Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Evaluation of the Impulsive Force Induced by the Snap-Through Buckling of Closed-Elastica

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Abstract: The present study proposes the evaluation of the impulsive force induced by the snap-through buckling mechanism of closed-elastica in terms of the momentum. A cable-mass system with slackness is developed as a benchmark setup, where the snap-through buckling mechanism imposes the impulsive force on the mass through the wire. The recorded time response of the mass provides the momentum of the snap-through buckling according to the conservation law. Experiments are carried out with several kinds of mass parameters, and the impulsive force induced by the snap-through buckling is successfully evaluated in the proposed context.

Keywords: Mechatronics, Design methodologies, Robotics technology, Elastic materials, Snap-through buckling, Impulsive force.

1. INTRODUCTION

Flexible and elastic materials have attracted the attention of the researchers in the robotics field (Pratt et al. 1995). Unlike conventional materials assumed to be high rigidness, these materials are deformed due to the external force and thus, in physical contact, they offer a function to store the received work as the elastic energy and to release it as the kinetic energy.

One of the robotic applications of elastic materials is the mechanism based on the snap-through buckling phenomenon of the closed-elastica (Yamada et al. 2007). The closed-elastica stores the work of the active joint torque as the elastic energy, and the snap-through buckling releases it instantaneously. These actions produce the impulsive motion of the closed-elastica and also yield the impulsive force when the elastica physically interacts with an object or environment.

The mechanism is mainly employed by small robots such as a swimming robot (Mochiyama et al. 2007) and a jumping robot (Tsuda et al. 2012) to acquire the creature-like agility. Recently the mechanism also plays a central role in the human joint impedance estimation. In (Yagi et al. 2018; Yagi et al. 2019), a wearable device utilizing the snap-through buckling is designed as shown in Fig. 1 to induce the dynamic response of a human joint. The ankle and wrist impedance are successfully estimated under the quasi-stationary condition.

While the impulsive force based on the snap-through buckling mechanism is utilized in a variety of applications, the design methodology has not been provided clearly due to a difficulty in measuring such force quantitatively. The impulsive force is hard to be measured directly by a conventional contact-based sensor. Therefore, rather than the force, another criterion is required to assess and design the snap-through buckling mechanism depending on the applications.

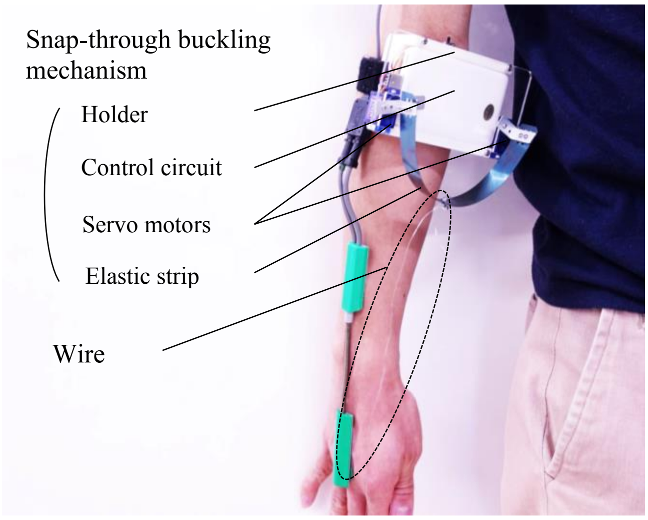

Fig. 1: Wearable type of the snap-through buckling mechanism for the human joint impedance estimation.

In this paper, we focus on the momentum to evaluate the actuation performance of the snap-through buckling mechanism. Since the momentum leads to the profile of the impulsive force at the impact, the pre-evaluated momentum allows us to simulate the dynamic response of an actuated system in advance. Therefore, the momentum can be employed to discuss the snap-through buckling mechanism, which is as well as other criteria employed to discuss actuator performance, such as the force, torque, or power.

