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Simultaneous Update of Controller and Model by Using Virtual Internal Model Tuning

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Abstract: In this paper, we consider data-driven approach to the simultaneous attainment of a controller and a model. Here, we utilize Virtual Internal Model Tuning (VIMT), which is proposed by the same authors, to update a controller with a virtual internal model. We clarify that the VIMT is effective for not only the update of a controller but also the attainment of the plant model so as to reflect a given tracking specification.

Keywords: Data-driven control, Virtual Internal Model Tuning, Modeling and Control

1. INTRODUCTION

Quite recently, there are many studies on control system design that directly utilizes the data without using a mathematical model of a plant. They are referred to as data-driven control approaches such as Iterative Feedback Tuning (IFT) by Hjalmarsson, et al. (2002), Virtual Reference iterative Tuning (VRFT) by Campi, et al. (2002), Fictitious Iterative Reference Tuning (FRIT) by Soma, et al. (2004) and Estimated Response Iterative Tuning (ERIT) by Kaneko and Nakamura (2017). Particularly, VRFT, FRIT and ERIT have practical advantages because these methods require only one-shot experiment data for the off-line optimization while IFT requires many experiments. ERIT minimizes the error between an estimated output and the desired one in the two-degree-of-freedom control system. The estimated output can be described by the initial output, the feedback controller used in the initial experiment, and the desired reference model. ERIT requires only the initial output while VRFT and FRIT requires the input and the output data. This is one of the reasons why ERIT is more effective than other two methods from the practical point of view. However, ERIT is applicable to only two-degree-of-freedom control system. On the other hand, one-degree-of-freedom control system is more widely used compared with two-degree-of-freedom control system. Quite recently, the authors have proposed a new controller parameter tuning by using only output data for a conventional one-degree-of-freedom control system. This method is referred as Virtual Internal Model Tuning (VIMT) in Ikezaki and Kaneko (2019a) and Ikezaki and Kaneko (2019b).

A mathematical model plays a crucial role even in the data-driven approach. For example, a mathematical model gives relevant information on the status of the plant under the operation. From the another viewpoint, a mathematical model is utilized to design more advanced controller in the system design scheme. Thus, it is prefer to not only update a controller but also to obtain a mathematical model of a plant. From the theoretical points of view, there exists a crucial interplay between a controller and a model, which is one of the important unsolved issues in systems and control. It is expected that data-driven approach may yield some relevant and meaningful results on this issue.

From these backgrounds, this paper considers data-driven approach to the simultaneous attainment of a controller and a model. Here, we utilize Virtual Internal Model Tuning (VIMT), which is proposed by the same authors, to update a controller. We also clarify that the VIMT is effective for not only an update of a controller but also the attainment of the plant model so as to reflect a given tracking specification.

2. PROBLEM SETTING

2.1 Notations

Let $\mathbb{R}$ and $\mathbb{R}^{\mathtt{n}}$ denote the set of real numbers and that of real vectors of size $\mathtt{n}$, respectively. Similarly, let $\mathbb{Z}$ denote the set of integers. For a continuous time signal w, we denote the value of w at the time $t \in \mathbb{R}$ as $w(t)$. For a discrete time signal w, we denote the value of w at the time $t \in \mathbb{Z}$ as $\boldsymbol{w}[t]$. For a continuous time signal $\boldsymbol{w}$, we introduce the norm computed by using $w(0), w(\Delta), w(2\Delta) \cdots, w(N\Delta)$ as $\|\boldsymbol{w}\|_{[0,N]} := \sqrt{\sum_{i=0}^{N} w(i\Delta)^2}$. Since the sampling time Δ does not appear in the discussions of this paper explicitly, we do not use the notation 'Δ' of this norm. Similarly, for a discrete time signal $\boldsymbol{w}$, we introduce the norm computed from the trancated signal as $\|\boldsymbol{w}\|_{[0,N]} := \sqrt{\sum_{i=0}^{N} w[i]^2}$. Let q denote the shift operator defined as $qw[i] = w[i+1]$. Let $\mathbb{R}(q)$ denote the set of the rational functions with real coefficients. Let a proper rational transfer function $G(s)$ or $G(q)$ denote a transfer function in the continuous time case or the discrete time case, respectively. Then we denote the output time signal of this system with respect to the input time signal $\boldsymbol{u}$ as $\boldsymbol{y} = G\boldsymbol{u}$ for the enhancement of the readability. Throughout this paper, we omit the notation 's' or 'q' from $G(s)$ or $G(q)$ if it is clearly follows from the context that this is a rational function with respect to the

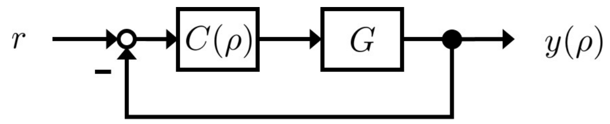

Fig. 1. The closed-loop system

continuous time case or the discrete time case, respectively. Although we focus on the discrete time case in this paper, except Section 6 where a numerical example is presented.

