Control design for interval type-2 Takagi-Sugeno singular systems with time delay

Salama Makni * **Mourad Kchaou** ** **Ahmed El Hajjaji** *** **Mohamed Chaabane** ****

* *University of Gabes, Department of electrical and automatic engineering, 6029, Gabes, Tunisia (e-mail: maknisalama@gmail.com).*
** *Department of Electrical Engineering, College of Engineering, University of Hail, PB 2440, Hail, Saudi Arabia (e-mail: mouradkchaou@gmail.com)*
*** *MIS Laboratory, University of Picardie Jules Verne, 33 Rue St Leu, 80000, Amiens, France (e-mail: hajjaji@u-picardie.fr)*
**** *University of Sfax, National school of engineering of Sfax, Lab-STA, LR11ES50, 3038, Sfax, Tunisia (e-mail: chaabane.ucpi@gmail.com)*

Abstract In this work, the admissibility analysis and the control problem are investigated for nonlinear singular systems represented by interval type-2 Takagi-Sugeno (T-S) fuzzy models with time delay. First, the interval type-2 fuzzy singular systems with time delay have been described. Second, the admissibility analysis of the autonomous singular systems is studied. Third, the control problem has been investigated. For this, an interval type-2 fuzzy control law is designed to guarantee the admissibility of the closed-loop system despite the presence of uncertainties and time delay. To demonstrate the existence of the proposed controller, by using generalised integral inequalities, sufficient delay-dependent conditions are given in terms of Linear Matrix Inequalities $(LMIs)$. Finally, an application to inverted pendulum system presented by interval type-2 fuzzy models is afforded to show the effectiveness of the suggested method.

Keywords: Interval type-2 fuzzy systems, singular systems, time delay, delay-dependent admissibility, $LMIs$.

1. INTRODUCTION

Recently, the control problem for singular systems has been studied in several researches due to their ability to describe greatly many applications such as power systems and robot manipulators. Studying singular systems is more complex because not only the stability should be guaranteed, but also the regularity and non impulsiveness need to be verified Dai (1989). Due to the effect of time delay on stability and system performance, many researchers have focused on control problem for singular systems with time delay Cui et al. (2013), Gassara et al. (2014), Kchaou (2019). Several methods have been proposed to deal with time delay such as Jensen inequality and Wirtinger inequality which are special cases of generalized integral inequality Park et al. (2018).
In the last decades, the T-S fuzzy representation Takagi and Sugeno (1985) has been considered to handle the control problem of nonlinear systems. Many results of controller synthesis have been proposed using type-1 fuzzy model Chang et al. (2012), Kchaou and Hajjaji (2017), Kchaou et al. (2018), Makni et al. (2019). However, research results which consider type-1 fuzzy model do not take into account uncertainties in the membership functions. Thus, we should study the type-2 fuzzy systems to solve the parametric uncertainties which are hidden in the membership functions (we can cite Lam et al. (2013), Li et al. (2017) and Tseng et al. (2017) and references therein). For example, authors in Lam et al. (2013) have studied the type-2 fuzzy systems where the time delay has not been taken into account. Furthermore, in Li et al. (2017), singular systems have not been considered for the investigation of control problem. Moreover, authors in Tseng et al. (2017) have focused on type-2 fuzzy control design for singular systems with time delay. However, they have considered a classical Lyapunov-Krasovskii functional which is a restrictive method.
In this work, the admissibility analysis and the control design problem are investigated for interval type-2 fuzzy singular systems in presence of time delay. Based on Lyapunov-Krasovskii theory and using generalised integral inequalities, delay-dependent conditions formulated in terms of convex optimization problem are proposed to ensure the admissibility of the closed-loop system. Furthermore, simulation results are illustrated to show the convergence of the inverted pendulum system which prove the efficiency of the proposed control design.
This paper is organised as follows. In section 2, interval type-2 T-S singular systems with time delay is introduced. In section 3, the admissibility analysis is presented. In section 4, the control design and delay-dependent conditions are given to guarantee the admissibility of closed-loop system in presence of time delay. Section 5 presents the simulation results to show the effectiveness of the proposed design method. To close this work, some comments are given in section 6.
Notations Throughout this paper, we note that X^T and X^{-1} are the transpose and the inverse of matrix X, respectively. $X > 0$ represents a positive definite matrix. $sym(X) = X + X^T$. In a matrix, $*$ denotes the transposed element in the symmetric position. $\star$ denotes elements that are not relevant to discussions.