The pre-evaluated momentum is especially useful for the joint impedance estimation because it can play a role of the system input and eliminate the need for force sensing. Thus, following the actuation manner presented in (Yagi et al. 2018; Yagi et al. 2019), we develop a benchmark setup so that is modeled by a cable-mass system with slackness (Brogliato, 2016).

In the experiments, the snap-through buckling mechanism physically interacts with an object through a metal wire, where the impulsive force is applied to the object. According to the energy conservation law, the response of the object gives the transferred velocity at the impact, which leads to the momentum generated by the mechanism. Some parameter conditions are examined, and the obtained results are discussed in the context of the impedance estimation.

2. SNAP-THROUGH BUCKLING MECHANISM AND EVALUATION SETUP

2.1 *The snap-through buckling mechanism*

Figure 1 also shows a typical setup of the snap-through buckling mechanism. The mechanism consists of an elastic strip (190 × 20 × 0.15 mm), two servo motors (SG92R: TowerPro), a custom-made holder, and a microprocessor-based control circuit (Arduino). The inward rotation of two servo motors induces the snap-through buckling phenomenon. The center of the elastic strip is extensively accelerated, and the impulsive motion is then produced. Interacting with the surrounding environment, the impulsive motion of the elastic strip generates the impulsive interaction force, and this force is utilized in some robotic applications.

2.2 *Benchmark setup*

To evaluate the actuation performance of the snap-through buckling mechanism, we developed a benchmark setup, as shown in Fig. 2. The benchmark setup aims at deriving the momentum transferred to a mass through the wire. The actuation in this setup can be modeled by the physical interaction between the first order buckling (Stoker, 1950) and the cable-mass system with slackness, which is described by

$$m\dot{v}(t) = -c(t) - F(x(t)) + f(\phi_m(t)) - \lambda(t) - mg\,, \tag{1}$$

$$M\dot{V}(t) = \lambda(t) - Mg\,, \tag{2}$$

$$0 \le \lambda(t) \perp L - X_0 - x(t) + X(t) \ge 0\,, \tag{3}$$

where the weight parameter is represented by m, and the mass parameter is by M. Also, $x(t)$ and $v(t)$ are the position and velocity of the weight, and $X(t)$ and $V(t)$ are those of the mass respectively. $F(x(t))$ is the nonlinear restoring force unique to the profile of the elastic strip. $f(\phi_m(t))$ is the lateral force depending on the servo motor angle, $\phi_m(t)$, in a manner of the series elastic actuator. g is the gravitational constant, which is set to 9.81. L is the length of the wire, and X_0 is the initial position of the mass. $\lambda(t)$ is the tensile force, which acts when the wire is completely elongated. That is, $\lambda(t) = \alpha\delta_{t_0}$, where α is constant and δ_{t_0} is Dirac measure (Brogliato, 2016).

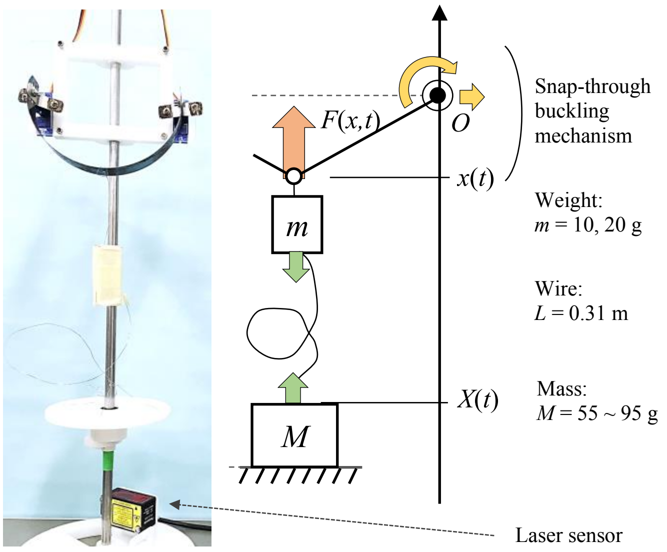

Fig. 2: Benchmark setup with an illustration. This setup consists of the snap-through buckling mechanism with a weight, a wire, a mass, and a position sensor. The guide rod is assumed to be frictionless.

