Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Inverse Shaper Based Active Vibration Control of Flexible Structures with a Collocated Sensor-Actuator Pair ⋆

Arzuman Can Kutlucan * **Baran Alikoc** **
Leyla Goren-Sumer *

* *Department of Control and Automation Eng., Faculty of Electrical and Electronics Eng., Istanbul Technical University, 34469 Istanbul, Turkey (e-mail: kutlucan@itu.edu.tr, leyla.goren@itu.edu.tr)*
** *Czech Institute of Informatics, Robotics, and Cybernetics, Czech Technical University in Prague, 16000 Prague, Czech Republic (Corresponding author, e-mail: baran.alikoc@cvut.cz)*

Abstract:
We propose a new control design for active vibration suppression of flexible structures with a collocated sensor-actuator pair. The proposed controller is based on the inverse form of a well-known input zero vibration (ZV) shaper. The inverse ZV shaper is utilized with a serially interconnected all-pass filter. This way, the appropriate controller frequency response properties for vibration suppression of collocated flexible systems is achieved, when the controller is applied in the positive feedback path. We propose two different cost functions to optimize the parameters of the proposed controller for efficient vibration suppression. The performance of the controller is investigated in both frequency and time domains through the vibration control of a cantilever beam model with a collocated piezoelectric sensor-actuator pair. Furthermore, its performance is compared with a recently improved version of the positive position feedback controller, which is a state-of-the-art work. It is shown via the simulations that the proposed controller suppresses vibrations more efficiently.

Keywords: Flexible systems, vibration control, collocated systems, input shapers

1. INTRODUCTION

In many engineering and scientific applications, flexible structures are attractive due to their advantages such as light weight and less energy consumption, see e.g. Song et al. [2006] for civil structures, Tokhi and Azad [2008] for robotic manipulators and Fleming et al. [2010] for a scanning probe microscope. However, they have highly resonant dynamics resulting in vibrations, which avoids even properly working of a device or machine. To suppress vibrations in flexible structures, there are two main techniques: (i) active vibration control that utilize sensors, actuators and controllers, (ii) passive techniques performed with damping elements without need of power supply. Active control approach is advantageous for suppressing the low-frequency vibrations, where passive systems are not capable due to limited damping characteristics.

An effective approach in active vibration control is to integrate (locate) *smart material* sensors and actuators on the flexible system, constructing a control architecture; see Chopra [2002]. Among various smart materials, piezoelectric patches, which can be used as both sensor and actuator, are usually preferable due to their fast response and large strain output (Orszulik and Shan [2012]). Piezoelectric sensor (PES) and piezoelectric actuator (PEA) pairs are commonly *collocated* on the flexible structure for active vibration control, since it provides better performance, robustness and easy implementation (Aphale et al. [2007]).

There are various active vibration feedback control methods for flexible structures with collocated sensors and actuators. One of the earliest and most well-known methods is the Positive Position Feedback (PPF) control, which is a second-order filter applied in the feedback path and was proposed by Goh and Caughey [1985]. Resonant Controllers and Integral Resonant Controllers (IRC) are other relatively recent well-known works, see Pota et al. [2002] and Aphale et al. [2007]. Also, delayed resonant controllers have been considered in Kim and Brennan [2013] and Kammer and Olgac [2016]. Among those methods, PPF approach is the commonly used/implemented one due to its (i) quick damping characteristics, (ii) insensitivity to spillover, (iii) easy implementation. Thus, the structure and design of PPF controllers have been widely studied and improved; see e.g. Moheimani et al. [2006], Mahmoodi and Ahmadian [2009] and Omidi and Mahmoodi [2015].

For vibration attenuation of flexible structures, input (command) shaping is a common technique conventionally applied in feed-forward path to suppress resonant modes. Various types of input shapers with lumped delays (e.g. ZV, ZVD, EI) have been proposed and investigated, see

⋆ This work was supported by the European Regional Development Fund under the project Robotics for Industry 4.0 (reg. no. CZ.02.1.01/0.0/0.0/15_003/0000470).

Singhose [2009]. Shapers with distributed delays have been studied recently since they provide better robustness, see e.g. Vyhlídal and Hromčík [2015], Alikoç et al. [2016]. Vyhlídal et al. [2016] proposed an efficient feedback architecture, the inverse of a signal shaper's dynamics in the feedback path, for flexible systems maneuvered by an actuator. Note that the proposed inverse shaper design architecture is capable of suppression of vibrations caused by disturbances on the actuator output. However, it is not able to attenuate the effects of disturbances acting on the flexible structure, since the flexible part is considered to be out of the feedback-loop.

In this study, we utilize a well-known input ZV shaper in its inverse form together with an all-pass filter interconnected in series, for active vibration suppression of flexible structures with a collocated PES-PEA pair. The proposed controller is applied in the positive feedback loop. The proposed control design is inspired from the similarity of inverse ZV shaper's frequency response to the classical PPF controller's. We provide two different optimization based methods to parametrize the controller for efficient vibration suppression. To the best of our knowledge, the proposed control design is new in the literature for active vibration control of collocated flexible structures. The proposed technique, with utilizing a genetic algorithm for solving optimization problem, is performed over a real system model via simulations, and compared with a recently proposed PPF controller. The results show that the inverse shaper design with the proposed optimized parametrization provides faster and better vibration attenuation.