2.2 Problem setting

Consider a closed loop system in Fig. 1. Assume that a plant G is a single input single output, minimum phase and linear time-invariant system. We also assume that G is unknown. We also assume that an effect of a noise is small so as to be neglected. A feedback controller $C(\boldsymbol{\rho})$ is described by a tunable parameter vector $\boldsymbol{\rho}$. For example, a discrete time linear controller $C(\boldsymbol{\rho})$ is parameterized by

$$C(\boldsymbol{\rho}) = \frac{\rho_{m+n+1}q^m + \cdots \rho_{n+2}q + \rho_{n+1}}{\rho_n q^n + \cdots + \rho_1 q + 1} \tag{1}$$

with a parameter vector

$$\boldsymbol{\rho} := [\rho_1 \ \rho_2 \ \cdots \rho_{n+m+1}]^{\mathrm{T}} \in \mathbb{R}^{\mathtt{n+m+1}}. \tag{2}$$

The discrete time PID controller

$$C(\boldsymbol{\rho}) = K_P + K_I \frac{q}{q-1} + K_D \frac{q-1}{q} \tag{3}$$

with a parameter vector

$$\boldsymbol{\rho} := [K_P, K_I, K_D]^{\mathrm{T}} \in \mathbb{R}^3. \tag{4}$$

In Fig. 1, the output and the input of the plant G in this closed loop system which includes a tunable controller $C(\boldsymbol{\rho})$ can be also regarded as a function of the parameter $\boldsymbol{\rho}$. Thus we denote them by $\boldsymbol{y}(\boldsymbol{\rho})$ and $\boldsymbol{u}(\boldsymbol{\rho})$, respectively. Similarly, we also denote the transfer function from the reference signal $\boldsymbol{r}$ to the output $\boldsymbol{y}(\boldsymbol{\rho})$ by

$$T(\boldsymbol{\rho}) := \frac{GC(\boldsymbol{\rho})}{1 + GC(\boldsymbol{\rho})}. \tag{5}$$

It is clear that $\boldsymbol{y}(\boldsymbol{\rho}) = T(\boldsymbol{\rho})\boldsymbol{r}$.

There are several specifications for a controller. In this paper, the aim of the control is to achieve the desired tracking property. Here, we give a desired tracking reference model from $\boldsymbol{r}$ to the output $\boldsymbol{y}$ as T_{d}. Then, the desired output is described by

$$\boldsymbol{y}_{\mathrm{d}} = T_{\mathrm{d}}\boldsymbol{r}. \tag{6}$$

Under these preparations, the problem we consider, here can be formalized as follows. For the closed-loop in Fig. 1, set the initial parameter $\boldsymbol{\rho}^0$ and perform the initial experiment with $C(\boldsymbol{\rho}^0)$. We obtain the initial data $\boldsymbol{u}(\boldsymbol{\rho}^0)$ and $\boldsymbol{y}(\boldsymbol{\rho}^0)$, respectively. Then, find a controller parameter $\boldsymbol{\rho}$ that achieves a given desired output $\boldsymbol{y}_{\mathrm{d}}$ and yields a mathematical model of a plant simultaneously by using only the data.

Throughout of this paper, the initial output data is measured from $t = 0$ until $t = N$, i.e., the data is regarded as the real vector of size $N + 1$ as

$$\boldsymbol{y}(\boldsymbol{\rho}^0) = [y(\boldsymbol{\rho}^0)(0), y(\boldsymbol{\rho}^0)(1), \cdots, y(\boldsymbol{\rho}^0)(N)]^{\mathrm{T}} \in \mathbb{R}^{\mathtt{N+1}}.$$

3. VIRTUAL INTERNAL MODEL TUNING

In this section, we give a brief explanation of Virtual Internal Model Tuning (VIMT) proposed by the authors. See details in Ikezaki and Kaneko (2019a) and Ikezaki and Kaneko (2019b).

Consider the closed-loop system in Fig. 1 by using the initial controller $C(\boldsymbol{\rho}^0)$ with the initial parameter $\boldsymbol{\rho}^0$. Suppose that we have already done the experiment with $C(\boldsymbol{\rho}^0)$ and we obtained that the initial output data

$$y(\boldsymbol{\rho}^0) = T(\boldsymbol{\rho}^0)r. \tag{7}$$

Consider a controller $C(\boldsymbol{\rho})$ with an arbitrary parameter $\boldsymbol{\rho}$. In this case, the output data can be written

$$\boldsymbol{y}(\boldsymbol{\rho}) = T(\boldsymbol{\rho})\boldsymbol{r}. \tag{8}$$

Focus on the relationship between $\boldsymbol{y}(\boldsymbol{\rho}^0)$ and $\boldsymbol{y}(\boldsymbol{\rho})$. From (5), (7) and (8), we simply obtain

$$\begin{aligned} \boldsymbol{y}(\boldsymbol{\rho}) &= T(\boldsymbol{\rho})T(\boldsymbol{\rho}^0)^{-1}\boldsymbol{y}(\boldsymbol{\rho}^0) \\ &= \frac{C(\boldsymbol{\rho})}{1 + GC(\boldsymbol{\rho})} \frac{1 + GC(\boldsymbol{\rho}^0)}{C(\boldsymbol{\rho}^0)} \boldsymbol{y}(\boldsymbol{\rho}^0). \end{aligned} \tag{9}$$

