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Position-based motion control for parallel manipulators under parametric uncertainties and with finite-time external disturbance rejection ⋆

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Abstract: We consider the task of motion control for non-prehensile manipulation using parallel kinematics mechatronic setup, in particular, stabilization of a ball on a plate under unmeasured external harmonic disturbances. System parameters are assumed to be unknown, and only a ball position is measurable with a resistive touch sensor. To solve the task we propose a novel passivity-based output control algorithm that can be implemented for unstable linearized systems of an arbitrary relative degree. In contrast to previous works, we describe a new way to parametrize harmonic signal generators and an estimation algorithm with finite-time convergence. This scheme enables fast disturbance cancellation under control signal magnitude constraints.

Keywords: Adaptive control, output control, motion control, disturbance rejection, finite-time estimation, ball-and-plate.

1. INTRODUCTION

The development of control algorithms for robot manipulators with a parallel kinematic scheme is an pressing challenge in the tasks of dynamic manipulation. Such systems have several advantages compared to manipulators with a serial scheme: kinematic chains are closed, which leads to robustness, as well as high accuracy of the positioning of the mechanism as a whole. Movable parallel parts reduce the load on the drive, which improves the dynamics and accuracy of the system Lynch and Mason (1999). Similar systems are used in flight simulators, in simulators for car drivers, in the production process. Parallel kinematics robots are also widely used in biomechatronics and rehabilitation of the neck, knee joints and foot joints.

This research is devoted to the output-feedback control of linear parametrically uncertain plants under unmeasured matching input harmonic disturbances. The paper presents a novel switching control algorithm that combines a passivity-based output controller with a finite-time harmonic disturbance parameters estimation algorithm that guarantees convergence of the disturbance parameters. Thus, the overall closed-loop system performance can be improved.

It is assumed that the frequency of the harmonic signal is unknown. In the majority of works devoted to the synthesis of frequency identification algorithms Landau et al. (2005); Marino and Tomei (2003); Amara et al. (1999); Bodson and Dopuglas (1997); Francis and Wonham (1976) the possibility of increasing the rate of convergence is not discussed, which can also be attributed to the open problems of control theory.

In the present work, the method of a 'consecutive compensator' Pyrkin et al. (2011) is used as a basic control approach. This method had been proven efficient in a number of parametrically uncertain robotic and mechatronic systems control applications, see e.g. Dobriborsci et al. (2018b,a,c). Moreover, the approach guarantees global convergence, can be applied to systems with an arbitrary relative degree, has a simple structure and is easy to configure. We implement this output-feedback controller to stabilize an unstable parametrically uncertain plant and further use its output for the input disturbance model identification.

The problem of cancelling external harmonic disturbances acting on unstable parametrically and structurally uncertain plants by identifying disturbance's internal model parameters as well as ways of increasing the rate of parametric convergence were studied in the authors' previously published works, e.g. Pyrkin et al. (2015a,b); Bobtsov et al. (2011).

A number of papers are devoted to finite-time estimators. In Ortega et al. (2019), Gerasimov et al. (2018) an estimator design which provides finite-time convergence is proposed. In Wang et al. (2019) an adaptive estimator of constant parameters without the hypothesis that regressor is Persistently Excited (PE) is proposed.

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However, finite-time estimation algorithms and controllers based on 'consecutive compensator' approach were never combined before to solve the output control problem under parametrical and signal disturbances. Such a fusion is quite promising, it provides convergence of the disturbance estimates with a finite amount of time.

The approach described in this paper is close to the monitoring function method presented in Roux Oliveira et al. (2017) for the problem of the output adaptive tracking control. The difference is that we use output adaptive robust controller with high-gain observer, which is different from sliding mode control.

The paper is structured as follows. After a short introduction, the problem statement together with important assumptions are given. Section 3 describes the output stabilizing controller based on 'consecutive compensator' method. In Section 4 we present the finite-time disturbance frequency estimation algorithm for input harmonic disturbance parameters, then in Section 5 a switching scheme that enables using these estimates for feedback controller adjustments. Finally, Section 6 is devoted to the case study of Ball-and-Plate robotic platform control using the presented technique. As an example we consider the problem of ball stabilization on a square platform. The task is complicated by the presence of harmonic disturbances in the system. Obtained results illustrate the overall improved performance of the system.