The paper is organized as follows. Section 2 provides the problem description with its model, and preliminaries for PPF control and inverse shaper design. We propose the new active vibration controller and the methods for its parametrization in Section 3. Section 4 provides the frequency and the time domain performance of the proposed controller over a case study. Section 5 concludes the paper.

2. PRELIMINARIES

In this section, we present the cantilever beam, which is a widely studied benchmark flexible structure, and its modeling. Then we provide a brief summary of the state-of-the-art work, PPF control design, for suppression of vibrations on flexible structures with collocated actuators and sensors. Finally, we present the conventional ZV shaper design and its inverse form application in the feedback loop, which is utilized in this work for the vibration suppression of flexible structures with a collocated PES-PEA pair.

2.1 Problem Description and Modeling

We study a typical smart flexible structure, in particular a flexible cantilever beam with a collocated PES-PEA pair, shown in Fig. 1. The beam is clamped at one end and is free at the other end, i.e. one end is fixed while the other is moving freely. The beam is assumed to be a Euler-Bernoulli beam, i.e. the beam deflection angle (or slope) is small enough and all of the beam sections are perpendicular to the neutral axis. The PES-PEA pair is attached on the beam. The displacement in y-axis, i.e. $y(r,t)$, which corresponds to the amplitude of vibrations

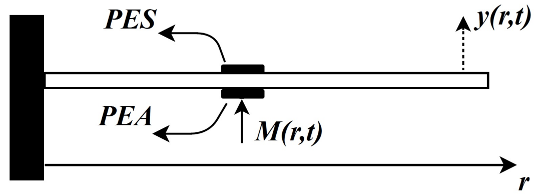

Fig. 1. The scheme of a cantilever beam with a collocated piezoelectric sensor-actuator pair

is measured by the induced voltage V_o by the PES. The bending moment $M(r,t)$, which is the input to the flexible system, is generated by the applied voltage V_i by the PEA to suppress the vibrations.

The general motion equation of a Euler-Bernoulli beam is a fourth-order partial differential equation, from which $y(r,t)$ can be solved numerically with respect to the relevant boundary conditions, see Moheimani et al. [2003]. The closed-form solution is possible when the beam is assumed to have a uniform mass distribution and a constant cross-sectional area. The exact solution is obtained utilizing the modal analysis technique in Halim [2002] with infinite number of mode shape functions described in Meirovitch [1975]. A more efficient approach based on finite element model, which is valid even for modeling non-homogeneous beams and includes the effect of collocated PES-PEA pair, is given in Orszulik and Shan [2011] and Orszulik and Shan [2012]. This way, the transfer function from V_i to V_o, i.e. from $M(r,t)$ to $y(r,t)$, is given as

$$G(s) = \sum_{i=1}^{\infty} \frac{k_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \tag{1}$$

where k_i is the gain, ζ_i is the damping ratio, ω_i is the frequency.

Indeed, the transfer function (1) of the considered flexible system has infinite number of modes. Obtaining such a model and control design over this model is theoretically challenging. However, in practice, a certain frequency range is of interest, and the modes in this range are usually considered for modeling and control design. In that manner, for practical modeling of flexible structures, (1) is truncated to N modes of interest as

$$G(s) = \sum_{i=1}^{N} \frac{k_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} + D, \tag{2}$$

where D is a constant feedthrough term.

Note that the term D in truncated transfer function (2) has an important role. Truncation process yields altering the actual zeros of (1) with the exclusion of high frequency modes. This undesired fact could be corrected with D for accurate modeling; see e.g. Moheimani [2000], Halim [2002] for different approaches on determination of D.

2.2 An Overview of PPF Control for Collocated Flexible Structures

PPF control is a pioneer design method for vibration suppression of flexible structures, proposed by Goh and Caughey [1985]. The block diagram of a PPF controlled flexible system with a collocated sensor-actuator pair is

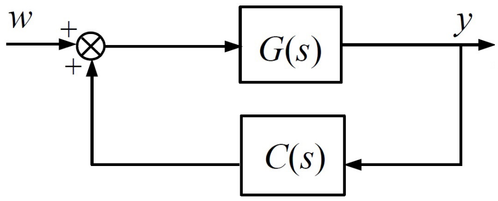

Fig. 2. Block diagram of a flexible system with a collocated sensor-actuator pair controlled with PPF

given in Fig. 2, see Mahmoodi et al. [2010], where w is the disturbance and y is the displacement of the beam. $G(s)$ represents the flexible structure transfer function given by (2). $C(s)$ is the PPF controller transfer function.

