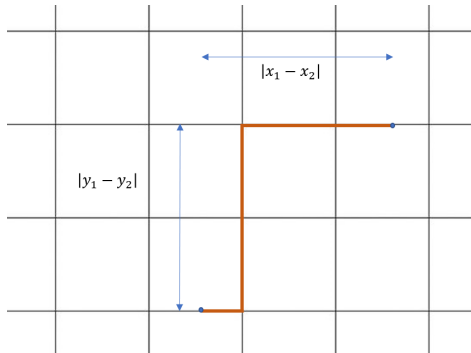


Fig. 3. Example of two points where the shortest route distance is given by the Manhattan norm.

For the sake of simplicity but with no loss of generality, let us define the streets of the city as a grid where the joints are given by natural numbers. In this case, the

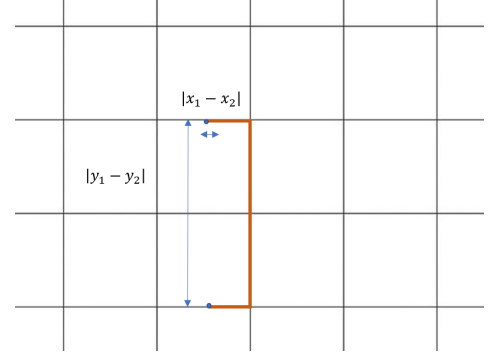


Fig. 4. Example of two points where the shortest route is not given by the Manhattan norm.

distance between the j -th and f -th spots can be computed as follows

$$d_{j,f} = |s_{x,j} - s_{x,f}| + |s_{y,j} - s_{y,f}|$$

$$\text{if } \left\{ |s_{y,j} - s_{y,f}| \geq 1 \cap |s_{x,j} - s_{x,f}| \geq 1 \right\}$$

$$\cup \left\{ |s_{y,j} - s_{y,f}| < 1 \cap ([s_{y,j}] = [s_{y,f}]) \right.$$

$$\cup [s_{y,j}] < s_{y,j} < [s_{y,j}] \cup [s_{y,f}] < s_{y,f} < [s_{y,f}] \cup$$

$$\left. [s_{y,j}] = [s_{y,f}] \cup [s_{y,j}] = [s_{y,f}] \right\}$$

$$\cup \left\{ |s_{x,j} - s_{x,f}| < 1 \cap ([s_{x,j}] = [s_{x,f}]) \right.$$

$$\cup [s_{x,j}] < s_{x,j} < [s_{x,j}] \cup [s_{x,f}] < s_{x,f} < [s_{x,f}] \cup$$

$$\left. [s_{x,j}] = [s_{x,f}] \cup [s_{x,j}] = [s_{x,f}] \right\} \quad (3)$$

$$d_{j,f} = |s_{x,j} - s_{x,f}| +$$

$$\min\{|s_{y,j} - [s_{y,j}]| + |s_{y,f} - [s_{y,f}]|,$$

$$|s_{y,j} - [s_{y,j}]| + |s_{y,f} - [s_{y,f}]|\}$$

$$\text{if } |s_{y,j} - s_{y,f}| < 1$$

$$\cap [s_{y,j}] = [s_{y,f}] \cap [s_{y,j}] < s_{y,j} < [s_{y,j}] \cap [s_{y,f}] < s_{y,f} < [s_{y,f}] \quad (4)$$

$$d_{j,f} = |s_{y,j} - s_{y,f}| +$$

$$\min\{|s_{x,j} - [s_{x,j}]| + |s_{x,f} - [s_{x,f}]|,$$

$$|s_{x,j} - [s_{x,j}]| + |s_{x,f} - [s_{x,f}]|\}$$

$$\text{if } |s_{x,j} - s_{x,f}| < 1$$

$$\cap [s_{x,j}] = [s_{x,f}] \cap [s_{x,j}] < s_{x,j} < [s_{x,j}] \cap [s_{x,f}] < s_{x,f} < [s_{x,f}] \quad (5)$$

where $\langle s_{x,j}, s_{y,j} \rangle$ and $\langle s_{x,f}, s_{y,f} \rangle$ represent the coordinates in the Euclidean space of the two spots.

Thus, the set of routes for the truck can be characterized as a symmetric graph $G = \langle V, E \rangle$ where V , the vertices of the graph, represent the location of the available launching spots and the edges E denote the minimum distance path $d_{j,f}$ between them by following the city constraints.