If the right hand side of (9) is completely known, the output $\boldsymbol{y}(\boldsymbol{\rho})$ of the closed loop with $C(\boldsymbol{\rho})$ can be also completely predicted. However, we can not utilize (9) to predict $\boldsymbol{y}(\boldsymbol{\rho})$ because it includes G which is assumed to be unknown. To overcome such a difficulty, we need to eliminate G from (9). We introduce the ideal controller C_{d} so as to satisfy

$$T_{\mathrm{d}} = \frac{GC_{\mathrm{d}}}{1 + GC_{\mathrm{d}}}. \tag{10}$$

This implies that C_{d} is the controller that achieves a given tracking response T_{d}. Eq. (10) also yields a representation of a plant as

$$G = \frac{T_{\mathrm{d}}}{(1 - T_{\mathrm{d}})C_{\mathrm{d}}}. \tag{11}$$

Substituting G in (11) into (9) yields

$$\boldsymbol{y}_{\mathrm{d}} = \left\{ T_{\mathrm{d}} + (1 - T_{\mathrm{d}}) \frac{C_{\mathrm{d}}}{C(\boldsymbol{\rho}^0)} \right\} \boldsymbol{y}(\boldsymbol{\rho}^0). \tag{12}$$

The goal is to obtain the controller that yields the desired output $\boldsymbol{y}_{\mathrm{d}} = T_{\mathrm{d}}\boldsymbol{r}$ in (6). Thus, from (12), if we can solve the following linear equation with respect to $C(\boldsymbol{\rho})$

$$\boldsymbol{y}_{\mathrm{d}} = \left\{ T_{\mathrm{d}} + (1 - T_{\mathrm{d}}) \frac{C(\boldsymbol{\rho})}{C(\boldsymbol{\rho}^0)} \right\} \boldsymbol{y}(\boldsymbol{\rho}^0), \tag{13}$$

then the solution of Eq.(13), say $C(\boldsymbol{\rho}^*)$, is the ideal controller C_{d}. This is the basic idea of the VIMT. As easily shown, C_{d} that satisfies (10) can be described by

$$C_{\mathrm{d}} = \frac{T_{\mathrm{d}}}{1 - T_{\mathrm{d}}} \frac{1}{G}. \tag{14}$$

This is the internal model controller (IMC) in Morari and Zafiriou (1989) where the internal model is equal to the actual plant G and the IMC filter is equal to T_{d}. G does not appear in our proposed method but it is only used for converting (9) to (12). This is the reason why we refer the above method as Virtual Internal Model Tuning (VIMT).

From the practical points of view, it is difficult to solve (13). Because the structure of the implemented tunable controller $C(\boldsymbol{\rho})$ is not equivalent to that of C_{d}. For example, if we use PID controller, then it is impossible to describe C_{d} by using only three PID parameters in most cases. Thus, we minimize the following cost function as

$$\begin{aligned} J(\boldsymbol{\rho}) &:= \|\boldsymbol{y}_{\mathrm{d}} - \boldsymbol{y}(\boldsymbol{\rho})\|_{[0,N]} \\ &= \left\| \boldsymbol{y}_{\mathrm{d}} - \left\{ T_{\mathrm{d}} + (1-T_{\mathrm{d}}) \frac{C(\boldsymbol{\rho})}{C(\boldsymbol{\rho}^0)} \right\} \boldsymbol{y}(\boldsymbol{\rho}^0) \right\|_{[0,N]} \end{aligned} \tag{15}$$

From the above discussion, the optimal parameter

$$\boldsymbol{\rho}^* := \arg\min_{\boldsymbol{\rho}} J(\boldsymbol{\rho}) \tag{16}$$

yields an controller that achieves the trajectory close to the desired output $\boldsymbol{y}_{\mathrm{d}}$ in the sense that the error between $\boldsymbol{y}_{\mathrm{d}}$ and the predicted output $\boldsymbol{y}(\boldsymbol{\rho}^*)$ by using $\boldsymbol{\rho}^*$ is minimized.

4. SIMULTANEOUS UPDATE OF A CONTROLLER AND A MODEL

In this section, we present a simultaneous update of a controller and a model by using VIMT. Since the cost function (15) describes the minimization between the desired output and the estimated one by using the tunable controller, it is intuitively natural to see that the minimization of $J(\boldsymbol{\rho})$ improves the performance of the tracking property. In fact, as shown in Ikezaki and Kaneko (2019b), the cost function (15) can be written by

$$J(\boldsymbol{\rho}) = \left\| \left(1 - \frac{C(\boldsymbol{\rho})}{C_{\mathrm{d}}}\right) T(1 - T(\boldsymbol{\rho}^0))\boldsymbol{y}_{\mathrm{d}} \right\|_{[0,N]}. \tag{17}$$

This implies that the minimization of $J(\boldsymbol{\rho})$ is effective for minimizing the relative error of $C(\boldsymbol{\rho})$ and C_{d} under the sensitivity function of the initial closed loop $1-T(\boldsymbol{\rho}^0)$ and the desired output $\boldsymbol{y}_{\mathrm{d}}$.