The main idea of PPF proposed in Goh and Caughey [1985] is to suppress the vibrations via designing a second-order filter $C(s)$ with a large damping as compared to the system damping. This approach was extended in Fanson and Caughey [1990] to multi-modal design, such that the PPF controller transfer function corresponds to

$$C(s) = \sum_{i=1}^{\tilde{N}} \frac{g_i \omega_{ci}^2}{s^2 + 2\zeta_{ci}\omega_{ci}s + \omega_{ci}^2}, \qquad (3)$$

where g_i, ζ_{ci} and ω_{ci} are the controller gain, damping ratio and frequency, respectively, determined to compensate the i^{th} mode of the flexible system (1). Note that $\tilde{N}$ is the number of targeted resonant modes in the controller, typically chosen $\tilde{N} < N$ for lower computational load in design. Design procedure for determining the parameters of (3) is summarized as follows: (i) Select $\zeta_{ci} > \zeta_i$, (ii) Select $\omega_{ci} \approx \omega_i$, (iii) Determine g_i's to maximize the closed-loop system damping by placing the closed-loop poles. An alternative approach to determine the parameters in (3) is H_∞-optimization based, namely

$$\min_{g_i, \zeta_{ci}, \omega_{ci}} \parallel T(j\omega) \parallel, \quad \forall \omega \in \mathbb{R} \qquad (4)$$

where $T(s)$ stands for the closed-loop transfer function from w to y in Fig. 2; see Moheimani et al. [2006] and Orszulik and Shan [2012] using a nonlinear search and a genetic algorithm, respectively, for the solution.

The vibration suppression performance of the PPF to steady-state disturbances has been enhanced by Mahmoodi and Ahmadian [2009], utilizing first-order filters in feedback path additionally to (3). This approach was generalized to optimization based multi-mode design in Omidi and Mahmoodi [2015], so called Multi-mode Modified PPF (MMPPF), with the controller transfer function

$$C(s) = \sum_{i=1}^{\tilde{N}} \frac{\alpha_i^2}{s^2 + 2\zeta_{ci}\omega_{ci}s + \omega_{ci}^2} + \frac{\beta_i^2}{s + \omega_{ci}}. \qquad (5)$$

Two different optimization methods for MMPPF controller design was proposed: LQR and M-norm optimization. It is shown that the M-norm optimized MMPPF is advantageous in terms of calculation simplicity and the power consumption. The idea is to determine the parameters in (5) which minimizes the cost function

$$M = \sum_{i=1}^{N} \parallel a_i \, T(j\omega_i) \parallel \qquad (6)$$

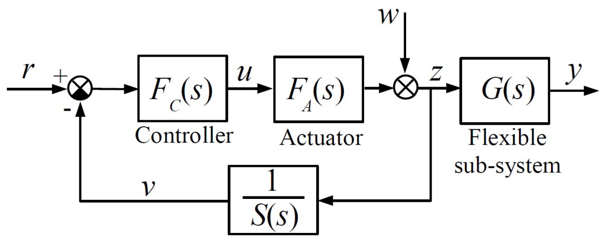

Fig. 3. Closed-loop system with inverse signal shaper

where a_i is the pre-selected weighting constant for the i^{th} resonant frequency ω_i, regarding to its importance for the designer. Clearly, by minimizing (6), it is aimed to reduce largest displacement amplitudes, i.e. vibrations, which occurs at system resonant frequencies.

2.3 Preliminaries on Input Shaper Design for Vibration Suppression

The main idea of input shaper design is feed-forward filtering (shaping) the reference command applied to an actuator mounted to a flexible structure. A classical example for such a system is a crane with a suspended load. The task of the shaper $S(s)$ is to move the actuator in such a way that the flexible system $G(s)$ is not excited. $S(s)$ is typically parametrized such that its dominant zeros are placed on a resonant mode,

$$r_{si} = -\zeta_{si}\omega_{si} \pm j\omega_{si}\sqrt{1-\zeta_{si}^2} \qquad (7)$$

of $G(s)$ in the open-loop.

A well-known and widely-used shaper is the ZV shaper represented by the transfer function

$$S(s) = A + (1-A)e^{-s\tau} \qquad (8)$$

where the gain and the delay are parametrized as

$$A = \frac{e^{\zeta_{si}\omega_{si}\pi/(\omega_{si}\sqrt{1-\zeta_{si}^2})}}{1 + e^{\zeta_{si}\omega_{si}\pi/\omega_{si}\sqrt{1-\zeta_{si}^2}}}, \; \tau = \frac{\pi}{\omega_{si}\sqrt{1-\zeta_{si}^2}}, \qquad (9)$$

respectively, to compensate the resonant mode r_{si} of $G(s)$, see Singhose et al. [1994].

Note that the shaper design in open-loop control is not able to cancel the vibrations caused by disturbances acting on the actuator. To overcome this disadvantage, the feedback architecture for the shaper design shown in Fig. 3 has been proposed in Vyhlídal et al. [2016]. The main motivation for this architecture is to cancel the effect of the disturbance w on the residual vibrations of the flexible sub-system $G(s)$, as well as the effect of the reference input r. To see this fact, consider the transfer functions,

$$T_{ry}(s) = \frac{S(s)F_C(s)F_A(s)}{S(s) + F_C(s)F_A(s)}G(s), \qquad (10)$$

$$T_{wy}(s) = \frac{S(s)}{S(s) + F_C(s)F_A(s)}G(s) \qquad (11)$$

from the reference and the disturbance, respectively, to the output y of the flexible sub-system. It is clear that the zeros of the shaper $S(s)$, which are also the zeros of the closed-loop transfer functions (10) and (11), can be assigned to cancel the resonant mode (7) of $G(s)$.