3.2 Local reduction and feasibility

The presented problem assumes non-predefined take-off and landing spots for each delivery. This assumption provides more flexibility in the path calculation but it can heavily increase the computation complexity of the problem if the size of the area and the number of available spots is too large.

To reduce the amount of evaluated points for large scenarios, and thus the complexity of the optimization problem, a reduction of the study area is used for each visiting point. The main idea of this area reduction lies in the fact that, given the limited autonomy of the fast vehicle, the number of streets from where the aerial vehicle can takeoff is limited. Being the flight autonomy and the grid structure parameters known in advance, the reduction of street constraints for each points can be thus computed beforehand. This concept is depicted in Fig. 5.

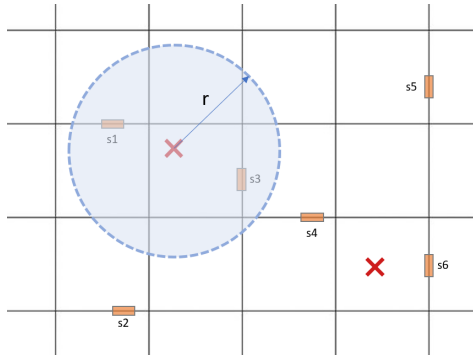


Fig. 5. Example of the reduction selection of available stops for the carrier, indicated by the light blue area.

Let us consider each visit point as the center $p_i \in \mathbb{R}^2$ of a circle with radius

$$r = V_v a \quad (6)$$

where V_v is the velocity of the drone and a its flight autonomy.

Let the location of the available stops be given by $S_i = \langle s_{x,i}, s_{y,i} \rangle \forall i = 1 \dots n$. Therefore, by defining the euclidean distance between each stop and the customer location as

$$d_i = \sqrt{(p_x - s_{x,i})^2 + (p_y - s_{y,i})^2} \quad (7)$$

it is enough to check if $d_i > r \forall i = 1 \dots n$ to discard the stops out of range for each target point and define the set V_i as the set of available stops for the i -th target. Additionally, this precomputation also provides a feasibility evaluation of the delivery targets based on the autonomy of the vehicle and the stops distribution.

4. MIXED-INTEGER FORMULATION

This section presents a mixed-integer formulation to solve the routing problem for the carrier-vehicle system. The problem is formulated in such a way that it is enough to select the optimal takeoff and landing points for the vehicle to determine the optimal routes of the truck and the drone.

Consider the two binary decision variables $\alpha_{j,f}^i$ and $\beta_{j,f}^i$, which define the choice of the take-off and landing spots for each visiting point i . In this case, $\alpha_{j,f}^i$ takes the value 1 when for the delivery point i the carrier stops at the j -th spot for the takeoff and at the f -th for the landing. Equivalently, $\beta_{j,f}^i$ defines the path between landing and next takeoff of the drone, being 1 if after the i -th delivery it lands in the j -th spot and the truck drives till f -th stop for the next flight.

These two variables define the path for each visited customer based on the two intervals characterized in (1) and (2) and therefore, the entire route followed by the system.

At every mission each customer must be served once and only once, which is expressed as follows

$$\sum_{f \in V_i} \sum_{j \in V_i} \alpha_{j,f}^i = 1 \quad \forall i = 1, \dots, N, \quad (8)$$

$$\sum_{f \in V_i} \sum_{j \in V_i} \beta_{j,f}^i = 1 \quad \forall i = 1, \dots, N-1, \quad (9)$$

where N denotes the number of delivery orders and V_i represents the precomputed set of available stops for the i -th customer. By doing so, we ensure that there is always a combination of spots used for each delivery.