Consider the attainment of the model. We construct the *virtual internal model* $\tilde{G}(\boldsymbol{\rho})$ described by

$$\tilde{G}(\boldsymbol{\rho}) := \frac{T_{\mathrm{d}}}{(1-T_{\mathrm{d}})C(\boldsymbol{\rho})}. \tag{18}$$

VIMT is a tuning method of a controller for achieving the desired output, i.e., this is used for improvement of the performance of the tracking property. Simultaneously, the obtained controller $C(\boldsymbol{\rho}^*)$ yields a mathematical model as

$$\tilde{G}(\boldsymbol{\rho}^*) = \frac{T_{\mathrm{d}}}{1-T_{\mathrm{d}}} \frac{1}{C(\boldsymbol{\rho}^*)} = G \frac{C_{\mathrm{d}}}{C(\boldsymbol{\rho}^*)}. \tag{19}$$

This implies that $\tilde{G}(\boldsymbol{\rho}^*)$ approaches to the actual plant G in the frequency where the updated controller $C(\boldsymbol{\rho}^*)$ approaches to the ideal controller C_{d}. A given specification T_{d} for a plant G directly relates to C_{d}. Thus, we see that update of the controller so as to achieve the specification also gives a mathematical model $\tilde{G}(\boldsymbol{\rho}^*)$ close to the actual G under the frequency range where T_{d} is dominant.

The attainment of a model can be explained in the different way as follows. The cost function in (15) can be written by

$$\begin{aligned} J(\boldsymbol{\rho}) &= \|\boldsymbol{y}_{\mathrm{d}} - \boldsymbol{y}(\boldsymbol{\rho})\|_{[0,N]} \\ &= \left\| T_{\mathrm{d}}\boldsymbol{r} - T_{\mathrm{d}}\boldsymbol{y}(\boldsymbol{\rho}^0) - T_{\mathrm{d}} \frac{(1-T_{\mathrm{d}})}{T_{\mathrm{d}}} \frac{C(\boldsymbol{\rho})}{C(\boldsymbol{\rho}^0)} \boldsymbol{y}(\boldsymbol{\rho}^0) \right\|_{[0,N]} \\ &= \left\| T_{\mathrm{d}} \left(\boldsymbol{r} - \boldsymbol{y}(\boldsymbol{\rho}^0) - \frac{(1-T_{\mathrm{d}})}{T_{\mathrm{d}}} \frac{C(\boldsymbol{\rho})}{C(\boldsymbol{\rho}^0)} \boldsymbol{y}(\boldsymbol{\rho}^0) \right) \right\|_{[0,N]}. \end{aligned} \tag{20}$$

By substituting the relationship (18) into the third term in the right hand side of (20), we rewrite it as

$$J(\boldsymbol{\rho}) = \left\| T_{\mathrm{d}} \left(\boldsymbol{r} - \boldsymbol{y}(\boldsymbol{\rho}^0) - \frac{1}{\tilde{G}(\boldsymbol{\rho})} \frac{1}{C(\boldsymbol{\rho}^0)} \boldsymbol{y}(\boldsymbol{\rho}^0) \right) \right\|_{[0,N]}. \tag{21}$$

Let focus on the last term of the right hand side of (21). From the relationship between the input and the output with respect to the initial controller, we see that

$$u(\boldsymbol{\rho}^0) = C(\boldsymbol{\rho}^0)(\boldsymbol{r} - \boldsymbol{y}(\boldsymbol{\rho}^0))$$

which is equivalent to

$$\frac{1}{C(\boldsymbol{\rho}^0)} y(\boldsymbol{\rho}^0) = G(\boldsymbol{r} - \boldsymbol{y}(\boldsymbol{\rho}^0)). \tag{22}$$

Here, we have used the trivial relationship $\boldsymbol{y}(\boldsymbol{\rho}^0) = G\boldsymbol{u}(\boldsymbol{\rho}^0)$. Substituting (22) into the last term of the right hand side of (21) yields

$$\begin{aligned} J(\boldsymbol{\rho}) &= \left\| T_{\mathrm{d}} \left(\boldsymbol{r} - \boldsymbol{y}(\boldsymbol{\rho}^0) - \frac{G}{\tilde{G}(\boldsymbol{\rho})} (\boldsymbol{r} - \boldsymbol{y}(\boldsymbol{\rho}^0)) \right) \right\|_{[0,N]} \\ &= \left\| \left(1 - \frac{G}{\tilde{G}(\boldsymbol{\rho})}\right) T_{\mathrm{d}}(\boldsymbol{r} - \boldsymbol{y}(\boldsymbol{\rho}^0)) \right\|_{[0,N]}. \end{aligned} \tag{23}$$

From (23), the minimization of $J(\boldsymbol{\rho})$ means the minimization of the relative error between the actual plant G and the virtual internal model $\tilde{G}(\boldsymbol{\rho})$ under the desired tracking reference model T_{d} and the initial error signal $\boldsymbol{r} - \boldsymbol{y}(\boldsymbol{\rho}^0)$. In other words, if the response when using the acquired $\boldsymbol{\rho}^*$ matches the target response, the virtual internal model $\tilde{G}(\boldsymbol{\rho}^*)$ approaches to G.