2. PROBLEM STATEMENT

Consider the linear SISO plant

$$a(p)y(t) = b(p)[u(t) + \delta(t)], \tag{1}$$

where $p = \frac{d}{dt}$ is the differentiation operator, $u(t)$ and $y(t)$ are input and output signals respectively, coefficients of the polynomials $a(p) = p^n + a_{n-1}p^{n-1} + ... + a_0$ and $b(p) = b_m p^m + b_{m-1}p^{m-1} + ... + b_0$ are unknown, and

$$\delta(t) = \bar{A}\sin(\omega t + \bar{\phi}) \tag{2}$$

is the input harmonic disturbance with the unknown amplitude $\bar{A}$, phase shift $\bar{\phi}$, and frequency

$0 < \omega_{min} < \omega < \omega_{max} < \infty$.

The control goal is to guarantee

$$\lim_{t\to\infty} y(t) = 0 \tag{3}$$

under the following assumptions:

(1) $b(p)$ is a Hurwitz polynomial;
(2) only the relative degree of the system $\rho = n - m$ is known, while degrees of the polynomials $a(p)$ and $b(p)$ are unknown.
(3) The lower bound $\omega_m in$ of frequency ω is known.

3. OUTPUT CONTROLLER DESIGN

Let us consider the output adaptive controller with modification for the input harmonic disturbance rejection introduced in Bobtsov et al. (2012)

$$u(t) = -k\frac{\alpha(p)(p+1)^2}{(p^2+\omega^2)}\xi_1(t), \tag{4}$$

$$\begin{cases} \dot{\xi}_1 = \sigma\xi_2, \\ \dot{\xi}_2 = \sigma\xi_3, \\ \dots \\ \dot{\xi}_{\rho_m-1} = \sigma\left(-k_1\xi_1 - \ldots - k_{\rho-1}\xi_{\rho-1} + k_1 y\right), \end{cases} \tag{5}$$

where $\alpha(p)$ is a Hurwitz polynomial of $(\rho - 1)$ degree, constant coefficient $k > 0$ is chosen such way that transfer function

$$H(p) = \frac{\alpha(p)b(p)(p+1)^2}{a(p)(p^2+\omega^2) + k\alpha(p)b(p)(p+1)^2}$$

is SPR, while $\sigma > k$ and parameters k_i are calculated for the system (5) to be asymptotically stable for $y(t) = 0$.

Proposition 1. The output feedback controller (4), (5) applied to the plant (1) guarantees achievement of the control goal (3) for the output variable $y(t)$.

The detailed proof of the Proposition 1 is given in Bobtsov et al. (2012).

4. FINITE-TIME DISTURBANCE FREQUENCY ESTIMATION

Here we introduce the algorithm for finite-time input harmonic disturbance parameters estimation.

At first, we parametrize the disturbance model.

Since the considered closed-loop system is linear and stable, the output variable $y(t)$ (when the transient time has elapsed) is tracking the external disturbance, i.e. $y(t) = A\sin(\omega t + \phi)$.

Consider two auxiliary signals

$$y_1(t) = y(t - \tau), \tag{6}$$
$$y_2(t) = y(t - 2\tau), \tag{7}$$

where $\tau \in \mathbb{R}_+$ are chosen values of the delay duration.

Rewrite (6) and (7)

$$y_1(t) = A\sin(\omega t + \phi)\cos\omega\tau - A\cos(\omega t + \phi)\sin\omega\tau, \tag{8}$$
$$y_2(t) = A\sin(\omega t + \phi)\cos 2\omega\tau - A\cos(\omega t + \phi)\sin 2\omega\tau. \tag{9}$$

Multiplying $y_1(t)$ by $\sin 2\omega\tau$ and $y_2(t)$ by $\sin\omega\tau$ and applying double angle formulas, we get

$$\begin{aligned} y_1(t)\sin 2\omega\tau - y_2(t)\sin\omega\tau &= A\sin(\omega t + \phi)\cos\omega\tau\sin 2\omega\tau \\ &- A\cos(\omega t + \phi)\sin\omega\tau\sin 2\omega\tau \\ &- A\sin(\omega t + \phi)\cos 2\omega\tau\sin\omega\tau \\ &+ A\cos(\omega t + \phi)\sin 2\omega\tau\sin\omega\tau = \\ &= 2A\sin(\omega t + \phi)\cos^2\omega\tau\sin\omega\tau \\ &- A\sin(\omega t + \phi)(2\cos^2\omega\tau - 1)\sin\omega\tau = \\ &= A\sin(\omega t + \phi)\sin\omega\tau = y(t)\sin\omega\tau \end{aligned} \tag{10}$$