To guarantee the continuity between flights, the stop chosen for the landing must coincide with the origin of the route till the next customer

$$\sum_{f \in V_i} \alpha_{f,j}^i = \sum_{f \in V_i} \beta_{j,f}^i \quad \forall i \in \{1, \dots, N\}, \forall j \in V_i. \quad (10)$$

Equivalently, the take-off must be the end of the route between the previous customer and the new one

$$\sum_{f \in V_i} \alpha_{j,f}^{i+1} = \sum_{f \in V_i} \beta_{j,f}^i \quad \forall i \in \{1, \dots, N-1\}, \forall j \in V_i. \quad (11)$$

The distances travelled between take-off and landings in the case of the aircraft are given by the sum of Euclidean norms between these spots and the location of the customer. This distance must be minor than the maximum range of the aircraft which can be stated as follows

$$\alpha_{j,f}^i (\|s_j - p_i\| + \|p_i - s_f\|) \leq v_a t_i^{to,l} \quad \forall i = 1 \dots N \quad \forall j, f \in V_i \quad (12)$$

where the variable $\alpha_{j,f}^i$ ensures that this constraint is based on the selected take-off and landing spots.

Given the possibility of using different spots for the takeoff and the landing of the drones, the carrier must ensure that the distance between these two locations is travelled sufficiently fast

$$\alpha_{j,f}^i d_{f,j} \leq v_a t_i^{to,l} \quad \forall i \in \{1, \dots, N-1\}, \forall j, f \in V_i. \quad (13)$$

Then, the period when the vehicle is part of the carrier system the carrier must provide the shortest path such that the time is minimized

$$\beta_{j,f}^i d_{f,j} \leq v_a t_i^{to,l} \quad \forall i \in \{1, \dots, N-1\}, \forall j, f \in V_i \quad (14)$$

At this point, the routing problem can be stated as the minimization of the sum of the time intervals for the deliveries and the routes between flights based on the

choice of the takeoff/landing spots. This optimization problem can be formulated as

$$\min_{\alpha_{j,f}^i, \beta_{j,f}^i} \sum_{i=1}^n t_i^{to,l} + \sum_{i=1}^{n-1} t_i^{l,to}, \quad (15)$$

s.t. (1) – (2), (8) – (14)

being a Mixed-Integer Linear Program (MILP).

The main virtue of this formulation is to provide optimal results for the route of both vehicles while being a class of optimization problem that can be efficiently solved (Vielma, 2015) by commercial solvers e.g. Gurobi or Cplex.

5. RESULTS

This section presents several numerical simulations that demonstrate the efficiency of the proposed formulation.

5.1 Illustrative example

In this example we consider the case of 5 customers. The carrier-vehicle system must depart from a given depot *D1* and finish the route at the depot *D2*. To assist the delivery, 35 random launching and recovery spots have been generated. These spots are assumed to belong to a given street but not necessarily to the joints of the graph. Table 1 describes the location of the 5 customers and 2 depots.

Table 1. Location of customers and depots.

Visit points	$x(km)$	$y(km)$
Depot 1	0	10
Customer 1	15.46	26.632
Customer 2	16.6	2.13
Customer 3	9.92	1.94
Customer 4	29.90	13
Customer 5	12.9	13.09
Depot 2	5	12

The maximum velocity considered for the truck is $20km/h$ while the drone has a maximum velocity of $40km/h$. In this example we compute the optimal route for two scenarios depending on the range of autonomy of the drone. The two ranges considered are based on common electrical drones examples, such as 15 and 21 minutes of flight autonomy. Fig. 6 depicts the results for the case of 15 minutes of autonomy and Fig. 7 depicts the case of 21 minutes. In these figures, the orange circles represent the available takeoff/landing points and the purple dots the location of the customers. Regarding the paths, the solid lines represent the ground displacements while the dashed lines depict the aerial route followed by the drone.

From Fig. 6 and Fig. 7, it can be seen how the optimal path for the truck heavily changes depending on the autonomy range of the drone. In Fig. 6 due to the small range of action, the take-off and landing are performed in the same spot while for the case of 21 minutes the optimal route combines different spots for the launch and the recovery of the vehicle.

The optimal route time for the case of 21 min autonomy is 2.95h. The same delivery scenario assuming the use of the

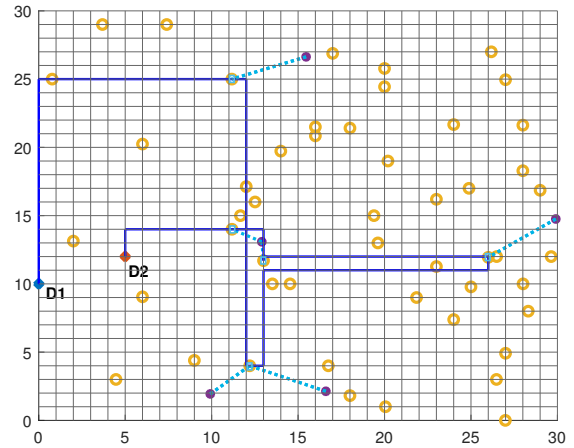


Fig. 6. Optimal route for an autonomy of 15 minutes.