5. ALGORITHM

We summarize the algorithm of our proposed method.

0. Give a desired reference model T_{d}.
1. For the closed-loop in Fig. 1, set the initial parameter $\boldsymbol{\rho}^0$ and perform the initial experiment with $C(\boldsymbol{\rho}^0)$. We obtain the initial output data $\boldsymbol{y}(\boldsymbol{\rho}^0)$. Suppose that $C(\boldsymbol{\rho}^0)$ is minimum phase and $T(\boldsymbol{\rho}^0)$ is stable.
2. By using T_{d}, $C(\boldsymbol{\rho}^0)$ and $\boldsymbol{y}(\boldsymbol{\rho}^0)$, we minimize the cost function (15) with respect to $\boldsymbol{\rho}$.
3. The optimal parameter $\boldsymbol{\rho}^* := \arg\min_{\boldsymbol{\rho}} J(\boldsymbol{\rho})$ yields the updated controller $C(\boldsymbol{\rho}^*)$ which leads to the improvement of the control performance and the attainment of the model in the sense that (17) and (23) (or (19)), are minimized.

In the following, we consider the optimization of (15). In the case where the structure of the tunable controller is described by (1), we need to apply a non-linear optimization because parameters are non-linearly involved with the cost function (15). In such a case, some algorithms, e.g., Gauss-Newton method, Particle Swarm Optimization in

Kenmedy and Eberhart (1995) and CMA-ES in Hansen (2005), are utilized. As an example of the case of non-convex optimization, we explain how the Gauss-Newton method is utilized for step 2. Let $C(\boldsymbol{\rho})$ is given as (1). when the error in the cost function is expressed

$$\boldsymbol{\xi}(\boldsymbol{\rho}) := \boldsymbol{y}_{\mathrm{d}} - \left\{ T_{\mathrm{d}} + (1 - T_{\mathrm{d}}) \frac{C(\boldsymbol{\rho})}{C(\boldsymbol{\rho}^0)} \right\} \boldsymbol{y}(\boldsymbol{\rho}^0) \in \mathbb{R}^{\mathtt{N+1}}, \quad (24)$$

The Jacobian $\lambda(\boldsymbol{\rho})$ of (24) can be written as

$$\lambda(\boldsymbol{\rho}) = \frac{\partial \boldsymbol{\xi}(\boldsymbol{\rho})}{\partial \boldsymbol{\rho}} = \frac{1 - T_{\mathrm{d}}}{C(\boldsymbol{\rho}^0)} \left[\frac{\partial C(\boldsymbol{\rho})}{\partial \rho_1}, \frac{\partial C(\boldsymbol{\rho})}{\partial \rho_2}, \cdots \frac{\partial C(\boldsymbol{\rho})}{\partial \rho_{m+n+1}} \right] \boldsymbol{y}(\boldsymbol{\rho}^0) \in \mathbb{R}^{(\mathtt{N+1}) \times (\mathtt{m+n+1})}. \quad (25)$$

In Gauss-Newton method, an initial value $\boldsymbol{\rho}^{(0)}$is set. In each step k, the variation $\delta\boldsymbol{\rho}^{(k)}$ is obtained as

$$\delta\boldsymbol{\rho}^{(k)} := -\{\lambda(\boldsymbol{\rho}^{(k)})^T \lambda(\boldsymbol{\rho}^{(k)})\}^{-1} \lambda(\boldsymbol{\rho}^{(k)})^T \boldsymbol{\xi}(\boldsymbol{\rho}^{(k)}) \in \mathbb{R}^{\mathtt{m+n+1}}, \quad (26)$$

and, update parameters by

$$\boldsymbol{\rho}^{(k+1)} := \boldsymbol{\rho}^{(k)} + \delta\boldsymbol{\rho}^{(k)} \in \mathbb{R}^{\mathtt{m+n+1}}. \quad (27)$$

By repeating this process until the value of $\|\boldsymbol{\xi}(\boldsymbol{\rho}^{(k)})\|_{[0,N]}$ becomes sufficiently small, $\boldsymbol{\rho}^*$ can be obtained.

