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Bounded-error target localization and tracking in presence of decoys using a fleet of UAVs

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Abstract: This paper addresses the problem of searching and tracking of an *a priori* unknown number of targets spread over some geographical area using a fleet of UAVs. State perturbations and measurement noises are assumed to belong to bounded sets. In the monitored geographical area, some decoys may be erroneously considered as targets when observed under specific conditions.
A robust bounded-error estimation approach is proposed to evaluate, at each time step, sets guaranteed to contain the actual state of already localized true targets and decoys. A set containing the states of targets still to be discovered is also evaluated. These sets are used to determine the control inputs of UAVs so as to minimize the estimation uncertainty at future time steps.
Simulations involving several UAVs show that the proposed robust set-membership estimator is able to estimate the state of all actual targets and to efficiently identify and eliminate decoys.

Keywords: Set-membership estimation, Target detection and tracking, Decoys detection, Cooperative guidance.

1. INTRODUCTION

Searching, detecting, and tracking mobile targets over some potentially large domain is a challenging task which can be addressed efficiently by cooperative agents such as fleets of UAVs, see Robin and Lacroix (2016); Khan et al. (2018). The search process, whereby the fleet collects and processes observations, is usually based on the use of probabilistic information. Various methods have been developed to determine the search strategy using optimal flight path design as in Moon (2008), using distributed model predictive control, see Yao et al. (2016), or using game-theoretic approaches, see Li and Duan (2017).

The efficiency of the selected strategy depends tightly on the availability, quality, and reliability of the information collected by UAVs. A target can be detected when a measurement of its state is available at a given UAV. The quality of the collected measurement determines the accuracy with which the target is localized. Measurement perturbation is mainly modelled as an additive noise, usually assumed to be a zero-mean Gaussian process. Its variance translates the quality and availability of the measurement, *e.g.*, the variance becomes very large when the range is above a given threshold, see Hu et al. (2014). As pointed out in Gu et al. (2015), the resulting performance may prove sensitive to the *a priori* assumptions on the probability density functions (pdfs) describing the process and measurement noises.

To overcome this issue, a set-membership description of uncertainties is suggested in Gu et al. (2015) or Drevelle et al. (2013). The only assumption made on the noises and uncertainties is that their realizations remain within known bounds. Using this description, one no longer searches for a single point estimate associated with *a posterior* density function but for sets guaranteed to contain the target states at each time step. Such an approach has been applied to cooperative guidance of a fleet of UAVs for target searching in Reynaud et al. (2018), and in Reboul et al. (2019), this former including the presence of obstacles in the field of target displacements.

The last origin of discrepancy of the collected information is the reliability of the measurements. A detected object may not necessarily correspond to a target. Several approaches have been considered to model the uncertainty on the decision of considering a detected object as a target. Several authors, see, *e.g.*, Bar-Shalom et al. (2011), introduce a false alarm probability to account for the imperfect processing of the information acquired by sensors. Another possibility consists in considering the presence of decoys, *i.e.*, static elements of environment (rocks, bushes, *etc.*) that can be considered as a true target when seen from a specific point of view. For example, Flint et al. (2004) introduces a Bayesian process for cooperative search when the sensors embedded on the UAVs are not able to determine whether a detected target is real or not. In He et al. (2017), random finite set probability density is used to model either the target-generated observation or false alarms. An interactive multi-model filter is then used to estimate the modes of the measured objects.

In this paper, we assume that each UAV is equipped with a sensor able to detect and localize targets in some compact subset of the search area. Contrary to Reynaud et al. (2018) and Reboul et al. (2019), one considers here the presence of several static decoys which could be erroneously interpreted as a true target under specific observation conditions. A distributed robust set-membership estimator is used to determine subsets where the targets may be located and subsets which are guaranteed to contain no target. A control input for each UAV is derived by predicting the impact of future measurements on the set estimates of target states using information provided by communicating neighbours. The control inputs minimizing the estimation uncertainty is determined using gradient search.

The paper is organized as follows. Section 2 describes the multi-target multi-UAV localization problem. Section 3 presents a distributed estimator that recursively provides sets that are guaranteed to contain the state of either true targets or decoys. Section 4 presents a control input design scheme to drive each UAV so as to minimize some measure of the estimation uncertainty. In Section 5, the set-membership estimator and the control input design algorithms are evaluated on simulations. Section 6 concludes this paper.