Dividing (10) by $\sin\omega\tau$, we get

$$2y_1(t)\cos\omega\tau = y(t) + y_2(t). \tag{11}$$

Now, we can derive the exact model (without assumption on exponential decaying terms due to unknown initial conditions) of the harmonic signal generator in the linear regression form

$$z(t) = \varphi(t)\theta, \tag{12}$$

where $z(t) = \frac{1}{2}\left(y(t) + y_2(t)\right)$, $\varphi(t) = y_1(t)$, and $\theta = \cos\omega\tau$.

Since the signals $y(t)$, $y_1(t)$, $y_2(t)$ are measurable, we can estimate disturbance frequency from the following algorithm

$$\dot{\hat{\theta}}(t) = K\varphi(t)(z(t) - \varphi(t)\hat{\theta}(t)), \tag{13}$$

$$\hat{\theta}_F(t) = \frac{1}{1 - w(t)}\left[\hat{\theta}(t) - w(t)\hat{\theta}_0\right], \tag{14}$$

$$\dot{w}(t) = -K\varphi^2(t)w(t), \tag{15}$$

where $K > 0$, $w(0) = 1$, and $\hat{\theta}_0$ is an initial guess on the disturbance parameters values.

It can be shown that $\hat{\theta}_F(t)$ converges to the real value of θ for finite time, which can be reduced by adjusting the gain K. The only issue with selecting very high values for K is that the presented scheme becomes very sensitive to measurements noise. But in any case we need to wait some small amount of time before $w(t) < 1$ such that (14) does not give division by zero. Detailed description and proof for (14)–(15) can be found in Ortega et al. (2019).

5. SWITCHING SCHEME

The last step in the proposed scheme is to set up a criterion, which would implement disturbance parameters' estimates substitution to the nominal controller (4) $\omega(t_i) = \bar{\omega}(t_i)$.

As it was outlined above, in contrast to Dobriborsci et al. (2019) we perform a single switching, i.e. the substitution of the disturbance frequency estimates will be performed at the moment of time when its already converged. The switching scheme can be analytically described by the relations below and applied by using a trigger scheme:

$$\begin{cases} \bar{\omega}(t) = \omega_{min}, & \text{where } t < T, \\ \bar{\omega}(t) = \dfrac{\arccos \hat{\theta}_F(T)}{\tau}, & \text{where } t \geq T, \end{cases} \tag{16}$$

where $\hat{\theta}_F(T)$ is obtained from (13)–(15).

A method reported above is quite similar to widely-used dwell-time switching logic and allows to avoid undesired jumps and oscillation in transients that can lead to loosing closed-loop system stability.

6. CASE-STUDY RESULTS

In this chapter we analyse how the proposed output controller and disturbance frequency estimation algorithm working in a loop can be applied for the mechatronic setup.

Here we consider a parallel kinematics Ball-and-Plate robotic platform as a plant (see Fig. 1). The goal is to stabilize a steel ball in user-defined coordinates on the square plate under input harmonic disturbances by applying voltages to the servo drives controlling the inclination of a plate, while kinematic and dynamic parameters of the system are unknown.

The ball and plate system can well be approximated by two linear decoupled systems. Therefore, this system with two inputs and two outputs can be treated as two decoupled SISO systems, therefore the proposed control approach can be implemented.

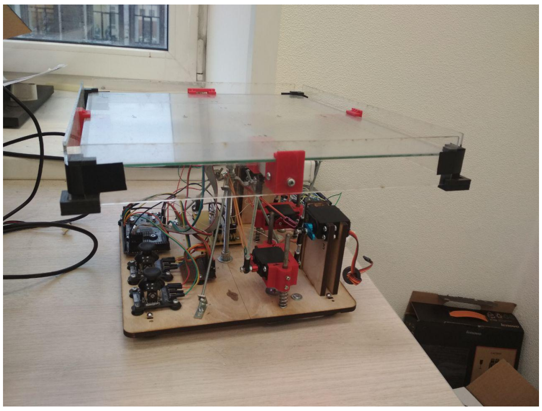


Fig. 1. Ball-and-plate lab setup

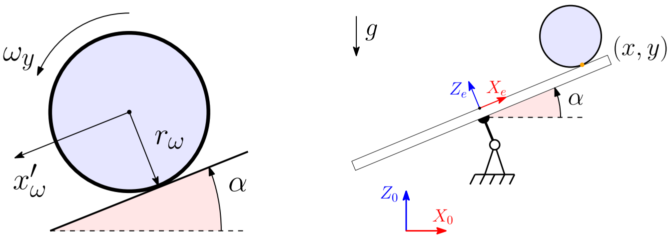


Fig. 2. Sketch of a ball on a plate pictured in OXZ plane

6.1 *Mathematical model*

Lets consider ball motion in OXZ plane (see Fig. 2). Here we derive the equations of motion for experimental setup under the following assumptions:

- There is no slipping for ball.
- The ball is completely symmetric and homogeneous.
- Friction forces are neglected.
- The ball and plate are in contact all time.

By assuming the generalized coordinates of system to be x_b and y_b for position of the ball in each direction and α and β the inclinations of the plate, i.e. $q = [x_b\, y_b\, \alpha\, \beta]^T$.

In accordance to previous works we obtain equation of motion Dobriborsci et al. (2018c,b); Dobriborsci and Kolyubin (2017)

$$\left(m_b + \frac{I_b}{r_b^2}\right)\ddot{x}_b - m\left(x_b\dot{\alpha}^2 + y_b\dot{\alpha}\dot{\beta}\right) + m_b g sin\alpha = 0 \tag{17a}$$

$$\left(m_b + \frac{I_b}{r_b^2}\right)\ddot{y}_b - m\left(y_b\dot{\beta}^2 + x_b\dot{\alpha}\dot{\beta}\right) + m_b g sin\beta = 0 \tag{17b}$$

The ball coordinates x and y are considered as the output.

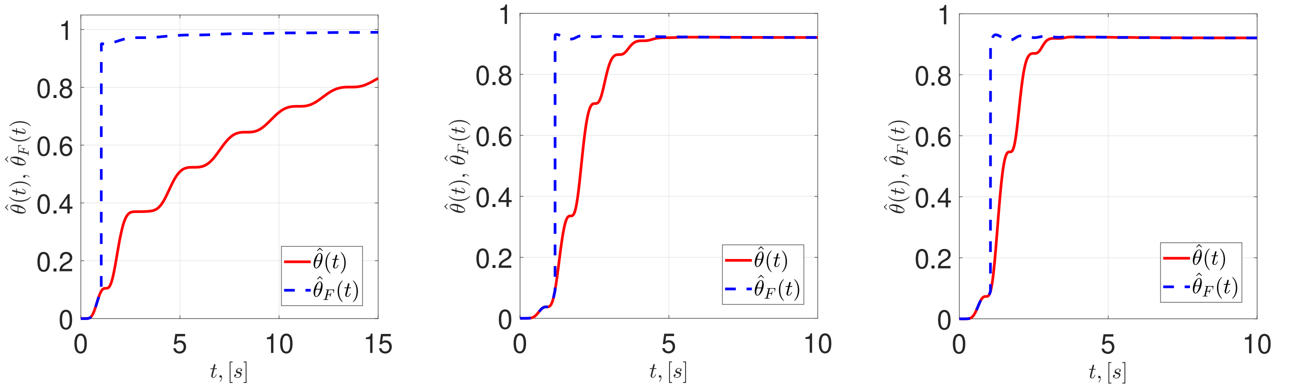
Taking into account servo drive dynamics, which is assumed to be captured by the 1st order aperiodic link transfer function, we can derive the following relations for the numerator and denominator of the system transfer functions for x and y control channels

$$b(p) = 2m_b g d r_b^2 K_m, \tag{18a}$$

$$a(p) = L(m_b r_b^2 T_m p^3 + I_b p^2), \tag{18b}$$

where K_m and T_m are servo drives gain and time constant respectively.

Parameters of the system are presented in the Table below.