In the case where the structure of the tunable controller is linearly parameterized such as (3), the optimization of (15) can be done by using a convex optimization, least squares and so on. Here, we explain how the least squares is utilized for the above step 2. Let $C(\boldsymbol{\rho})$ be a linearly-parameterized controller described by

$$C(\boldsymbol{\rho}) = \rho_1\alpha_1(q) + \rho_2\alpha_2(q) + \cdots + \rho_n\alpha_n(q) = \sum_{i=1}^{n} \rho_i\alpha_i(q) \quad (28)$$

where $\boldsymbol{\rho} := [\rho_1, \rho_2, \cdots, \rho_n]^T \in \mathbb{R}^{\mathtt{n}}$ and $\alpha_i(q) \in \mathbb{R}(q)$, $i = 1, 2, \cdots, n$, are fixed rational functions. For example, in the case of the PID controller, they are written by $\alpha_1 = 1$, $\alpha_2 = \frac{q}{q-1}$ and $\alpha_3 = \frac{q-1}{q}$. Apply (28) to the error signal to be minimized in (15) as

$$\boldsymbol{\xi}(\boldsymbol{\rho}) := \boldsymbol{y}_{\mathrm{d}} - \left\{ T_{\mathrm{d}} + (1 - T_{\mathrm{d}}) \frac{C(\boldsymbol{\rho})}{C(\boldsymbol{\rho}^0)} \right\} \boldsymbol{y}(\boldsymbol{\rho}^0) = \boldsymbol{y}_{\mathrm{d}} - \left\{ T_{\mathrm{d}} + \frac{(1 - T_{\mathrm{d}})}{C(\boldsymbol{\rho}^0)} \sum_{i=1}^{n} \rho_i\alpha_i(q) \right\} \boldsymbol{y}(\boldsymbol{\rho}^0) = \boldsymbol{w} - \sum_{i=1}^{n} \boldsymbol{v}_i\rho_i \quad (29)$$

where

$$\boldsymbol{w} := \boldsymbol{y}_{\mathrm{d}} - T_{\mathrm{d}}\boldsymbol{y}(\boldsymbol{\rho}^0) \in \mathbb{R}^{\mathtt{N+1}} \quad (30)$$

and

$$\boldsymbol{v}_i := \left(\frac{1 - T_{\mathrm{d}}}{C(\boldsymbol{\rho}^0)} \alpha_i(q) \right) \boldsymbol{y}(\boldsymbol{\rho}^0) \in \mathbb{R}^{\mathtt{N+1}}, \quad i = 1, \cdots, n. \quad (31)$$

It is easily seen from the conventional least squares method that the minimization of $J(\boldsymbol{\rho}) = \|\boldsymbol{\xi}(\boldsymbol{\rho})\|_{[0,N]}$ can be done by computing

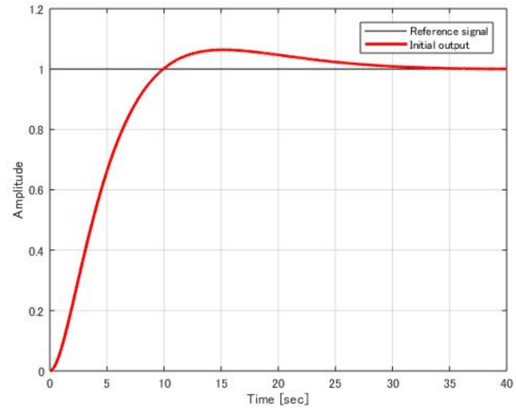

Fig. 2. The initial output

$$\boldsymbol{\rho}^* = (\boldsymbol{V}^{\mathrm{T}}\boldsymbol{V})^{-1}\boldsymbol{V}^{\mathrm{T}}\boldsymbol{w} \in \mathbb{R}^{\mathtt{n}} \quad (32)$$

$$\boldsymbol{V} := [\, \boldsymbol{v}_1 \ \boldsymbol{v}_2 \ \cdots \ \boldsymbol{v}_n \,] \in \mathbb{R}^{(\mathtt{N+1}) \times \mathtt{n}}. \quad (33)$$

6. NUMERICAL EXAMPLE

Consider a plant

$$G = \frac{1}{(s+1)(10s+1)}. \quad (34)$$

The controller is a tunable PID controller as

$$C(\boldsymbol{\rho}) = \rho_1 + \frac{\rho_2}{s} + \rho_3 \frac{s}{0.1s+1}. \quad (35)$$

The reference signal $\boldsymbol{r}$ is the unit step signal and the sampling time is 0.01[sec].

We perform the initial numerical experiment by using the initial controller parameter $\boldsymbol{\rho}^0 := [2, 0.3, 0]$. The output in the initial setting is shown in Fig. 2 where we see that the initial output $\boldsymbol{y}(\boldsymbol{\rho}^0)$ has an over shoot.