The ZV shaper (8) can be utilized in the feedback loop in Fig. 3 with its inverse form, which has the transfer function

$$S^{-1}(s) = \frac{z(s)}{v(s)} = \frac{1}{A + (1-A)e^{-s\tau}}, \tag{12}$$

and is easy to implement, see Vyhlídal et al. [2016]. Indeed, the parametrization of Inverse ZV (IZV) shaper is also as given in (9) to cancel a single resonant mode (7).

In the following section, we present the reasoning of utilization of IZV shaper with an all-pass filter and its parametrization via optimization for active vibration suppression of flexible structures with a collocated sensor-actuator pair, which are modeled by (2).

3. INVERSE ZV SHAPER BASED CONTROL FOR COLLOCATED FLEXIBLE STRUCTURES

In this section, we first present over an example, the similarity in the frequency responses of the PPF controller and the IZV shaper. Inspired from that, we propose a new controller, namely a filtered IZV shaper, also applied in the positive feedback path to suppress vibrations in flexible structures with a collocated sensor-actuator pair. Then, we present two different optimization approaches, with newly introduced cost functions, to parametrize the proposed controller for efficient vibration suppression.

3.1 Motivating Example and the Proposed Controller

Consider the transfer function of the PPF controller in (3) for a single mode ($\tilde{N} = 1$) with the gain $g_{ci} = 1$, the damping ratio $\zeta_{c1} = 0.1$ and the frequency $\omega_{c1} = 5$ rad/s. Moreover, consider the IZV shaper in (12), where A and τ are parametrized as in (9) for the same damping ratio and frequency values. The frequency responses of the considered PPF controller and IZV shaper are given in Fig. 4.

Notice that the magnitude peak of the IZV shaper appears at the design frequency $\omega_{c1} = 5$ rad/s as it happens for the PPF controller. This is the main desired fact in control design for Fig. 2 to suppress vibrations at the resonant frequency caused by disturbances. On the other hand, the phase of the IZV shaper is 0° while the phase of the PPF controller is $-90°$ at $\omega = \omega_{c1}$. It is shown in Kwak and Heo [2007] that the phase of the controller $C(s)$ should be around $-90°$ at the resonant frequency. To do so, we utilize an all-pass filter interconnected to the IZV shaper in series, which only shifts the phase response of IZV shaper to $-90°$ at a resonant frequency ω_i, while keeping the magnitude response same; see dashed lines in Fig. 4. Thus, we propose the Filtered IZV (FIZV) controller described by the transfer function

$$C(s) = \sum_{i=1}^{M} S_i^{-1}(s) \frac{-s + \omega_i}{s + \omega_i}, \tag{13}$$

where

$$S_i^{-1}(s) = \frac{\gamma_i}{A_i + (1-A_i)e^{-s\tau_i}}, \tag{14}$$

applied in the positive feedback path as in Fig. 2, for multi-mode vibration suppression of the considered collocated flexible structure modeled by (2) with resonant frequencies ω_i. The IZV shaper to be designed for the corresponding resonant modes are interconnected in parallel similarly as in the multi-mode design of PPF controller.

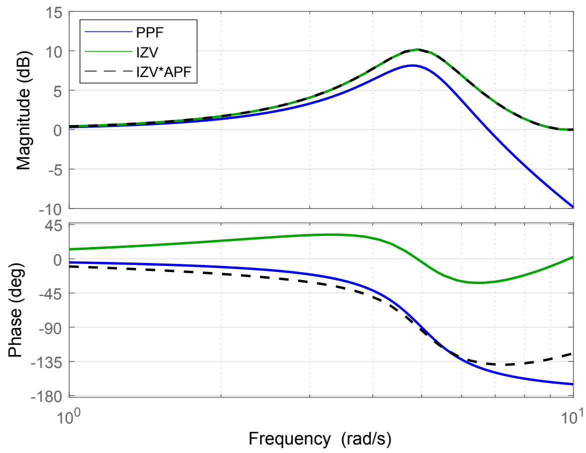

Fig. 4. The frequency responses of the PPF controller, IZV shaper and its interconnection with an all-pass filter

Note that the feedback architectures for active vibration control design in Fig. 2 and for the conventional inverse shaper application in Fig. 3 are different. For flexible structures with collocated PES-PEA pair, the flexible system transfer function is in the feedback loop, thus the closed-loop dynamics are affected by the flexible system modes. Besides, the architecture in Fig. 3 is not capable of suppressing the vibrations caused by disturbances acting on flexible structure and exciting its resonant modes at most, which is actually the main goal of this paper for the described problem. Thus, the IZV shaper terms (14) in the proposed FIZV controller (13) additionally have the gain parameters γ_i compared to the classic IZV shaper (12). In design, at least those gains should be determined properly to suppress vibrations, even if A_i and τ_i are parametrized as described in Section 2.3.