2. PROBLEM FORMULATION

Consider a fleet of N_u identical UAVs which aim is to search and track N_t potentially moving targets within some limited geographical area. The environment of the UAVs is cluttered with N_d decoys, each of which may be erroneously interpreted as a target when observed under specific conditions. N_t and N_d are constant but not known *a priori*.

2.1 UAV and target states

Time is assumed to be sampled with a constant period T. At time k (time instant $t = kT$), let $\mathbf{x}^\mathrm{u}_{i,k} \in \mathbb{R}^{n_\mathrm{u}}$ be the state vector of UAV i, $\mathbf{x}^\mathrm{t}_{j,k} \in \mathbb{R}^{n_\mathrm{t}}$ the state vector of target j, and $\mathbf{x}^\mathrm{d}_{\ell,k} \in \mathbb{R}^{n_\mathrm{t}}$ the state vector of decoys ℓ. The evolution with time of the state of UAVs and true targets is modelled as

$$\mathbf{x}^\mathrm{u}_{i,k+1} = \mathbf{f}^\mathrm{u}_k\left(\mathbf{x}^\mathrm{u}_{i,k}, \mathbf{u}_{i,k}\right) \tag{1}$$

and

$$\mathbf{x}^\mathrm{t}_{j,k+1} = \mathbf{f}^\mathrm{t}_k\left(\mathbf{x}^\mathrm{t}_{j,k}, \mathbf{v}_{j,k}\right), \tag{2}$$

where $\mathbf{u}_{i,k}$ is the control input for UAV i, to be chosen in a set $\mathbb{U}$ of admissible control inputs; $\mathbf{v}_{j,k}$ are unknown target state perturbations belonging to the known box $[\mathbf{v}]$. In this paper, one assumes that the decoys are static, thus:

$$\mathbf{x}^\mathrm{d}_{\ell,k+1} = \mathbf{x}^\mathrm{d}_{\ell,k}. \tag{3}$$

An extension to moving false targets would require the introduction of a dynamical model for the decoys similar to (2).

At time $k = 0$, all $\mathbf{x}^\mathrm{t}_{j,0}$ and $\mathbf{x}^\mathrm{d}_{\ell,0}$ are assumed to belong to some *a priori* known compact set $\mathbb{X}_0 \subset \mathbb{R}^{n_\mathrm{t}}$. Moreover, one assumes that $\mathbf{x}^\mathrm{t}_{j,k} \in \mathbb{X}_0$ for all $k > 0$.

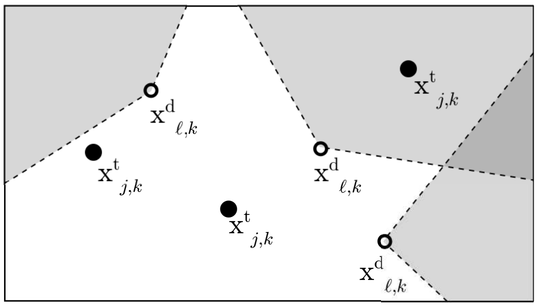

Fig. 1. Projection of the 2D plane (x_1, x_2) of the search area, of the state of true targets $\mathbf{x}^\mathrm{t}_{i,k}$ (filled circles) and decoys $\mathbf{x}^\mathrm{d}_{\ell,k}$ (empty circles); the projection of the conic subspace within which UAVs may be confused with decoys is also represented in gray.

2.2 Measurements

Each UAV is equipped with a sensor able to observe a subset of the target state space $\mathbb{X}_0$ and to acquire information on targets belonging to this subset. For a given value $\mathbf{x}^\mathrm{u}_{i,k}$ of the state of UAV i, the observed subset (field-of-view) is denoted as $\mathbb{F}_i(\mathbf{x}^\mathrm{u}_{i,k}) \subset \mathbb{R}^{n_\mathrm{t}}$. From the analysis of $\mathbb{F}_i(\mathbf{x}^\mathrm{u}_{i,k})$ at time k, one assumes that UAV i is able to get a list $\mathcal{D}_{i,k}$ of indices of *detected* targets, *i.e.*,

$$\mathbf{x}^\mathrm{t}_{j,k} \in \mathbb{F}_i\left(\mathbf{x}^\mathrm{u}_{i,k}\right) \Rightarrow j \in \mathcal{D}_{i,k}. \tag{4}$$