2. SYSTEM DESCRIPTION

In this paper, singular systems with time delay using the interval type-2 T-S fuzzy approach have been investigated. This class of systems can be represented as follows:

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + A_{hi} x(t-\tau) + B_i u(t)) \\ x(t) = \phi(t),\ t \in [-\tau, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^m$ denote system state and control input vectors, respectively. $\phi(t)$ denotes the initial condition for $-\tau \leq t \leq 0$. A_i, A_{hi} and B_i are system matrices. E is a singular matrix which satisfies $rank(E) = q < n$, τ represents a constant delay and ξ is the premise variable. Membership functions are defined as

$$\begin{cases} \mu_i(\xi(t)) = \underline{v}_i(\xi(t))\underline{\mu}_i(\xi(t)) + \bar{v}_i(\xi(t))\bar{\mu}_i(\xi(t)) \\ \sum_{i=1}^{r} \mu_i(\xi(t)) = 1 \end{cases} \tag{2}$$

where $0 \leq \underline{v}_i(\xi(t)), \bar{v}_i(\xi(t)) \leq 1,\ \underline{v}_i(\xi(t)) + \bar{v}_i(\xi(t)) = 1$ in which $\underline{v}_i(\xi(t))$ and $\bar{v}_i(\xi(t))$ are weighting functions.
We have:

$$\begin{aligned} \tilde{\mu}_i(\xi(t)) &= [\underline{\mu}_i(\xi(t)), \bar{\mu}_i(\xi(t))] \\ &= [\prod_{j=1}^{p} \underline{h}_{W_{ij}}(f_j(\xi(t))), \prod_{j=1}^{p} \bar{h}_{W_{ij}}(f_j(\xi(t)))] \end{aligned}$$

with

$$\bar{h}_{W_{ij}}(f_j(\xi(t))) \geq \underline{h}_{W_{ij}}(f_j(\xi(t))) \geq 0$$

in which $0 \leq \underline{h}_{W_{ij}}(f_j(\xi(t))), \bar{h}_{W_{ij}}(f_j(\xi(t))) \leq 1$ represent the lower and upper membership functions and W_{ij} are interval type-2 fuzzy set of rules i; $i = 1, \ldots, r,\ \ j = 1 \ldots, p$.
Consider the following singular system with time delay.

$$\begin{cases} E\dot{x}(t) = Ax(t) + A_h x(t-\tau) \\ x(t) = \phi(t),\ t \in [-\tau, 0] \end{cases} \tag{3}$$

Before starting, we recall the following definition and lemmas which will be considered throughout the paper:

Definition 1. Dai (1989)
1. System (3) is said to be regular if $det(sE - A)$ is not identically zero.
2. System (3) is said to be impulse free if $deg(det(sE - A)) = rank(E)$.
3. System (3) is admissible if it is regular, impulse free and stable.

Lemma 1. Park et al. (2018)
For a symmetric matrix $M > 0$, scalars a and b with $a < b$, vector z, $k = 1, \ldots, 3$ and by selecting $p_{0,0}(s) = 1, p_{1,0}(s) = s - a - \frac{b-a}{2}$ and $p_{2,0} = (s-a)^2 - (b-a)(s-a) + \frac{(b-a)^2}{6}$, the following result holds:

$$(b-a)\int_a^b \dot{z}(s)^T E^T M E \dot{z}(s) ds \geq \sum_{k=1}^{3} (2k-1)\Gamma_k^T(z) M \Gamma_k(z) \tag{4}$$

with
$\Gamma_1(z) = E(z(b) - z(a))$
$\Gamma_2(z) = E(z(b) + z(a)) - \frac{2}{b-a}\int_a^b Ez(s)ds$

$$\begin{aligned} \Gamma_3(z) &= E(z(b) - z(a)) + \frac{6}{b-a}\int_a^b Ez(s)ds \\ &\quad - \frac{12}{(b-a)^2}\int_a^b \int_s^b Ez(u)duds \end{aligned}$$

Lemma 2. Uezato and Ikeda (1999)
Singular matrix E can be written as $E = E_L E_R^T$ where E_L and E_R are full column ranks. Let U be full row rank and V be full column rank such that $UE = 0$ and $EV = 0$. For symmetric matrix P such that $E_L^T P E_L > 0$ and non singular matrix X, we have $PE + U^T X V^T$ is non singular and the following result holds:

$$(PE + U^T X V^T)^{-1} = \bar{P}E^T + V\bar{X}U \tag{5}$$

with $\bar{P}$ and $\bar{X}$ are symmetric and non singular matrices, respectively such that
$E_R^T \bar{P} E_R = (E_L^T P E_L)^{-1}$ and $\bar{X} = (V^T V)^{-1} X^{-1} (UU^T)^{-1}$.

Lemma 3. Zhang et al. (2018)
Based on Lemma 2, we have:

$$\begin{aligned} (PE + U^T X V^T)^T E (PE + U^T X V^T)^{-1} = E^T \\ (PE + U^T X V^T)^{-T} E^T (PE + U^T X V^T) = E \end{aligned} \tag{6}$$

In the following section, the admissibility analysis is studied.

3. ADMISSIBILITY ANALYSIS

In this part, the admissibility analysis of system (1) with $u(t) = 0$ has been studied. For this, we propose the following Theorem.