3.2 Parametrization of the FIZV Controller

Our aim is to parametrize the FIZV controller given by (13)-(14) applied in positive feedback path to suppress the resonant modes of (2). Note that each all-pass filter in (13) is parametrized with the corresponding resonant mode ω_i due to the needed phase shift explained in the previous subsection. Thus, it remains to parametrize the IZV shaper terms given by (14). For this purpose, we present two norm-based optimization approaches for the closed-loop transfer function $T(s)$ of the positive feedback loop in Fig. 2 with $G(s)$ in (2) and $C(s)$ in (13)-(14).

First, we consider a modified version of the M-norm optimization method with the given cost function (6). For parametrization of (13)-(14), we describe the optimization problem

$$\min_{\gamma_i} \quad \sum_{i=1}^{N} a_i \parallel T(j\omega_i) \parallel + a_0 T(0) \tag{15}$$

where A_i and τ_i are parametrized as in (9) for $\omega_{si} = \omega_i$ and a pre-selected small ζ_{si} value. Such selection of controller frequency and damping ratio was originally offered in Omidi and Mahmoodi [2015] for MMPPF design. Differently, we introduce an additional term, the weighted DC-gain term -$a_0 T(0)$- in the cost function of (15) to achieve better vibration suppression at low frequencies.

Table 1. System parameters in (2)

	k_i	ζ_i	ω_i
Mode 1	5.4681	0.0006	10.5833
Mode 2	108.9400	0.0033	65.5031
Mode 3	322.9974	0.0091	181.8815
Mode 4	153.8421	0.0177	354.3648

Table 2. Parameters of MMPPF (5) and IZV (14) controllers found by (15) and (16).

M-norm	**i = 1**	**i = 2**
MMPPF	$\alpha_1 = 5.5165$ $\beta_1 = 0.1015$	$\alpha_2 = 32.9111$ $\beta_2 = 1.0510$
FIZV	$k_1 = 0.2231$	$k_2 = 0.2750$
$\mathbf{H_\infty}$-norm	**i = 1**	**i = 2**
MMPPF	$\alpha_1 = 10.041$ $\beta_1 = 0.1365$ $\zeta_{c1} = 0.1971$	$\alpha_2 = 46.3298$ $\beta_2 = 0.0008$ $\zeta_{c2} = 0.1160$
FIZV	$\gamma_1 = 0.2044$ $\zeta_{s1} = 0.2044$	$\gamma_2 = 0.6377$ $\zeta_{s2} = 0.1441$

Second, we consider a modified version of the H_∞-norm optimization given by (4). For parametrization of (14), we describe the optimization problem

$$\min_{\gamma_i,\zeta_{si}} \quad \sum_{i=1}^{\tilde{N}} b_i \parallel T(s)H_i(s) \parallel_\infty + b_0\, T(0) \qquad (16)$$

where A_i and τ_i are parametrized as in (9) for $\omega_{si} = \omega_i$ and the optimization variable ζ_{si}, b_i is the pre-selected weighting constant for the i^{th} resonant mode, and $H_i(s)$ is a pre-determined band-pass filter for the i^{th} resonant mode. The weighted DC-gain term -$b_0T(0)$- is introduced here for the same reason in M-norm optimization. Moreover, the band-pass filters are utilized in the cost function of (16) additionally, compared to (4). $H_i(s)$ is chosen in such a way that the resonant mode ω_i is in the range of its bandwidth. This way, the desired frequency range is dominated to achieve a better sub-optimal solution of the non-convex problem (16) searched by an algorithm. We use a genetic algorithm for the parameter search to solve the described optimization problems (15) and (16) in the following section.

4. NUMERICAL SIMULATIONS

In this section, we consider the model of a real cantilever beam with a collocated PES-PEA pair studied in Orszulik and Shan [2012]. The flexible system has been modeled precisely by (2) for $N = 4$, i.e. for four resonant modes, with the parameters given in Table 1.

For the given system, we perform the design of proposed FIZV controller (13)-(14) and also the design of the MMPPF controller (5) proposed in Omidi and Mahmoodi [2015], with the presented M-norm and H_∞-norm optimization methods in Section 3.2. In design, we target the first two resonant modes of the system given in Table 1, thus $\tilde{N} = 2$ for both controllers. The controller parameters found by M-norm optimization in (15) and H_∞-norm optimization in (16) using the standard genetic algorithm of MATLAB, are given in Table 2. All the controller frequencies are determined as $\omega_{ci} = \omega_{si} = \omega_i$ in the same way of the conventional design approach. For the M-norm optimization based controllers, $\zeta_{ci} = \zeta_{si} = 0.01$ are pre-

