## **Robust Controller Design For Modified Smith Predictor**

P. Princes Sindhuja\*, V. Vijayan\*, Rames C. Panda\*\*

 \* E&I Department, St. Joseph's College of Engineering, Chennai – 600119, India.{Email: sindhujanelson@gmail.com;vinvpn@gmail.com}
 \*\*Chemical Engineering Department, CSIR-CLRI, Adyar, Chennai – 600020, Email: <u>panda@clri.res.in</u>

**Abstract:** The present paper presents control of dead time processes using Sliding Mode Controller (SMC) in a Modified Smith Predictor (MSP) structure. The SMC working principle relies on the reaching phase and the sliding phase where the tuning parameters  $K_D$  and  $\delta$  play major roles in the speed, overshoot and chattering effects. Here in this paper, model based tuning rule is used to modify the  $K_D$  rule/equation and is implemented to different processes with modified smith predictor (MSP) structure to counteract the constant load disturbance for an integrating process. It has been found that the proposed MSP-SMC combination provides improved performance and robustness behavior. The analyses are done for servo and regulatory response cases. Moreover, the robustness and invariance property of the SMC has been analyzed against parametric uncertainty and other various disturbances (noise). The results obtained are compared with that of available methods.

Keywords: Sliding mode control, Smith Predictor, Time delay Systems, Process Control, PID control.

#### 1. INTRODUCTION

Dead time is a very common occurrence in most of the industrial chemical processes due to transportation lag in pipeline, time lag during computation or any other. The desired closed loop performance utterly degrades if the process has relatively large dead time compared to dominant time constant. Estimation of dead-time poses uncertainty giving rise to model-uncertainty. Hence the control of dead time process is a challenging task. In earlier days, conventional PID controllers were used in controlling the processes with small dead time. However, Smith in 1957 proposed that the Smith predictor (SP) structure for dead time compensation processes with long time delay, but the structure was not applicable for integrating systems as it results in constant offset due to load disturbance [1]. Hence many MSP structures are proposed for dead time and integrating processes to overcome the steady state errors [2]. Many other researchers [3] [4] [5] [6] [7] have designed modified Smith predictor for improving the performance of the closed loop system using PID or IMC controllers. Later various researches tend towards implementing the feature of both the Smith predictor and the cascade controller were designed for improved closed loop performance and fast rejection of disturbance [8], [9]. Smith predictor is basically a model-based structure hence it is affected by parametric uncertainties. These IMC-PID controller parameters rely on the process parameters which are obtained from identification of model. There might be modelling errors due to parametric uncertainties or due to unmodelled dynamics. These uncertainties affect the performance of the controller drastically. In order to assure finite time stability of the closed loop system, there is a need for Sliding Mode Controller, a robust controller insensitive to uncertainties.

Sliding Mode Controller is a variable structure controller designed for linear and non-linear systems [10] [11] [12]. In the literature work of Sliding Mode Controller Oscar Camacho and Ruben Rojas has done an extensive work on SMC for stable, unstable, integrating and inverse processes for SISO and MIMO system [13] [14] [15] [16] [17]. Sliding Mode Controller was designed using the empirical lower order models mostly by a FOPDT system which contained uncertainties due to unmodelled dynamics due to linearization of the nonlinear system. All the works had shown improved performance and the robustness. To enhance the performance of the integrating system with dead time, Camacho and Cruz [18] [19] had proposed a Modified Smith predictor using Sliding Mode Controller for elevated dead time.

The tuning parameters  $K_D$  and  $\delta$  play a vital role in SMC with regard to speed, overshoot and chattering. This insight has urged to improve the MSP-SMC with the modification in the  $K_D$  parameter which is the sole responsible in bringing the states from initial states to the sliding surface. Hence in this paper, a modified  $K_D$  law/equation has been proposed for SMC combined with MSP structure. Thus, by changing only the  $K_D$  equation the performance has been improved considerably. The modified equation has also been tested to a IFOPDT process

The paper is organized as follows, section 2 explains the mathematical formulation of the SMC control law and the tuning parameters of IFOPDT process. Section 3 discusses the results for IFOPDT. The performances with proposed control law are compared with that of other available controllers in terms of servo and regulatory performances and robustness.

# 2. MATHEMATICAL FORMULATION OF SLIDING MODE CONTROL LAW IN MSP STRUCTURE

The formulation of the control law of the SMC in an MSP / SP structure for IFOPDT process is analyzed below.

Let's consider an IFOPDT type of process.