Theorem 1. System (1) with $u(t) = 0$ is admissible if there exist symmetric matrices $P > 0$, $X_{11} > 0$, $Q > 0$, $R > 0$, $W_1 > 0$ and $W_2 > 0$, for constant delay τ, such that the following $LMIs$ hold:

$$\Psi_i = \begin{bmatrix} \Psi_{11i} & \Psi_{12i} & \Psi_{13i} & \Psi_{14i} & \tau A_i^T R \\ * & \Psi_{22} & \Psi_{23i} & \Psi_{24i} & \tau A_{hi}^T R \\ * & * & \Psi_{33} & \Psi_{34} & 0 \\ * & * & * & \Psi_{44} & 0 \\ * & * & * & * & -R \end{bmatrix} < 0, \forall i = 1:r \tag{7}$$

with

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ * & P_{22} & P_{23} \\ * & * & P_{33} \end{bmatrix}$$

$$\begin{aligned} \Psi_{11i} &= sym\big((P_{11}E + U^T X_{11} V^T)^T A_i\big) + sym(E^T P_{12} E) \\ &\quad + sym(\tau E^T P_{13} E) + Q + \tau^2 W_1 + \frac{\tau^4}{4} W_2 - 9E^T R E \\ \Psi_{12i} &= (P_{11}E + U^T X_{11} V^T)^T A_{hi} - E^T P_{12} E + 3E^T R E \\ \Psi_{13i} &= \tau A_i^T P_{12} E + \tau E^T P_{22} E + \tau^2 E^T P_{23}^T E - \tau E^T P_{13} E \\ &\quad -24E^T R E \\ \Psi_{14i} &= \tfrac{\tau^2}{2} A_i^T P_{13} E + \tfrac{\tau^2}{2} E^T P_{23} E + \tfrac{\tau^3}{2} E^T P_{33}^T E + 30E^T R E \\ \Psi_{22} &= -Q - 9E^T R E \\ \Psi_{23i} &= \tau A_{hi}^T P_{12} E - \tau E^T P_{22} E + 36E^T R E \\ \Psi_{24i} &= \tfrac{\tau^2}{2} A_{hi}^T P_{13} E - \tfrac{\tau^2}{2} E^T P_{23} E - 30E^T R E \\ \Psi_{33} &= -\tau^2 E^T P_{23} E - \tau^2 E^T P_{23}^T E - 192E^T R E - \tau^2 W_1 \\ \Psi_{34} &= -\tfrac{\tau^3}{2} E^T P_{33} E + 180E^T R E \\ \Psi_{44} &= -180E^T R E - \tfrac{\tau^4}{4} W_2 \end{aligned}$$

where U and V are defined in Lemma 2.

Proof. The proof is divided into two parts. The first one deals with the regularity and impulse freeness and the second one analyses the stability of the system. Since $rank(E) = q < n$, there exist two invertible matrices M and N such that

$$MEN = \begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}$$

Define

$$MA_iN = \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix}$$

$$M^{-T}RM^{-1} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}$$

$$M^{-T}P_{1j}M^{-1} = \begin{bmatrix} P_{1j11} & P_{1j12} \\ P_{1j21} & P_{1j22} \end{bmatrix},\ j = 1, 2, 3$$

$$M^{-T}U^T = \begin{bmatrix} 0 \\ U_1^T \end{bmatrix},\ V^TN = \begin{bmatrix} 0 & V_1^T \end{bmatrix}$$

such that U_1 and V_1 are invertible.
From condition (7), it is easy to declare that $\Psi_{11i} < 0$. Considering positive matrices Q, W_1 and W_2, we get:

$$\Psi_{11i} - (Q + \tau^2 W_1 + \frac{\tau^4}{4} W_2) < 0 \tag{8}$$

Pre and post-multiplying equation (8) by N^T and N, we obtain:

$$\begin{bmatrix} \star & \star \\ \star & sym(A_{i22}^T U_1^T X_{11} V_1^T) \end{bmatrix} < 0 \tag{9}$$

Knowing that $\sum_{i=1}^{r} \mu_i(\xi(t)) = 1$ and $\mu_i(\xi(t)) \geq 0$, we get:

$$\sum_{i=1}^{r} \mu_i(\xi(t)) sym(A_{i22}^T U_1^T X_{11} V_1^T) < 0 \tag{10}$$

So, $\sum_{i=1}^{r} \mu_i(\xi(t)) A_{i22}$ is non singular. Thus, according to Definition 1, the autonomous singular type-2 T-S system is regular and impulse free. To study the stability, we consider the following Lyapunov-Krasovskii functional:

$$V(x(t)) = V_1(x(t)) + V_2(x(t)) + V_3(x(t)) + V_4(x(t)) \tag{11}$$

where
$V_1(x(t)) = \Delta(t)^T P \Delta(t)$

$$\Delta(t) = \left[x(t)^T E^T \ \int_{t-\tau}^{t} x(s)^T E^T ds \ \int_{-\tau}^{0}\int_{t+\theta}^{t} x(s)^T E^T ds d\theta \right]^T$$