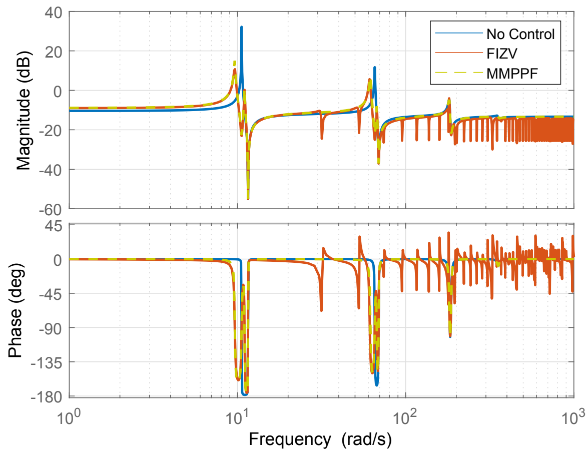

Fig. 5. The frequency responses of the flexible system, and the closed-loop systems controlled with FIZV and MMPPF controllers parametrized via M-norm optimization (15)

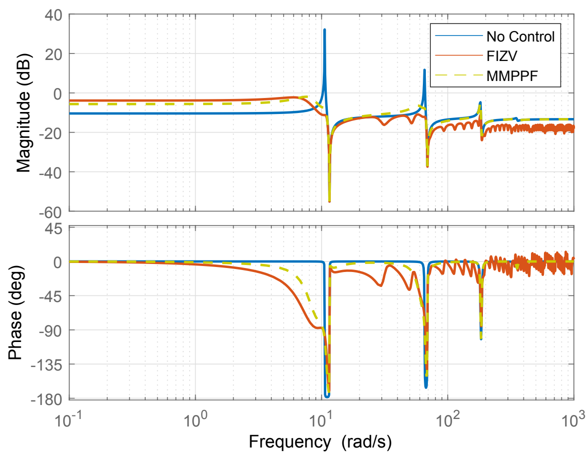

Fig. 6. The frequency responses of the flexible system, and the closed-loop systems controlled with FIZV and MMPPF controllers parametrized via H_∞-norm optimization (16)

selected. The frequency responses of the open-loop system $G(s)$, and the closed-loop systems controlled with FIZV and MMPPF parametrized via the presented M-norm and H_∞-norm optimization techniques are given in Fig. 5 and Fig. 6, respectively. The following was observed from the frequency responses in Fig. 5 and Fig. 6.

- All controllers designed via the given optimization approaches are able to suppress the targeted two resonant modes.
- M-norm based design provides better suppression at targeted resonant modes, while H_∞-norm provides suppression for wider range of frequencies.
- There is not a remarkable difference of M-norm based designed FIZV and MMPPF controllers in suppressing the resonant modes.
- Both controllers designed via M-norm optimization do not suppress the 3^{rd} resonant mode. However, for H_∞-norm based design, FIZV suppresses the 3^{rd} resonant mode while MMPPF does not.
- Considering H_∞-norm based designs, the FIZV controller results in better suppression at resonant fre-

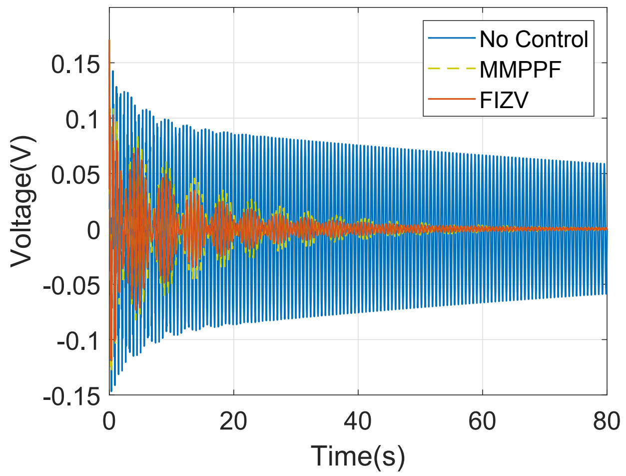

Fig. 7. The system responses under impulsive disturbance without control and with controls designed via M-norm optimization

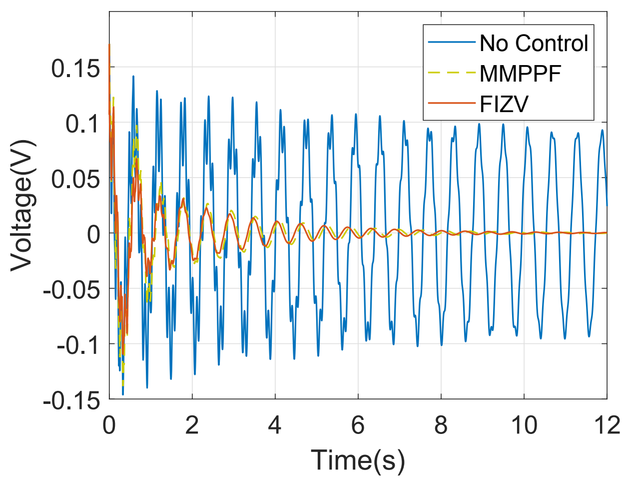

Fig. 8. The system responses under impulsive disturbance without control and with controls designed via H_∞-norm optimization

quencies and generally all over the frequency range compared to the MMPPF controller.