(a) Standard gradient descent method and finite-time modification, for $\omega = 1.2\frac{rad}{s}$, $K = 0.5$

(b) Standard gradient descent method and finite-time modification, $\omega = 4\frac{rad}{s}$, $K = 0.9$

(c) Standard gradient descent method and finite-time modification, where $\omega = 4\frac{rad}{s}$, $K = 1.8$

Fig. 3. Transients for disturbance parameter estimation

Table Parameters of the Ball-and-Plate system

m_b	$0.05\ kg$	L	$0.11\ m$
g	$9.81\ m/s^2$	K_m	0.25
d	$0.02\ m$	T_m	$0.018\ s$
r_b	$0.0125\ m$	I_b	$3.13 \cdot 10^{-5} kg \cdot m^2$

6.2 Simulation results

We will verify the proposed approach for two cases. At first, consider system performance assuming that the disturbance signal is directly measurable. We demonstrate results for two harmonic disturbance signals with different parameters

$$\delta_1(t) = 3sin(1.2t + \frac{\pi}{2}), \tag{19}$$

and

$$\delta_2(t) = 3sin(4t + \frac{\pi}{2}). \tag{20}$$

Transients for the signals of the closed-loop system are presented in Fig.3, which illustrate the convergence of the input harmonic disturbance parameter to the real value in case we use finite-time algorithm modification and its comparison to standard gradient-descent method. The increase of the gain coefficient parameter K leads to the more accurate convergence of the estimates. The gradient-descent method provides the convergence of the parameters in more than 30 seconds with frequency $\omega = 1.2$, $K = 0.5$, $\tau = 0.1$ without modification and in about 8 seconds with $K = 3.8$ and with finite-time modification, whereas with frequency $\omega = 4$, $K = 0.9$, $\tau = 0.1$ in 6 seconds without modification and in 3 seconds with $K = 1.8$ with finite-time modification.

Now, consider the more realistic case, when external disturbance is not directly measurable, and we estimate its parameters by measuring system output only, while plant parameters are a priori unknown.

For the dynamical model of the ball-and-plate setup that we use (relative degree $\rho = 3$), the proposed output controller with tuned parameters can be described as

$$u(t) = -\kappa \frac{\alpha(p)(p+1)^2}{p(p^2 + \bar{\omega})} \xi_1, \tag{21}$$

$$\begin{cases} \dot{\xi}_1(t) = \sigma \xi_2(t), \\ \dot{\xi}_2(t) = \sigma(-k_1 \xi_1(t) - k_2 \xi_2(t) + k_1 y(t)), \end{cases} \tag{22}$$

where $\kappa = 1.2, \sigma = 35, k_1 = 2, k_2 = 5, \alpha(p) = p^2 + 3p + 1$.

Again, we demonstrate results for two harmonic disturbance signals with different parameters (19) and (20).

The simulation results of the disturbed plant behavior are presented in Fig.4 - 5, where identification of the frequency can be observed. The finite-time algorithm modification proves its efficiency and provides faster convergence than gradient-descent method. The transient time constitutes 8 seconds with frequency $\omega = 1.2$, $K = 7.1$, $\tau = 0.1$ and 5 seconds with $\omega = 4$, $K = 2.8$, $\tau = 0.1$.

7. CONCLUSION

This work presented a modification of the output adaptive control algorithm based on the "consecutive compensator" method, where the unknown input harmonic disturbance rejection is organized via the finite-time disturbance parameters estimation algorithm.

The proposed controller remains a simple structure, but guarantees better closed-loop system performance because of the finite-time convergence of the disturbance's parameters estimates. At the same time, this approach simplifies the switching rule used for parameters' estimates substitution to the feedback controller.

Possible directions for future work include extension of the obtained results for the case when an input disturbance is approximated by the Fourier series, i.e. dealing with multi-harmonic signals, solving trajectory tracking tasks, including MIMO cases, and modifications of the finite-time algorithms for better robustness to measurement noise.