The transfer function of an IFOPDT system is given as in equation (1)

$$G_{P}(s) = \frac{Y(s)}{U(s)} = \frac{K_{P}e^{-D_{P}s}}{s(\tau_{P}s+1)}$$
(1)

The transfer function of the model  $G_m(s)$  without dead time in the MSP structure is given as,

$$G_{P}(s) = \frac{X_{1}(s)}{U(s)} = \frac{K_{m}}{s(\tau_{m}s+1)}$$
(2)

Equation (8) is simplified in differential form as,

$$X_1(s)\left[s^2\tau_m + s\right] - K_m U(s) = 0 \tag{3}$$

On taking inverse Laplace transform,

$$\frac{d^2 X_1(t)}{dt} = \frac{K_m}{\tau_m} U(t) - \frac{1}{\tau_m} \frac{dX_1(t)}{dt}$$
(4)

In the first step as discussed before in the design of Sliding Mode Controller is to select the sliding surface. In the IFOPDT system considered, the order of the system n=2. The sliding surface in equation (5) is designed for the process model taken here.

$$S(t) = \left(\frac{d}{dt} + \lambda\right)^2 \int_0^t e(t)dt$$
(5)

$$S(t) = \left(\frac{d^2t}{dt^2} + \lambda^2 + 2\lambda \frac{dt}{dt}\right) \int_0^t e(t)dt$$
(6)

$$S(t) = 2\lambda e(t) + \lambda^2 \int_0^t e(t)dt + \frac{de(t)}{dt}$$
(7)

Let  $\lambda_1 = 2\lambda$  and  $\lambda_0 = \lambda^2$ 

The equation (7) is the sliding surface S(t) which is in the form of PID. Next is to design the control law consisting of continuous and discontinuous parts. In order to obtain that, the control law should satisfy the sliding condition.

Thus, to obtain the continuous part of the controller, differentiate equation (7) and equate to zero.

$$\frac{dS}{dt} = \lambda_1 \frac{de(t)}{dt} + \frac{d^2 e(t)}{dt^2} + \lambda_0 e(t) = 0$$
(8)

The error signal e(t) is given as  $e(t) = R(t)-X_1(t)$ . Substituting in equation (8) we get

$$\lambda_{1} \frac{d(R(t) - X_{1}(t))}{dt} + \frac{d^{2}(R(t) - X_{1}(t))}{dt^{2}} + \lambda_{0}e(t) = 0$$
(9)

Differentiation of R(t) is ignored since it's a constant value, Hence

$$-\lambda_1 \frac{dX_1}{dt} - \frac{d^2 X_1}{dt^2} + \lambda_0 e(t) = 0$$
 (10)

Substituting equation (4) in (10) we obtain,

$$-\lambda_1 \frac{dX_1}{dt} - \left[\frac{K_m}{\tau_m} U(t) - \frac{1}{\tau_m} \frac{dX_1}{dt}\right] + \lambda_0 e(t) = 0$$
$$U(t) = Uc(t) = \frac{1}{K_m} \left[ \left(1 - \lambda_1 \tau_m\right) \frac{dX_1}{dt} + \tau_m \lambda_0 e(t) \right]_{(12)}^{(11)}$$

$$U_D(t) = K_D \frac{S(t)}{|S(t)| + \delta}$$
(13)

Thus equation (12) obtained is the continuous part of the controller. The discontinuous part of the controller is given in equation (13). Thus, the combined control law can be written as,

$$U(t) = \frac{1}{K_m} \left[ \left( 1 - \lambda_1 \tau_m \right) \frac{dX_1}{dt} + \tau_m \lambda_0 e(t) \right] + K_D \frac{S(t)}{|S(t)| + \delta}$$
(14)

with the sliding surface S(t) defined as,

$$S(t) = \operatorname{sgn}(K) \left[ \lambda_1 e(t) + \lambda_0 \int_0^t e(t) dt - \frac{dX_1(t)}{dt} \right]$$
(15)

#### **Modified K**<sub>D</sub> equation:

In the proposed method the  $K_D$  equation has been modified. It is stated that the  $K_D$  value is inversely proportional to the ultimate gain and the gain of the process. This equation is obtained by checking the performance for nominal cases of IFOPDT processes with different controllability ratios (CR) by changing the  $K_D$  values and hence the  $\delta$  values. In order to improve the robustness in terms of parametric uncertainties of the system the

factor 
$$\left(\frac{\tau_i}{\lambda + D_m}\right)$$
 has been multiplied with the proposed K<sub>D</sub>

equation. [For IFOPDT]:

$$\mathbf{K}_{\mathrm{D}} = \left(\frac{1}{\left|\mathbf{K}_{\mathrm{m}}\right| \mathbf{K}_{\mathrm{U}}}\right) \left(\frac{\tau_{\mathrm{i}}}{\lambda + \mathbf{D}_{\mathrm{m}}}\right) \tag{16}$$