2.3 *Role of the momentum and its evaluation*

Without loss of generality, the time when the tensile force $\lambda(t)$ occurs is set to the initial time, thus $t_0 = 0$. Also, the initial position of the mass is supposed to be the origin.

Let the non-zero value of $V(0^+)$ be denoted by V_0. Provided that $V(0^-) = 0$, the definition of Dirac measure leads to

$$\alpha = MV_0\,. \tag{4}$$

Intuitively, this is because the impulse yields the jump of the velocity and its difference depends on the magnitude α. Thus, the pre-evaluated momentum implies the profile of the impulsive force and thus can be employed as a criterion to consider the actuation performance.

According to (1) – (3), the mass is subjected to the impulsive force through the wire and then starts free motion in the vertical axis. Based on the energy conservation law, the instantaneous velocity of the mass at the initial time can be derived from its apex. Let the apex of the induced 1-DOF motion be denoted by X_t. The velocity V_0 can then be derived by

$$V_0 = \sqrt{2gX_t}\,. \tag{5}$$

Thus, the momentum of the object, denoted by P, is given by

$$P = MV_0 = M\sqrt{2gX_t}\,. \tag{6}$$

3. EXPERIMENTS

3.1 *Conditions*

The experiments are conducted to obtain the initial velocity of the mass, and then the momentum is calculated. The position of the mass is measured using the laser sensor (HG-C1200: Panasonic). The sensing algorithm is developed in MATLAB/Simulink environment and implemented with the Digital Signal Processor (sBOXII: MIS Corporation). The sampling interval is set to T = 1 ms. The recorded data are processed by the fourth-order lowpass filter with the cut-off frequency of 100 Hz.

The weight and the mass parameters are set to m = 10, 20 g and M = 55, 65, 75, 85, 95 g. Five trials are carried out for each condition.

Separately, the snap-through motion of the closed-elastica is captured by a high-speed camera with 1000 fps. The time evolution of the trajectory is analyzed by MATLAB. The velocity of the center of the closed-elastica is obtained by the forward difference of the trajectory in the vertical axis.

3.2 Results

According to (5) and (6), the obtained apex gives the velocity V_0 and the momentum P generated by the snap-through buckling mechanism. The obtained apex and the calculated momentum for each experimental condition are illustrated in Figs. 3 and 4 respectively. Table 1 shows the mean value of the momentum and the maximum velocity of the elastic strip in the snap motion for each weight parameter.

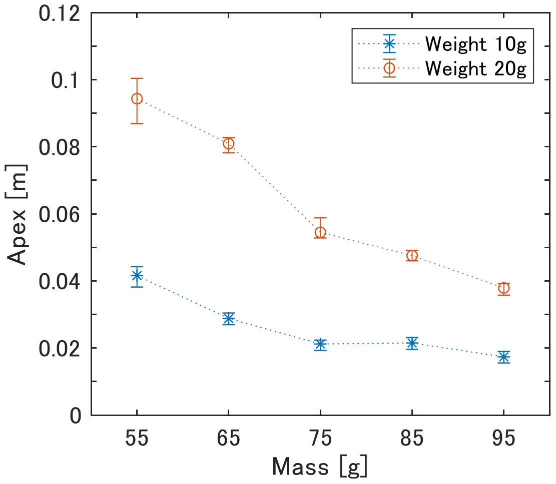

Fig. 3: The apex obtained by each experimental condition.