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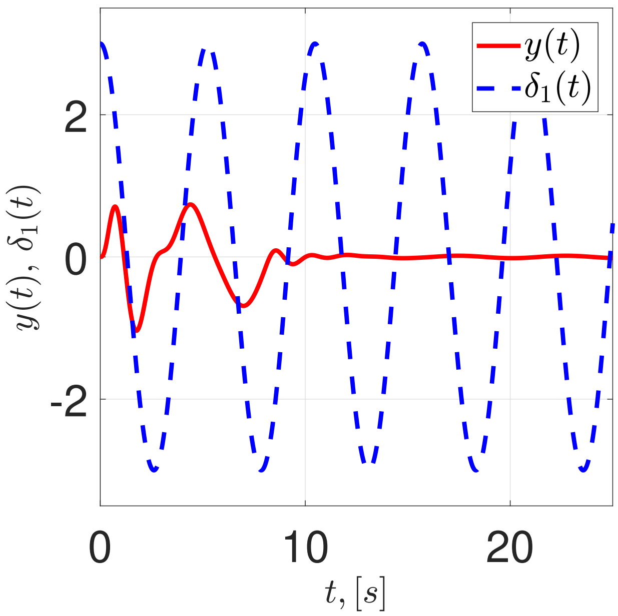
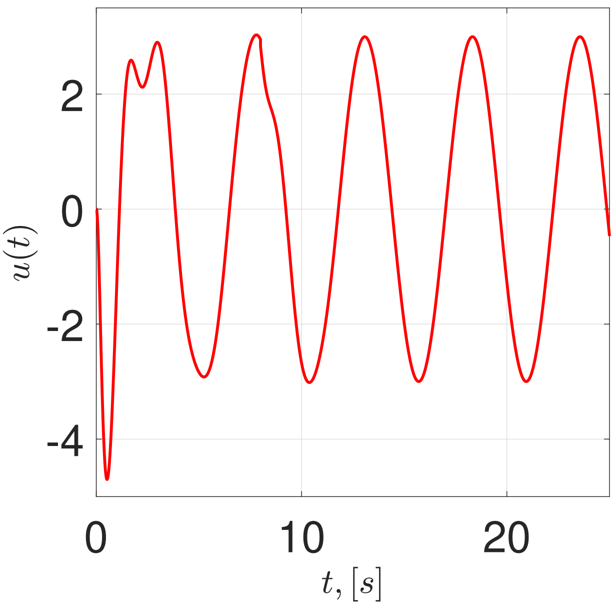
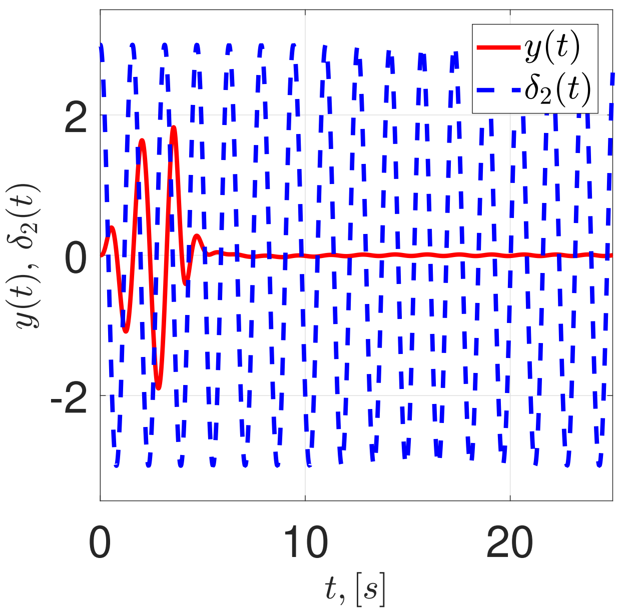


(a) Output and the disturbance signals with finite-time modification

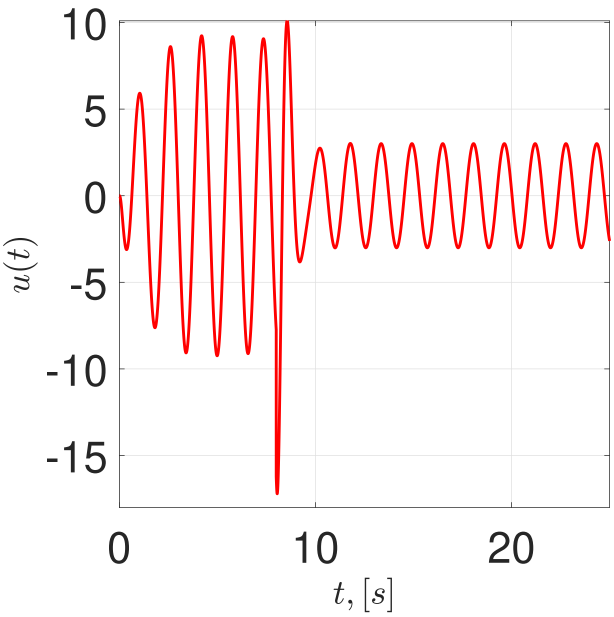


(b) Control signal with finite-time modification

Fig. 4. For $\omega = 1.2\frac{rad}{s}$, $K = 7.1$



(a) Output and the disturbance signals with finite-time modification



(b) Control signal with finite-time modification

Fig. 5. For $\omega = 4\frac{rad}{s}$, $K = 2.8$

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