$V_2(x(t)) = \int_{t-\tau}^{t} x(s)^T Q x(s) ds$
$V_3(x(t)) = \tau \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}(s)^T E^T R E \dot{x}(s) ds d\theta$

$$\begin{aligned} V_4(x(t)) &= \tau \int_{-\tau}^{0}\int_{t+\theta}^{t} x(s)^T W_1 x(s) ds d\theta \\ &+ \frac{\tau^2}{2} \int_{-\tau}^{0}\int_{u}^{0}\int_{t+\theta}^{t} x(s)^T W_2 x(s) ds d\theta du \end{aligned}$$

The derivative of $V(x(t))$ along the trajectory of autonomous system (1) is given as
$\dot{V}(x(t)) = \dot{V}_1(x(t)) + \dot{V}_2(x(t)) + \dot{V}_3(x(t)) + \dot{V}_4(x(t))$

$$\dot{V}_1(x(t)) = sym(\dot{\Delta}(t)^T P \Delta(t)) = \Delta_1(t)^T \pi \Delta_2(t) \tag{12}$$

where

$$\Delta_1(t) = \begin{bmatrix} E\dot{x}(t) \\ E(x(t) - x(t-\tau)) \\ \tau E x(t) - \int_{t-\tau}^{t} E x(s) ds \end{bmatrix}$$

$$\Delta_2(t) = \begin{bmatrix} x(t) \\ \int_{t-\tau}^{t} x(s) ds \\ \int_{-\tau}^{0}\int_{t+\theta}^{t} x(s) ds d\theta \end{bmatrix}$$

$$\pi = \begin{bmatrix} sym(P_{11}E + U^T X_{11} V^T) & P_{12}E & P_{13}E \\ * & sym(P_{22}E) & P_{23}E \\ * & * & sym(P_{33}E) \end{bmatrix}$$

and

$$\dot{V}_2(x(t)) = x(t)^T Q x(t) - x(t-\tau)^T Q x(t-\tau) \tag{13}$$

$$\dot{V}_3(x(t)) = \tau^2 \dot{x}(t)^T E^T R E \dot{x}(t) - \tau \int_{t-\tau}^{t} \dot{x}(s)^T E^T R E \dot{x}(s) ds \tag{14}$$

$$\begin{aligned} \dot{V}_4\ (x(t)) &= \tau^2 x(t)^T W_1 x(t) - \tau \int_{t-\tau}^{t} x(s)^T W_1 x(s) ds \\ &+ \frac{\tau^4}{4} x(t)^T W_2 x(t) - \frac{\tau^2}{2} \int_{-\tau}^{0}\int_{t+\theta}^{t} x(s)^T W_2 x(s) ds d\theta \end{aligned} \tag{15}$$

From (14) and using Lemma 1, we get:

$$-\tau \int_{t-\tau}^{t} \dot{x}(s)^T E^T R E \dot{x}(s) ds \leq \zeta(t)^T \Lambda \zeta(t) \tag{16}$$

with

$$\zeta(t) = \left[x(t)^T \ x(t-\tau)^T \ \frac{1}{\tau}\int_{t-\tau}^{t} x(s)^T ds \ \frac{2}{\tau^2}\int_{-\tau}^{0}\int_{t+\theta}^{t} x(s)^T ds d\theta \right]^T \tag{17}$$

$$\Lambda = \begin{bmatrix} -9E^T RE & 3E^T RE & -24E^T RE & 30E^T RE \\ * & -9E^T RE & 36E^T RE & -30E^T RE \\ * & * & -192E^T RE & 180E^T RE \\ * & * & * & -180E^T RE \end{bmatrix} \tag{18}$$

From (15) and using Jensen inequality, we have:

$$-\tau \int_{t-\tau}^{t} x(s)^T W_1 x(s) ds \leq -\Big(\int_{t-\tau}^{t} x(s) ds\Big)^T W_1 \Big(\int_{t-\tau}^{t} x(s) ds\Big) \tag{19}$$

and

$$\begin{aligned} -\frac{\tau^2}{2} &\int_{-\tau}^{0}\int_{t+\theta}^{t} x(s)^T W_2 x(s) ds d\theta \\ &\leq -\Big(\int_{-\tau}^{0}\int_{t+\theta}^{t} x(s) ds d\theta\Big)^T W_2 \Big(\int_{-\tau}^{0}\int_{t+\theta}^{t} x(s) ds d\theta\Big) \end{aligned} \tag{20}$$