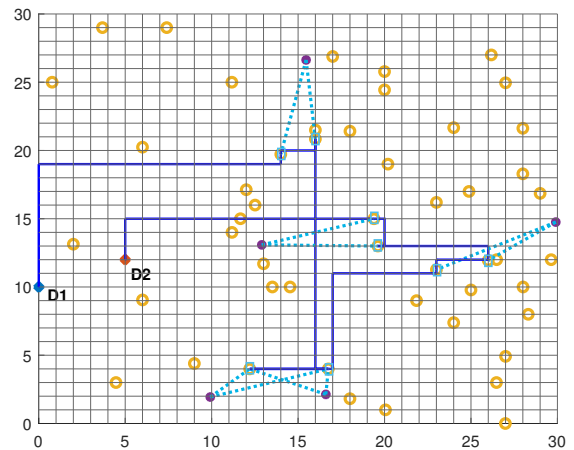


Fig. 7. Optimal route for an autonomy of 21 minutes.

same spot for the takeoff and landing for each customer provides a final completion time of 3.33h. This comparison shows an improvement of 13.3% when allowing the truck to use a second spot for the landing of the vehicle.

These results support the main claim of this paper that, depending on the autonomy and speeds of the vehicles, the optimal drop-off spots may vary and that using the same spot for takeoff and landing can lead to sub-optimal results.

5.2 Computational analysis

This section evaluates the computational performance of the presented solution. Given a confined area of $30 \times 30 km$ and 50 available launch spots, several random scenarios have been generated. This analysis accounts for cases from 3 to 10 customers and evaluates the solving time of the optimization problem.

For this test, the characteristics of the vehicles are similar to the previous example, with $V_c = 20km/h$ and $V_v = 40km/h$ as the maximum velocities for each of the vehicles. Regarding the autonomy of the drone, the simulations consider an autonomy of 21 min per flight.

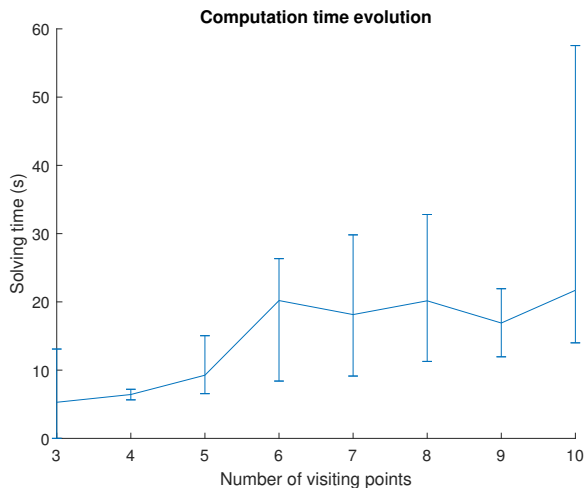


Fig. 8. Evolution for the computational time of the solver.

The numerical simulations have been performed using GUROBI solver in YALMIP environment for Matlab. For each number of customers 10 random cases are generated, obtaining the maximum, minimum and average time to solve them. Fig. 8 shows the evolution of the computational time of the mixed-integer optimization problem by the solver.

In Fig. 8, it can be seen how the computation of the solution does not increase too heavily due to the fact that the number of spots remains the same.

6. CONCLUSIONS

This paper considers the problem of delivery in an urban area based on the use of a carrier-vehicle system. This system combines the capabilities of a carrier, such as its large autonomy, with the speed and maneuverability of a drone. The problem is characterized in such a way that the optimal routes can be computed based on the selection of the optimal launching and recovery points for the drone.

The authors propose a mixed-integer formulation which optimally solves the presented problem. Several simulations complete the paper to support the efficiency of the solution and the relevance of the presented scenario, where take-off and landing points do not necessarily coincide.

Future works will extend the problem to the case where several vehicles are involved. Another interesting future scenario considers the use of dynamic launching spots. These spots will be based on predefined routes provided by the city public transportation such as tram or bus lines.

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