We take the following four desired tracking models as difference specifications to see how a specification effects the update of the control performance and how works on the attainment of the virtual internal model.

$$\begin{cases} T_{\mathrm{d}1} = \dfrac{1}{0.2s+1}, & T_{\mathrm{d}2} = \dfrac{1}{(0.2s+1)^2} \\ T_{\mathrm{d}3} = \dfrac{1}{2s+1}, & T_{\mathrm{d}4} = \dfrac{1}{(2s+1)^2}. \end{cases} \quad (36)$$

$T_{\mathrm{d}1}$ and $T_{\mathrm{d}2}$ have a cut-off frequency 5.0×10^0[rad/sec] that is higher frequency than the frequency range of G. While $T_{\mathrm{d}3}$ and $T_{\mathrm{d}4}$ have a cut-off frequency 5.0×10^{-1}[rad/sec] that is between the two cutoffs of the plant system G.

For these $T_{\mathrm{d}i}$, we apply VIMT. In the following, the notation PIDi corresponds to the results for each tracking model $T_{\mathrm{d}i}$, $i = 1, 2, 3, 4$, respectively. In the update calculation, since the controller is PID, parameters are obtained by the least square method.

As results, we obtain the optimal parameters as shown in Table 1.

The outputs of the obtained PID parameters by using VIMT are shown in Fig. 3, Fig. 5, Fig. 7, Fig. 9, respectively. In these figures, the reference signal, the desired output and the output of the closed loop with the updated PID are drawn by the black line, the red line,

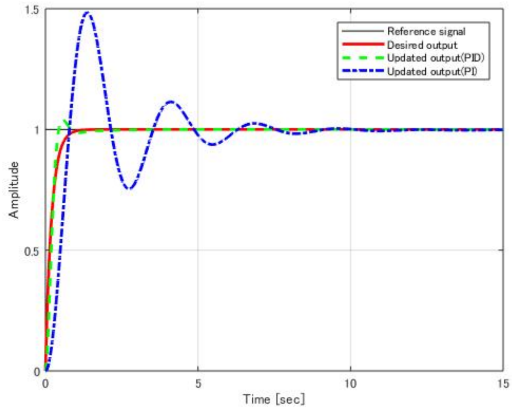

Fig. 3. The output response and the desired output for T_{d1}.

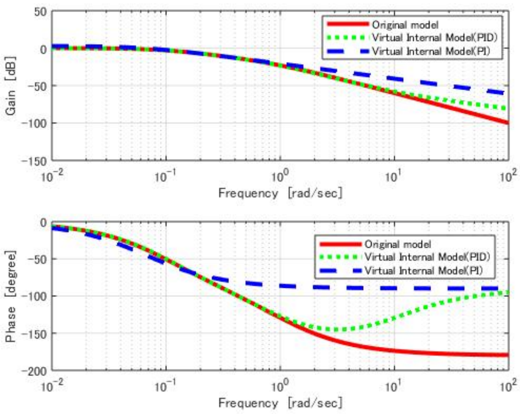

Fig. 4. Comparison of the Bode diagrams of Virtual Internal Model $\tilde{G}(\boldsymbol{\rho}^*)$ and the original G for T_{d1}.

and the green chain line, respectively. Simultaneously, the frequency characteristics of the obtained virtual internal model $\tilde{G}(\boldsymbol{\rho}^*)$ by using (18) and the original plant G are shown in Fig. 4, Fig. 6, Fig. 8, and Fig. 10. In these figures, the bode plots of the original model and the obtained virtual model are drawn by the red dotted line and the green solid line, respectively.

In Fig. 3, the updated output can not achieve the desired output due to the fact that the desired reference model T_{d1} requires the control performance for wide frequency range (the cut off frequency is 5[rad/sec] while those of T_{d3} and T_{d4} are 0.5[rad/sec]) and the fast transient response (the relative degree is 1 while those of T_{d2} and T_{d4} are two). The reason for this is that the initial data does not include information of G to achieve the specification given by T_{d1}. Except the case 1, the improvement of the controller performance is better than the initial output.

Table 1. PID controller parameters obtained by VIMT

Controller	Proportional	Integral	Differential
PID1	53.7851	4.9999	50.1434
PID2	27.2503	2.5000	22.2804
PID3	5.4440	0.5000	5.0059
PID4	2.5000	0.2500	0.0000

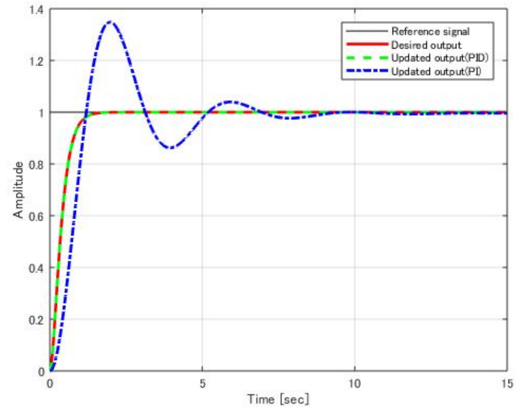

Fig. 5. The output response and the desired output for T_{d2}.