Considering equations (12), (13), (16), (19) and (20), we obtain:

$$\dot{V}(x(t)) \leq \sum_{i=1}^{r} \mu_i(\xi(t)) \zeta(t)^T (\Psi_i^* + \tau^2 \Omega_i^T R \Omega_i) \zeta(t) \tag{21}$$

where

$$\Psi_i^* = \begin{bmatrix} \Psi_{11i} & \Psi_{12i} & \Psi_{13i} & \Psi_{14i} \\ * & \Psi_{22} & \Psi_{23i} & \Psi_{24i} \\ * & * & \Psi_{33} & \Psi_{34} \\ * & * & * & \Psi_{44} \end{bmatrix},$$

and

$$\Omega_i = \begin{bmatrix} A_i & A_{hi} & 0 & 0 \end{bmatrix},\ \forall i = 1, 2, \ldots, r$$

According to equation (7) and Schur complement lemma, for $i = 1, 2, \ldots, r$, $\dot{V}(x(t)) \leq 0$. Thus, system (1) is admissible.

□

Remark 1. Comparing with Tseng et al. (2017) and Li et al. (2017), authors have chosen a classical Lyapunov-Krasovskii functional. However, we have considered an augmented vector and a triple summation to construct a new functional which reduce the conservatism of the proposed methods.

4. CONTROL DESIGN AND ADMISSIBILITY SYNTHESIS

In this section, a type-2 fuzzy control law has been designed to not only compensate non linearities but also to stabilize the closed loop singular type-2 fuzzy systems despite the presence of time delay. For this, we construct the following control law:

$$u(t) = \sum_{j=1}^{r} \mu_j(\xi(t))K_j x(t) \tag{22}$$

in which $\mu_j(\xi(t))$ is defined in equation (2). After that, considering equations (1) and (22) the singular closed loop system is given as follows:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(\xi(t))\mu_j(\xi(t))\big((A_i + B_iK_j)x(t) \\ &\quad + A_{hi}x(t-\tau)\big) \end{aligned} \tag{23}$$

System (23) is admissible if the condition presented in the next Theorem holds.

Theorem 2. System (23) is admissible if there exist symmetric matrices $\bar{P}_{11} > 0$, $\bar{P}^* > 0$, $\bar{X}_{11} > 0$, $\bar{Q} > 0$, $\bar{R} > 0$, $\bar{W}_1 > 0$ and $\bar{W}_2 > 0$ and matrices H_j, L_j, for constant delay τ, such that the following $LMIs$ hold:

$$\bar{\Psi}_{ii} < 0 \tag{24}$$

$$\bar{\Psi}_{ij} + \bar{\Psi}_{ji} < 0 \tag{25}$$

with

$$\bar{\Psi}_{ij} = \begin{bmatrix} \bar{\Psi}_{11ij} & \bar{\Psi}_{12i} & \bar{\Psi}_{13} & \bar{\Psi}_{14} & \bar{\Psi}_{15ij} \\ * & \bar{\Psi}_{22} & \bar{\Psi}_{23} & \bar{\Psi}_{24} & \bar{\Psi}_{25i} \\ * & * & \bar{\Psi}_{33} & \bar{\Psi}_{34} & 0 \\ * & * & * & \bar{\Psi}_{44} & 0 \\ * & * & * & * & \bar{\Psi}_{55} \end{bmatrix} \tag{26}$$

$$\bar{P}^* = \begin{bmatrix} \bar{P}_{22} & \bar{P}_{23} \\ * & \bar{P}_{33} \end{bmatrix}$$

$$\begin{aligned} \bar{\Psi}_{11ij} &= sym\big((\bar{P}_{11}E^T + V\bar{X}_{11}U)^T A_i^T\big) + \tau^2\bar{W}_1 + \frac{\tau^4}{4}\bar{W}_2 \\ &\quad + sym\big((H_jE^T + L_jU)^T B_i^T\big) + \bar{Q} - 9E\bar{R}E^T \end{aligned}$$

$\bar{\Psi}_{12i} = A_{hi}(\bar{P}_{11}E^T + V\bar{X}_{11}U) + 3E\bar{R}E^T$
$\bar{\Psi}_{13} = \tau E\bar{P}_{22}E^T + \tau^2 E\bar{P}_{23}^T E^T - 24E\bar{R}E^T$
$\bar{\Psi}_{14} = \frac{\tau^2}{2}E\bar{P}_{23}E^T + \frac{\tau^3}{2}E\bar{P}_{33}E^T + 30E\bar{R}E^T$
$\bar{\Psi}_{15ij} = (\bar{P}_{11}E^T + V\bar{X}_{11}U)^T A_i^T + (H_jE^T + L_jU)^T B_i^T$
$\bar{\Psi}_{22} = -\bar{Q} - 9E\bar{R}E^T$
$\bar{\Psi}_{23} = -\tau E\bar{P}_{22}E^T + 36E\bar{R}E^T$
$\bar{\Psi}_{24} = -\frac{\tau^2}{2}E\bar{P}_{23}E^T - 30E\bar{R}E^T$
$\bar{\Psi}_{25i} = (\bar{P}_{11}E^T + V\bar{X}_{11}U)^T A_{hi}^T$
$\bar{\Psi}_{33} = -\tau^2 E\bar{P}_{23}E^T - \tau^2 E\bar{P}_{23}^T E^T - 192E\bar{R}E^T - \tau^2\bar{W}_1$
$\bar{\Psi}_{34} = -\frac{\tau^3}{2}E\bar{P}_{33}E^T + 180E\bar{R}E^T$
$\bar{\Psi}_{44} = -180E\bar{R}E^T - \frac{\tau^4}{4}\bar{W}_2$
$\bar{\Psi}_{55} = -(\bar{P}_{11}E^T + V\bar{X}_{11}U)^T - (\bar{P}_{11}E^T + V\bar{X}_{11}U) + \bar{R}$