Each decoy is confused with one of the N_t true targets. For each decoy of index $\ell = 1, \ldots, N_\mathrm{d}$, the index $j(\ell) \in [1, N_\mathrm{t}]$ of the true target with which it is confused is assumed to be constant. When a decoy is present in the field-of-view $\mathbf{x}^\mathrm{d}_\ell \in \mathbb{F}_i(\mathbf{x}^\mathrm{u}_{i,k})$, one assumes that it is only detected when some additional observation condition is satisfied, *i.e.*,

$$\mathbf{x}^\mathrm{d}_{\ell,k} \in \mathbb{F}_i\left(\mathbf{x}^\mathrm{u}_{i,k}\right) \text{ and } g_{i,\ell}\left(\mathbf{x}^\mathrm{u}_{i,k}, \mathbf{x}^\mathrm{d}_{\ell,k}\right) \geqslant 0 \Rightarrow j(\ell) \in \mathcal{D}_{i,k}. \tag{5}$$

The UAVs are not aware of the structure of $g_{i,\ell}$. The function $g_{i,\ell}(\mathbf{x}^\mathrm{u}_{i,k}, \mathbf{x}^\mathrm{d}_{\ell,k}) \geqslant 0$ indicates, for example, that the ℓ-th object is confused with target $j(\ell)$ only if it is observed from specific points of view belonging to some polyhedral cone whose apex is $\mathbf{x}^\mathrm{d}_{\ell,k}$.

Fig. 1 illustrates the 2D projection of the search space, as well as the projection of the state of true targets $\mathbf{x}^\mathrm{t}_{i,k}$ (filled circles) and of decoys $\mathbf{x}^\mathrm{d}_{\ell,k}$ (empty circles). The projection of the cones, defined by $g_{i,\ell}$, in which the UAVs may be confused by decoys is represented in gray.

For each (true) target $j \in \mathcal{D}_{i,k}$, a noisy observation of the state $\mathbf{x}^\mathrm{t}_{j,k}$ is obtained as

$$\mathbf{y}_{i,j,k} = \mathbf{h}_i\left(\mathbf{x}^\mathrm{u}_{i,k}, \mathbf{x}^\mathrm{t}_{j,k}\right) + \mathbf{w}_{i,j,k}, \tag{6}$$

where $\mathbf{h}_i$ is the observation equation of UAV i and $\mathbf{w}_{i,j,k}$ represents some measurement noise, bounded in some known box $[\mathbf{w}]$. When a decoy with state $\mathbf{x}^\mathrm{d}_\ell$ is detected, a similar observation equation is obtained

$$\mathbf{y}_{i,j(\ell),k} = \mathbf{h}_i\left(\mathbf{x}^\mathrm{u}_{i,k}, \mathbf{x}^\mathrm{d}_\ell\right) + \mathbf{w}^\mathrm{d}{}_{i,j(\ell),k} \tag{7}$$

where $\mathbf{w}^\mathrm{d}{}_{i,j(\ell),k}$ is again some measurement noise, bounded in some known box $[\mathbf{w}]$. As many observations $\mathbf{y}_{i,j,k}$ as true targets or decoys present in $\mathbb{F}_i(\mathbf{x}^\mathrm{u}_{i,k})$ are obtained.

2.3 Communications

When two UAVs with indexes i_1 and i_2 come in vicinity, they are able to exchange their respective information. Assume that the communication topology of the fleet at time k is described by an undirected graph $\mathcal{G}_k = (\mathcal{N}_\mathrm{u}, \mathcal{E}_k)$. $\mathcal{N}_\mathrm{u} = \{1, 2, ..., N_\mathrm{u}\}$ is the set of nodes and $\mathcal{E}_k \subset \mathcal{N}_\mathrm{u} \times \mathcal{N}_\mathrm{u}$ the set of edges of the network at time k. The set of neighbours of UAV i at time k is $\mathcal{N}_{i,k} = \{j \in \mathcal{N}_\mathrm{u} | (i,j) \in \mathcal{E}_k,\ i \neq j\}$. When $(i,j) \in \mathcal{E}_k$, then UAVs i and j are able to communicate without delay and without error. When $(i,j) \notin \mathcal{E}_k$, then UAVs i and j are unable to communicate.