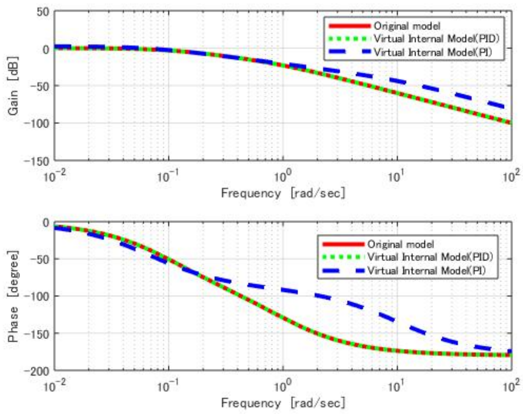

Fig. 6. Comparison of the Bode diagrams of Virtual Internal Model $\tilde{G}(\boldsymbol{\rho}^*)$ and the original G for T_{d2}.

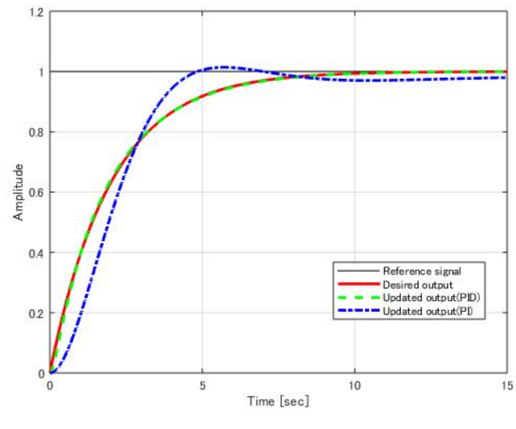

Fig. 7. The output response and the desired output for T_{d3}.

From Fig. 4, Fig. 6, Fig 8 and Fig 10, the frequency properties of each obtained virtual model approaches to that of the original plant G in the frequency range which is less than each cut off frequency. The relative degree of T_{d1} and T_{d3} are smaller than that of G, their frequency responses do not match for that of G over their cut-off frequency. Summing up, we see that the proposed method

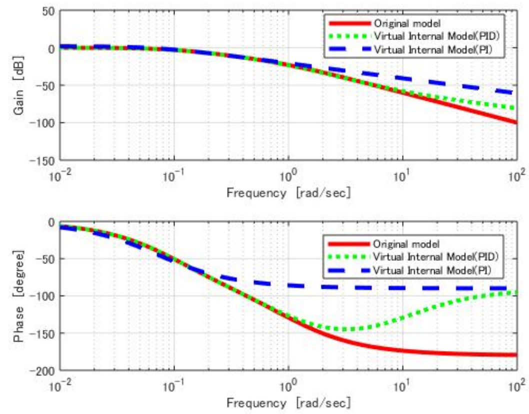

Fig. 8. Comparison of the Bode diagrams of Virtual Internal Model $\tilde{G}(\boldsymbol{\rho}^*)$ and the original G for $T_{\rm d3}$.

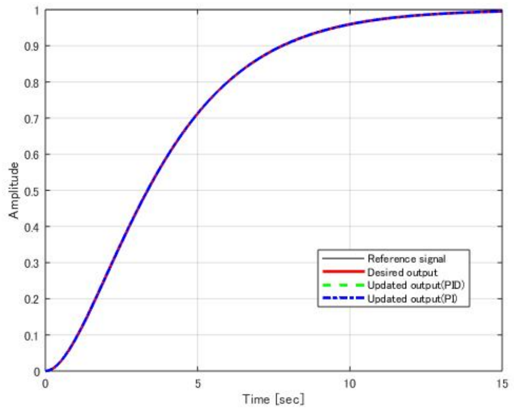

Fig. 9. The output response and the desired output for $T_{\rm d4}$.

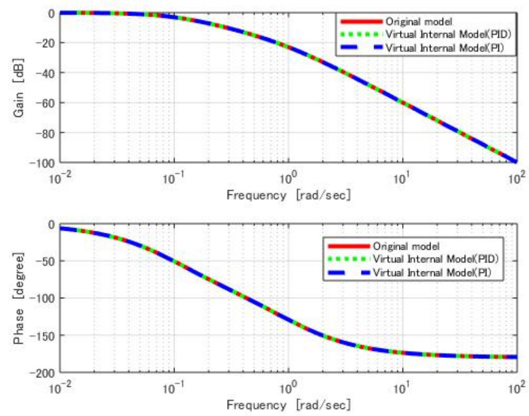

Fig. 10. Comparison of the Bode diagrams of Virtual Internal Model $\tilde{G}(\boldsymbol{\rho}^*)$ and the original G for $T_{\rm d4}$.

gives not only controller but also an appropriate model for each case within each required frequency range to achieve the desired property.

7. CONCLUSION

In this paper, we have provided data-driven parameter tuning of a controller together with the attainment of the model of a plant. Here, we have utilized Virtual Internal Model Tuning (VIMT), which is proposed by the same authors, to update a controller with a model. We have clarified that the VIMT is effective for not only an update of a controller but also attainment of the plant model in the control frequency. Future issues include selection of reasonable specifications, evaluation of update performance considering noise, and improvement of control performance by extending the controller class.

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