Proof. According to Theorem 1, by setting $P_{12} = 0$ and $P_{13} = 0$ and by replacing A_i with $A_i + B_iK_j$ in equation (7), we obtain the matrix Ψ_{ij}. Let $\Upsilon = (P_{11}E + U^T X_{11}V^T)^{-T}$, $H_j = K_j\bar{P}_{11}$ and $L_j = K_jV\bar{X}_{11}$. Pre and post-multiplying Ψ_{ij} by $diag(\Upsilon, \Upsilon, \Upsilon, \Upsilon, R^{-1})$ and its transpose, equation (26) holds using Lemmas 2 and 3 and the following expressions :
$\bar{Q} = (P_{11}E + U^T X_{11}V^T)^{-T} Q (P_{11}E + U^T X_{11}V^T)^{-1}$
$\bar{R} = (P_{11}E + U^T X_{11}V^T)^{-1} R (P_{11}E + U^T X_{11}V^T)^{-T}$
$\bar{W}_1 = (P_{11}E + U^T X_{11}V^T)^{-T} W_1 (P_{11}E + U^T X_{11}V^T)^{-1}$
$\bar{W}_2 = (P_{11}E + U^T X_{11}V^T)^{-T} W_2 (P_{11}E + U^T X_{11}V^T)^{-1}$
Then, the controller gains can be computed through the following equation:
$K_j = (H_jE^T + L_jU)(\bar{P}_{11}E^T + V\bar{X}_{11}U)^{-1}$.

□

5. SIMULATION RESULTS

In this section, the pendulum system (Lam et al. (2013), Su et al. (2013)) described in this work by a Type 2 T-S fuzzy singular model is provided to further demonstrate the performance of the developed control scheme :

$$\ddot{\theta}(t) = \frac{-3am_pL\dot{\theta}^2(t)sin(2\theta(t))/2 + 3gsin(\theta(t)) - 3acos(\theta(t))u(t)}{4L - 3am_pLcos^2(\theta(t))}$$

with $\theta(t)$ represents the angular displacement of the pendulum, $g = 9.8m/s^2$, m_c represents the mass of the cart, m_p represents the mass of the pendulum, $a = 1/(m_c + m_p)$, $L = 0.5m$ which is the length of the pendulum and $u(t)$ represents the force applied to the cart. The system can be described as:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = f_1(t)x_1(t) + f_2(t)u(t) \\ 0 = Lsin(x_1(t)) - x_3(t) \end{cases} \tag{27}$$

where

$$f_1(t) = \frac{\big(g - am_pLx_2^2(t)cos(x_1(t))\big)sin(x_1(t))}{\big(4L/3 - am_pLx_2^2(t)cos^2(x_1(t))\big)x_1(t)}$$

$$f_2(t) = \frac{-acos(x_1(t))}{4L/3 - am_pLcos^2(x_1(t))}$$

$x_1(t)$ denotes the angle of the pendulum from the vertical, $x_2(t)$ denotes the angular velocity, $x_3(t)$ denotes the relative horizontal distance from the pendulum center to cart. m_p and m_c are uncertain parameters which satisfy $m_{c\,min} = 2Kg \leq m_c \leq m_{c\,max} = 3Kg$ and $m_{p\,min} = 1Kg \leq m_p \leq m_{p\,max} = 2Kg$.
The interval type-2 fuzzy model with time delay is presented as follows :

$$E\dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\big((1-\gamma)A_ix(t) + \gamma A_ix(t-\tau) + B_iu(t)\big) \tag{28}$$

with

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 9.2902 & 0 & 0 \\ -0.483 & 0 & -1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 9.2902 & 0 & 0 \\ 0.483 & 0 & -1 \end{bmatrix},\ A_3 = \begin{bmatrix} 0 & 1 & 0 \\ 23.52 & 0 & 0 \\ -0.483 & 0 & -1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0 & 1 & 0 \\ 23.52 & 0 & 0 \\ 0.483 & 0 & -1 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ -0.6667 \\ 0 \end{bmatrix}$$