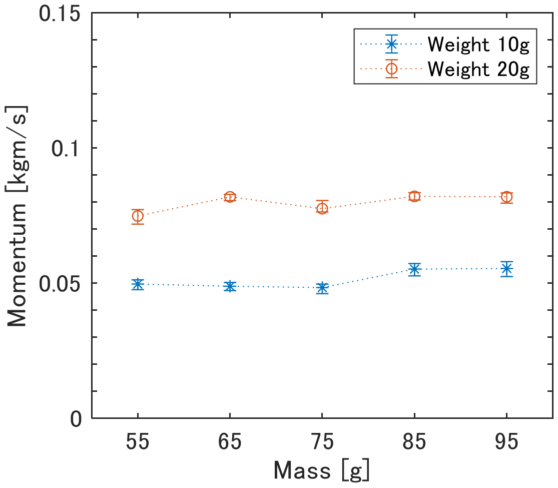

Fig. 4: The calculated momentum for each mass parameter. The marker indicates the mean value of 5 trials, and the error bar means the maximum and the minimum values.

Table 1: The mean value of the momentum of the object generated by the snap-through buckling mechanism, and the maximum velocity of the center of the elastic strip in the snap motion.

Weight [g]	Momentum [kgm/s]	Snap motion [m/s]
10	0.0515	6.286
20	0.0797	4.706

3.3 Discussion

As illustrated in Fig. 3, the apex decreases as the mass parameter increases. Since the momentum of the weight is expected to be constant, the decrease of the apex is reasonable. On the other hand, Fig. 4 illustrates that the calculated momenta of the mass are almost constant and their dispersion is trivial. Although the actuation based on the impulsive motion intuitively seems to contain an indeterminacy, the results indicate that the momentum to be generated by the snap-through buckling through the wire is predictable if the settings, such as the wire length, are the same.

Because of the discussion above, the mean values shown in Table 1 describe the actuation performance of the snap-through buckling mechanism and therefore, they are the main results of this study. The velocity of the snap motion captured by the high-speed camera provides the possible maximum of the momentum together with the attached weight. The maxima are 0.0629 and 0.0941 kgm/s for the weight m = 10 and 20 g respectively. The results of momentum shown in Table 1 are reasonable compared to these maximum values. Furthermore, according to anothr observation based on the high-speed video, they are consistent with the behavior of the benchmark setup during experiments.

The present study successfully evaluates the impulsive force generated by the elastic mechanism in terms of the momentum. A priori knowledge to the momentum transferred to the object allows us to simulate the dynamic response that corresponds to the impulse response. When the momentum is converted into the angular fashion using a moment arm parameter, it is also available in the human joint impedance estimation. The inertia of a joint can be calculated in advance (de Leva, 1996). Thus, using the pre-evaluated momentum shown in Table 1 and the angular response induced by the snap-through buckling, we can search the impedance parameters minimizing an error function defined by the difference between the simulated response and the actual response. That is, the evaluation of the impulsive force in terms of the momentum leads to the force-sensorless impedance estimation method, which is one of the remarkable applications of this study.

The experimental results also provide one of the criteria capable of used to design the snap-through buckling mechanism for robotic systems such as jump and swimming robots and a catapult system. Since the impulsive motion induced by the snap-through buckling mechanism depends on the profile of the elastic strip, we are planning to investigate the momentum produced by another profile of the elastic strip in order to establish the design methodology of the mechanism.

4. CONCLUSION

In the present study, we evaluated the impulsive force induced by the snap-through buckling of closed-elastica in terms of the momentum. The evaluated momentum allows us to simulate the dynamic response of an object to be actuated and thus can be employed as the criterion to design the snap-through buckling mechanism depending on a robotic application.

Further investigation on the relationship between the profile of the elastic strip and the generated momentum is needed to establish the design methodology. Also, the experimental verification of the impedance estimation method that is suggested in this study and is capable of eliminating the need for a force sensor is expected to be conducted in future works.

ACKNOWLEDGMENT

This work was supported by JSPS KAKENHI Grant Number 19K14933.

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