From the observations above, it is concluded that the FIZV controller provides good suppression of resonant modes. Especially for the H_∞-norm optimization based design case, the FIZV controller performs better than the MMPPF controller. Besides, less parameters are to be optimized for FIZV controller design compared to the MMPPF controller design, in both optimization methods.

Remark 1. Note that the frequency response of the system with FIZV controllers has troughs continuously, and also more frequently for higher frequencies. This fact, which occurs due to the delay term, i.e. the periodicity of $e^{-s\tau_i}$ terms in the closed-loop transfer functions, may yield in better suppression of high frequency resonant modes.

We also illustrate the time-domain performance of the designed controllers when an impulsive and a periodic disturbance act on the flexible system. First, to simulate

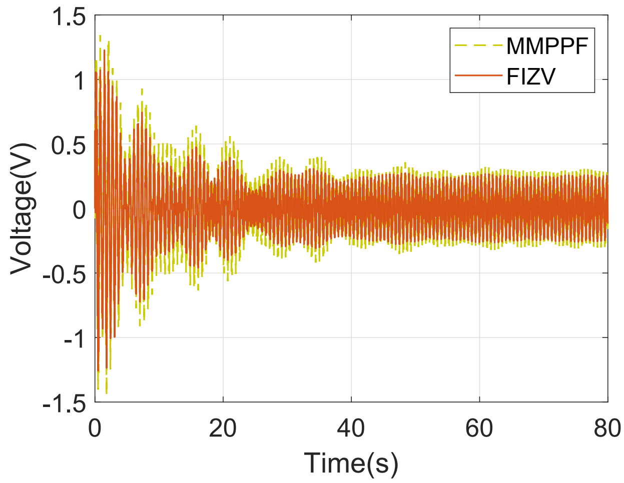

Fig. 9. The closed-loop system responses under periodic disturbance with controls via M-norm optimization

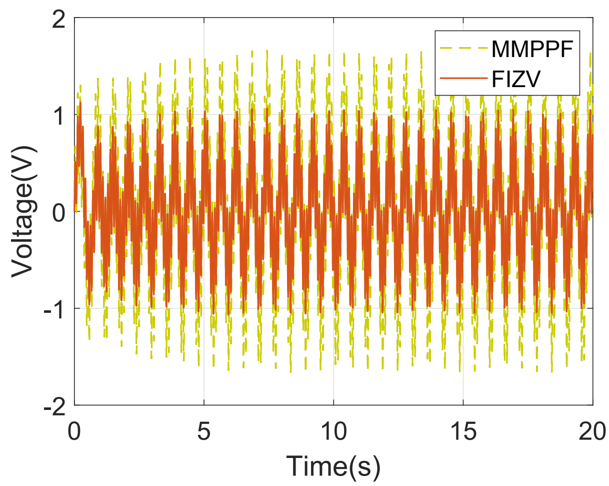

Fig. 10. The closed-loop system responses under periodic disturbance with controls via H_∞-norm optimization

the impulsive disturbance, we assume that an impulsive voltage of 0.17 V is applied at $t = 0$ to the PEA on the beam. The system response without control and the closed-loop system responses are given in Fig. 7 and Fig. 8 with controllers designed via M-norm based and H_∞-norm based optimization methods, respectively. For both methods, but especially for M-norm optimization, FIZV controller suppresses the vibrations faster.

To simulate the periodic disturbance, we assume that a periodic voltage disturbance is applied to the PEA on the beam. The periodic disturbance is characterized as $w(t) = 2\sin(\omega_1 t) + 2\sin(\omega_2 t)$, where ω_1 and ω_2 are the frequency values of the first two resonant modes of the flexible system given in Table 1. The closed-loop system responses are given in Fig. 9 and Fig. 10 for controllers designed via M-norm based and H_∞-norm based optimization methods, respectively. For both methods, but especially for H_∞-norm optimization, FIZV controller suppresses the vibrations better. Moreover, FIZV controller suppresses the vibrations faster than MMPPF controller in the M-norm optimization design case.

From the simulations in time-domain provided above, we conclude that the proposed FIZV controller has also good time-domain characteristics, and dominates the MMPPF controller.

Remark 2. Note that MMPPF controllers designed via the proposed optimization criteria in (15) and (16) provide better suppression performance compared to the previously proposed optimization criteria in (6) and (4). However, we skip to demonstrate that contribution over MMPPF controller design due to space limitations and our main focus on demonstrating the FIZV controller proposed in this study.