$$B_1 = B_3,\ B_2 = B_4 = \begin{bmatrix} 0 \\ -0.0792 \\ 0 \end{bmatrix}$$

Assume that $x_1(t)$ is measurable, the lower and upper membership functions are given as follows:

$$
\begin{aligned}
&\underline{h}_{W_{11}}(x_1) = 1 - e^{-(x_1^2/1.5)},\ \underline{h}_{W_{12}}(x_1) = 0.4e^{-(x_1^2/0.2)}\\
&\underline{h}_{W_{21}}(x_1) = \underline{h}_{W_{11}}(x_1),\ \underline{h}_{W_{22}}(x_1) = 1 - \bar{h}_{W_{12}}(x_1)\\
&\underline{h}_{W_{31}}(x_1) = 1 - \bar{h}_{W_{11}}(x_1),\ \underline{h}_{W_{32}}(x_1) = \underline{h}_{W_{12}}(x_1)\\
&\underline{h}_{W_{41}}(x_1) = \underline{h}_{W_{31}}(x_1),\ \underline{h}_{W_{42}}(x_1) = \underline{h}_{W_{22}}(x_1)\\
&\bar{h}_{W_{11}}(x_1) = 0.25e^{-(x_1^2/0.3)},\ \bar{h}_{W_{12}}(x_1) = e^{-(x_1^2/2.5)}\\
&\bar{h}_{W_{21}}(x_1) = \bar{h}_{W_{11}}(x_1),\ \bar{h}_{W_{22}}(x_1) = 1 - \underline{h}_{W_{12}}(x_1)\\
&\bar{h}_{W_{31}}(x_1) = 1 - \underline{h}_{W_{11}}(x_1),\ \bar{h}_{W_{32}}(x_1) = \ h_{W_{12}}(x_1)\\
&\bar{h}_{W_{41}}(x_1) = \bar{h}_{W_{31}}(x_1),\ \bar{h}_{W_{42}}(x_1) = \bar{h}_{W_{22}}(x_1)
\end{aligned}
$$

For $\gamma = 0.1$, $\tau = 0.2$, if equations (24) and (25) hold, we get via yalmip toolbox the following controller gain matrices:

$$
\begin{aligned}
K_1 &= [\,61.1636\ \ 14.0391\ \ 0.0645\,]\\
K_2 &= [\,283.7181\ \ 69.2988\ \ 0.0022\,]\\
K_3 &= [\,92.0017\ \ 16.7493\ \ 0.0792\,]\\
K_4 &= [\,394.3674\ \ 96.6673\ \ 0.1035\,]
\end{aligned}
$$

For the simulation results, the initial conditions are chosen as $x(0) = [\pi/6 \ \ -1.4 \ \ 0.25]^T$.
Figures 1-3 show the simulation results. Figure 1 illustrates the state responses of the open-loop system. Figure 2 presents the responses of the system state and the convergence of the control force is demonstrated in Figure 3. Figure 1 shows that the autonomous system is unstable. However, from Figure 2, it is clear that the closed-loop interval type-2 fuzzy singular system is stable. As a result, the designed interval type-2 fuzzy control law can completely compensate the non linearities. Moreover, the closed-loop system converges to zero in a small time even the presence of time delay and the existence of uncertainties considered in the membership functions. Thus, our control design scheme is effective.