Thus, the K<sub>D</sub> equation for the IFOPDT are stated and the results are verified. It can be seen that the peak overshoot has been reduced by adding a set point filter for which the filter time constant  $\tau_f$  is equal to the process time constant  $\tau_p$ 

[21]. The tuning parameters proposed by Camacho (2004) and that the modification of  $K_D$  equation proposed in this paper are tabulated in Table 1 [22].

#### 4. RESULTS:

The MSP-SMC structure is implemented for IFOPDT process

Case 1: In case 1 an IFOPDT model structure 
$$G_p(s) = \frac{e^{-6s}}{s(3s+1)}$$
 has been considered which has a

Controllability Ratio (CR) =2 [18]. The tuning parameters given by Camacho (2004) are obtained as  $\lambda_1$ =1.333,  $\lambda_0$  =0.222,  $K_D$ =0.4429,  $\delta$  = 1.5017. The  $K_D$  value by the proposed method is  $K_D$ =26.40,  $\delta$  = 9.8111. The performance measure for the proposed method and the Camacho's method are tabulated in Table 2 and 3. The setpoint and regulatory responses are depicted in Figures (1) and (2).

#### Table 1 Tuning parameters of MSP-SMC control law PROPOSED METHOD

# **MODIFIED SMC TUNING PARAMETER IFOR IFOPDTI:** $K_{D} = \left(\frac{1}{|K_{m}|K_{U}}\right) \left(\frac{\tau_{i}}{\lambda + D_{m}}\right)$

Where

 $K_u = \frac{4 \times h}{\prod \times a}$  (obtained by relay auto-tuning method)

 $\tau_i$  obtained by Laurent's method of tuning PID parameters.

$$\tau_{i} = \frac{1}{\beta} \left( \left( \tau_{m} + \beta \right) + \frac{D_{m}^{2}}{2\left( \lambda + D_{m} \right)} \right) \text{ [For IFOPDT]}$$

### **SET POINT FILTER**

$$G_{f}(s) = \frac{1}{(\tau_{f}s+1)}$$
$$\tau_{f} = \tau_{p}$$

The performance measures for the proposed method are  $IAE_R$  =10.33,  $IAE_L$  =7.46 and that of the Camacho's (2004) are  $IAE_R$  = 13.55,  $IAE_L$  =10.55. Moreover, the peak overshoot has been efficiently reduced by means of a set point filter with its filter constant equal to the time constant of the model. Also, when disturbance of L=0.1at t=50sec. has been introduced to the process the proposed method has rejected the disturbances efficiently. The robustness of the proposed method in terms of dead time is 20% of Dp and -20% of Dp.



Figure 1 Servo response of IFOPDT process



Figure 2 Regulatory response of IFOPDT process.

Table 2: Servo response performance measures of IFOPDT

Servo Response					
Camacho's method		Proposed method			
IAE <sub>R</sub>	PO <sub>R</sub> %	IAEL	POL%		
13.55	15.13	10.33	6.06		

**Table 3:** Regulatory response performance measures ofIFOPDT

Regulatory Response					
Camacho's method		Proposed method			
IAE <sub>R</sub>	PO <sub>R</sub> %	IAEL	PO <sub>L</sub> %		
10.55	6.06	7.46	58.55		

#### **5. CONCLUSION**

In this paper, a novel sliding mode controller (SMC) has been designed for IFOPDT type model structure and is implemented in modified smith predictor structure. The SMC control law has two parts, continuous and discontinuous. The discontinuous part is governed by a use defined parameter,  $K_D$ , which has been designed to provide a faster and stable closed-loop response. It has been found that tuning parameters  $K_D$  and  $\delta$  are able to control the speed, overshoot and chattering of the response. Present algorithm is implemented on IFOPDT to study their performances and has been found to be satisfactory. The performances have been compared with available literature.

Acknowledgement: The first author is willing to acknowledge and thank CLRI for providing infrastructure and facility for carrying out this research. There is no conflict of interest.