5. CONCLUSION

A new active vibration control design applied with positive feedback is proposed for flexible structures with a collocated PES-PEA pair. The proposed controller is based on the inverse form of a well-known ZV shaper, namely its serial interconnection with an all-pass filter, which we named as FIZV controller. The implementation of the FIZV controller is easy due its simple structure with lumped delays, as known from the classical input shaper design. New cost functions are introduced for the M-norm and H_∞-norm optimization based designs to parametrize the FIZV controller for better vibration suppression. Note that the proposed cost functions for optimization approaches can also be utilized to parametrize the other kinds of active vibration controllers. The effective vibration suppression characteristics of the FIZV controller is demonstrated over a real flexible system model, and compared with a recently proposed MMPPF controller. Numerical simulations show also that the FIZV controller provides better performance for vibration suppression as compared to the MMPPF controller, even it has a simpler form. In feature research, the effects of the FIZV controller on the closed-loop stability and on the robustness will be studied.

REFERENCES

B. Alikoç, T. Vyhlídal, and A.F. Ergenç. Closed-form smoothers and shapers with distributed delay for damped oscillatory modes. *IET Control Theory A.*, 10 (18):2534–2542, 2016.

S.S. Aphale, A.J. Fleming, and S.O.R. Moheimani. Integral resonant control of collocated smart structures. *Smart Mater. Struct.*, 16(2):439–446, 2007.

I. Chopra. Review of state of art of smart structures and integrated systems. *AIAA J.*, 40(11):2145–2187, 2002.

J.L. Fanson and T.K. Caughey. Positive position feedback control for large space structures. *AIAA J.*, 28(4):717–724, 1990.

A.J. Fleming, S.S. Aphale, and S.O.R. Moheimani. A new method for robust damping and tracking control of scanning probe microscope positioning stages. *IEEE T. Nanotechnol.*, 9(4):438–448, 2010.

C.J. Goh and T.K. Caughey. On the stability problem caused by finite actuator dynamics in the collocated control of large space structures. *Int. J. Control*, 41 (3):787–802, 1985.

D. Halim. *Vibration Analysis and Control of Smart Structures.* PhD thesis, The University of Newcastle, 01 2002.

A.S. Kammer and N. Olgac. Delayed resonator concept for vibration suppression using piezoelectric networks. *Smart Mater. Struct.*, 25(11):115008, 2016.

S.M. Kim and M.J. Brennan. Active vibration control using delayed resonant feedback. *Smart Mater. Struct.*, 22(9):095013, 2013.

M.K. Kwak and S. Heo. Active vibration control of smart grid structure by multiinput and multioutput positive position feedback controller. *J. Sound Vib.*, 304(1-2): 230–245, 2007.

S.N. Mahmoodi and M. Ahmadian. Active vibration control with modified positive position feedback. *J. Dyn. Syst.-T ASME*, 131(4):1–8, 2009.

S.N. Mahmoodi, M. Ahmadian, and Da.J. Inman. Adaptive modified positive position feedback for active vibration control of structures. *J. Intel. Mat. Syst. Str.*, 21 (6):571–580, 2010.

L. Meirovitch. *Elements of Vibration Analysis.* McGraw-Hill, 1975.

S.O.R. Moheimani. Minimizing the effect of out of bandwidth modes in truncated structure models. *J. Dyn. Syst.-T. ASME*, 122(1):237–239, 2000.

S.O.R. Moheimani, D. Halim, and A. J. Fleming. *Spatial control of vibration: theory and experiments.* World Scientific, 2003.

S.O.R Moheimani, B.J.G Vautier, and B. Bhikkaji. Experimental implementation of extended multivariable PPF control on an Active Structure. *IEEE T. Contr. Syst. T.*, 14(3):443–445, 2006.

E. Omidi and S.N. Mahmoodi. Multimode modified positive position feedback to control a collocated structure. *J. Dyn. Syst.-T. ASME*, 137(5), 2015.

R.R. Orszulik and J. Shan. Vibration control using input shaping and adaptive positive position feedback. *J. Guid. Control Dynam.*, 34(4):1031–1044, 2011.

R.R. Orszulik and J. Shan. Active vibration control using genetic algorithm-based system identification and positive position feedback. *Smart Mater. Struct.*, 21(5): 055002, 2012.

H.R. Pota, S.O.R. Moheimani, and M. Smith. Resonant controllers for smart structures. *Smart Mater. Struct.*, 11(1):1–8, 2002.

W. Singhose. Command shaping for flexible systems: A review of the first 50 years. *Int. J. Precis. Eng. Man.*, 10(4):153–168, 2009.

W. Singhose, W. Seering, and N. Singer. Residual vibration reduction using vector diagrams to generate shaped inputs. *J. Mech. Des.-T. ASME*, 116(2):654–659, 1994.

G. Song, V. Sethi, and H. Li. Vibration control of civil structures using piezoceramic smart materials: A review. *Eng. Struct.*, 28(11):1513–1524, 2006.

M.O. Tokhi and A.K.M Azad. *Flexible Robot Manipulators: modelling, simulation and control.* IET, London, United Kingdom, 2008.

T. Vyhlídal and M. Hromčík. Parameterization of input shapers with delays of various distribution. *Automatica*, 59:256–263, 2015.

T. Vyhlídal, M. Hromčík, V. Kučera, and M. Anderle. On feedback architectures with zero-vibration signal shapers. *IEEE T. Automat. Contr.*, 61(8):2049–2064, 2016.