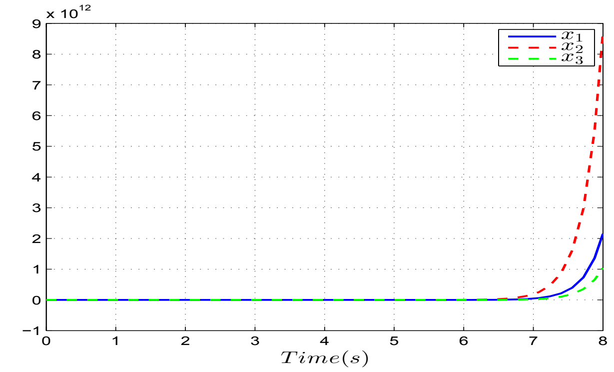

Figure 1. State responses of autonomous system

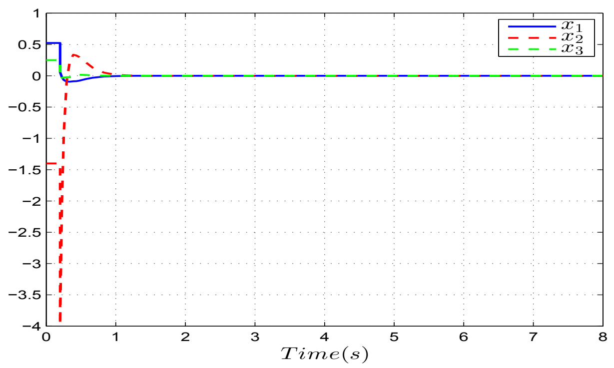

Figure 2. State responses

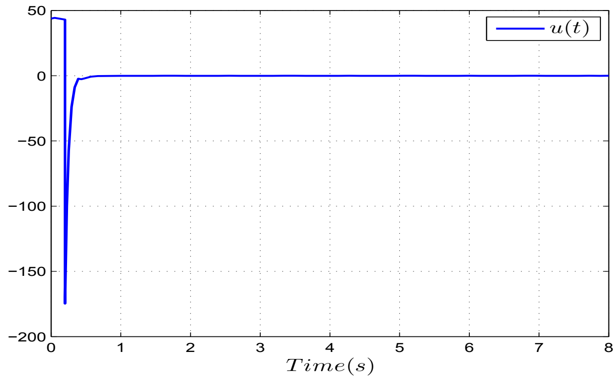

Figure 3. Control force of $u(t)$

6. CONCLUSION

In this work, a control design method has been elaborated for an interval type-2 T-S fuzzy singular systems with time delay. The proposed method allows the convergence of the state of closed-loop system. For the admissibility analysis, sufficient conditions have been presented in terms of $LMIs$. Finally, the performance of the suggested method has been illustrated via an application to inverted pendulum. As future work, we will focus on the control problem for the same class of systems while considering a time varying delays and mismatched membership functions.

REFERENCES

Chang, Y.H., Chan, W.S., and Chang, C.W. (2012). TS fuzzy model-based adaptive dynamic surface control for ball and beam system. *IEEE transactions on industrial electronics*, 60(6), 2251–2263.

Cui, W.X., Fang, J.A., Shen, Y.L., and Zhang, W.B. (2013). Dissipativity analysis of singular systems with markovian jump parameters and mode-dependent mixed time-delays. *Neurocomputing*, 110, 121–127.

Dai, L. (1989). *Singular control systems*, volume 118. Springer.

Gassara, H., Kchaou, M., Hajjaji, A.E., and Chaabane, M. (2014). Control of time delay fuzzy descriptor systems with actuator saturation. *Circuits, Systems, and Signal Processing*, 33(12), 3739–3756.

Kchaou, M. (2019). Robust observer-based sliding mode control for nonlinear uncertain singular systems with time-varying delay and input non-linearity. *European Journal of Control*, 49, 15–25.

Kchaou, M., Gassara, H., and El-Hajjaji, A. (2018). Adaptive sliding mode control for fuzzy singular systems with time delay and input nonlinearity. *International Journal of Adaptive Control and Signal Processing*, 32(3), 464–479.

Kchaou, M. and Hajjaji, A.E. (2017). Resilient H_∞ sliding mode control for discrete-time descriptor fuzzy systems with multiple time delays. *International journal of systems science*, 48(2), 288–301.

Lam, H.K., Li, H., Deters, C., Secco, E., Wurdemann, H., and Althoefer, K. (2013). Control design for interval type-2 fuzzy systems under imperfect premise matching. *IEEE Transactions on Industrial Electronics*, 61(2), 956–968.

Li, H., Wang, J., Wu, L., Lam, H.K., and Gao, Y. (2017). Optimal guaranteed cost sliding-mode control of interval type-2 fuzzy time-delay systems. *IEEE Transactions on Fuzzy Systems*, 26(1), 246–257.

Makni, S., Bouattour, M., Hajjaji, A.E., and Chaabane, M.. (2019). Robust observer based Fault Tolerant Tracking Control for T-S uncertain systems subject to sensor and actuator faults. *ISA Transactions*, 88, 1 – 11.

Park, M.J., Kwon, O., and Ryu, J. (2018). Generalized integral inequality: Application to time-delay systems. *Applied Mathematics Letters*, 77, 6–12.

Su, Z., Zhang, Q., and Ai, J. (2013). Practical and finite-time fuzzy adaptive control for nonlinear descriptor systems with uncertainties of unknown bound. *International Journal of Systems Science*, 44(12), 2223–2233.

Takagi, T. and Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control. *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-15(1), 116–132.

Tseng, C.L., Wang, S.Y., Jean, F.R., Wu, T.Y., Chou, J.H., and Liu, F.Y. (2017). Type-2 TS fuzzy multiple feedback-loop guaranteed cost controller design for uncertain singular time-delay system. In *2017 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*, 3002–3007. IEEE.

Uezato, E. and Ikeda, M. (1999). Strict LMI conditions for stability, robust stabilization, and H_∞ control of descriptor systems. In *Proceedings of the 38th IEEE Conference on Decision and Control*, volume 4, 4092–4097. IEEE.

Zhang, F., Zhang, Q., and Li, J. (2018). Networked control for T-S fuzzy descriptor systems with network-induced delay and packet disordering. *Neurocomputing*, 275, 2264–2278.