#### REFERENCES

- [1] O. J. M. Smith, "Closer control of loops with time delays," *Chem Eng Prog*, vol. 53, pp. 216-219, 1957.
- [2] K. J. Astrom, C. C. Hang and B. C. Lim, "A new Smith predictor for controlling a process with an integrator and long dead-time," *IEEE transactions on Automatic Control*, vol. 39, no. 2, pp. 343-345, 1994.
- [3] M. R. Matausek and A. D. Micic, "A modified Smith predictor for controlling a process with an integrator and long dead-time," *IEEE transactions on automatic control*, vol. 41, no. 8, pp. 1199-1203, 1996.
- [4] H. J. Kwak, S. W. Sung and I. B. Lee, "Modified Smith predictors for integrating processes: Comparisons and proposition," *Industrial and Engineering Chemistry Research*, vol. 40, no. 6, pp. 1500-1506, 2001.
- [5] C. C. Hang, Q. G. Wang and X. P. Yang, "A modified Smith predictor for a process with an integrator and long dead time," *Industrial and Engineering Chemistry Research*, vol. 42, no. 3, pp. 484-489, 2003.
- [6] R. C. Panda and M. O. Tade, "Modified Smith predictor with PID structure for control of multivariable processes," *International Journal of Process Systems Engineering*, vol. 1, no. 2, p. 197, 2011.
- [7] R. C. Panda, S.-b. Hung and C.-c. Yu, "An Integrated Modified Smith Predictor with PID Controller for Integrator Plus Deadtime Processes Controller for Integrator Plus Deadtime Processes," no. December, pp. 1397-1407, 2016.
- [8] I. Kaya, N. Tan and D. P. Atherton, "Improved cascade control structure for enhanced performance," *Journal of Process Control*, vol. 17, no. 1, pp. 3-16, 2007.
- [9] S. Uma, M. Chidambaram, A. Seshagiri Rao and C. K. Yoo, "Enhanced control of integrating cascade processes with time delays using modified Smith predictor," *Chemical Engineering Science*, vol. 65, no. 3, pp. 1065-1075, 2010.
- [10] J.-J. E. Slotine, W. Li and others, Applied nonlinear control, vol. 199, Prentice hall Englewood Cliffs, NJ, 1991.
- [11] V. I. Utkin, "Survey Paper: Variable Structure Systems with Sliding Modes," *IEEE Transactions on Automatic Control*, vol. 22, no. 2, pp. 212-222, 1977.
- [12] J. Y. Hung, W. Gao and J. C. Hung, "Variable Structure Control: A Survey," *IEEE Transactions on Industrial Electronics*, vol. 40, no. 1, pp. 2-22, 1993.
- [13] O. Camacho, "Controller for Nonlinear Chemical," vol. 122, no. December, 2000.
- [14] O. E. Camacho, C. Smith and E. Chacon, "Toward an implementation of Sliding Mode Control to chemical

processes," *IEEE International Symposium on Industrial Electronics*, vol. 3, no. March 2016, pp. 1101-1105, 1997.

- [15] O. Camacho and C. A. Smith, "Sliding mode control: An approach to regulate nonlinear chemical processes," *ISA Transactions*, vol. 39, no. 2, pp. 205-218, 2000.
- [16] O. Camacho, R. Rojas and W. García, "Variable structure control applied to chemical processes with inverse response," *ISA Transactions*, vol. 38, no. 1, pp. 55-72, 1999.
- [17] O. Camacho, R. Rojas and W. García-Gabín, "Some long time delay sliding mode control approaches," *ISA Transactions*, vol. 46, no. 1, pp. 95-101, 2007.
- [18] O. Camacho and F. De la Cruz, "Smith predictor basedsliding mode controller for integrating processes with elevated deadtime," *ISA Transactions*, vol. 43, no. 2, pp. 257-270, 2007.
- [19] I. Kaya, "Sliding-mode control of stable processes," *Industrial and Engineering Chemistry Research*, vol. 46, no. 2, pp. 571-578, 2007.
- [20] R. C. Panda, "Synthesis of PID tuning rule using the desired closed-loop response," *Industrial and Engineering Chemistry Research*, vol. 47, no. 22, pp. 8684-8692, 19 11 2008.
- [21] V. Vijayan and R. C. Panda, "Design of a simple setpoint filter for minimizing overshoot for low order processes," *ISA Transactions*, vol. 51, no. 2, pp. 271-276, 2012.
- [22] O. a. R. R. Camacho, "A general sliding mode controller for nonlinear chemical processes," *Journal of Dynamic Systems, Measurement, and Control*, vol. 122, no. 4, pp. 650-655, 2000.