2.4 Estimates

$\mathbb{I}_{i,k}$ gathers the information available to UAV i at time k. From $\mathbb{I}_{i,k}$, UAV i is able to evaluate $\mathcal{L}_{i,k}$, the *list* of indices of true targets or decoys already detected or which presence has been signaled by an other UAV of the fleet to UAV i. $\mathbb{I}_{i,k}$ is used to evaluate a list of *set estimates* $\mathcal{X}_{i,k} = \{\mathbb{X}_{i,j,k}\}_{j\in\mathcal{L}_{i,k}}$ containing the state values of the already detected true targets or decoys. $\mathbb{X}_{i,j,k}$ contains all possible values of $\mathbf{x}^\mathrm{t}_{j,k}$ that are consistent with the information available to UAV i at time k. If some decoys with index $j(\ell) = j$ has been detected, $\mathbb{X}_{i,j,k}$ may not necessarily contain $\mathbf{x}^\mathrm{t}_{j,k}$. Finally, UAV i also maintains a set $\overline{\mathbb{X}}_{i,k}$ containing the possible state values of *true targets* not yet detected.

2.5 Evaluation of the estimation uncertainty

To quantify the estimation uncertainty, one has to account for the set estimates of the already detected targets as well as for $\overline{\mathbb{X}}_{i,k}$. The estimation uncertainty for UAV i is defined as follows

$$\Phi\left(\mathcal{X}_{i,k}, \overline{\mathbb{X}}_{i,k}\right) = \frac{1}{\max\{1, |\mathcal{L}_{i,k}|\}} \sum_{j\in\mathcal{L}_{i,k}} \phi\left(\mathbb{X}_{i,j,k}\right) + \lambda\phi\left(\overline{\mathbb{X}}_{i,k}\right)$$

where $\phi(\mathbb{X}_{i,j,k})$ represents the volume of the set $\mathbb{X}_{i,j,k}$, $|\mathcal{L}_{i,k}|$ is the cardinal number of $\mathcal{L}_{i,k}$, and λ is some parameter to adjust the relative importance of the average state estimation uncertainty of detected targets and of not yet detected targets. The estimation uncertainty for the fleet at time k is obtained as

$$\Phi_k = \frac{1}{N_\mathrm{u}} \sum_{i=1}^{N_\mathrm{u}} \Phi\left(\mathcal{X}_{i,k}, \overline{\mathbb{X}}_{i,k}\right), \tag{8}$$

which corresponds to the average estimation uncertainty among all UAVs. In Section 4, the sequence of control inputs for each UAV is evaluated so as to minimize the estimation uncertainty Φ_k for each k.

3. SET ESTIMATES FOR A GIVEN UAV

This section describes the evolution with time of the set estimates $\mathcal{L}_{i,k}$, $\mathcal{X}_{i,k}$, and $\overline{\mathbb{X}}_{i,k}$ managed by a given UAV i. These estimates are evaluated considering a generalization of the nonlinear recursive set-membership state estimator introduced in Kieffer et al. (2002). As the classical Kalman filter, it alternates prediction and correction steps. For the initialization, at time $k = 0$, one has $\mathcal{L}_{i,0} = \emptyset$, $\mathcal{X}_{i,0} = \emptyset$, and $\overline{\mathbb{X}}_{i,0} = \mathbb{X}_0$ for $i = 1, \ldots, N_\mathrm{u}$.

3.1 Prediction step

Assume that at time k, $\mathcal{L}_{i,k}$, $\mathcal{X}_{i,k}$, and $\overline{\mathbb{X}}_{i,k}$ are available to UAV i. At time $k+1$, without additional information, the predicted value of $\mathcal{L}_{i,k}$ is

$$\mathcal{L}_{i,k+1|k} = \mathcal{L}_{i,k}, \tag{9}$$

since, with the information available at time k, one is unable to determine whether UAV i will detect new targets at time $k+1$.

For each target considered as detected with index $j \in \mathcal{L}_{i,k+1|k}$, UAV i has to evaluate the set of possible target state values at time $k+1$. For that purpose, two hypotheses have to be considered. In case of a true target, one has to determine the set of all target state values that are consistent with $\mathbb{X}_{i,j,k}$, with the dynamics (2), and the bounded state perturbation. In case of a decoy, one has to account for (3). Consequently,

$$\begin{aligned}\mathbb{X}_{i,j,k+1|k} &= \mathbf{f}^\mathrm{t}_k\left(\mathbb{X}_{i,j,k}, [\mathbf{v}_k]\right) \cup \mathbb{X}_{i,j,k} \\ &= \left\{\mathbf{f}^\mathrm{t}_k(\mathbf{x}, \mathbf{v}) \mid \mathbf{x} \in \mathbb{X}_{i,j,k}, \mathbf{v} \in [\mathbf{v}_k]\right\} \cup \mathbb{X}_{i,j,k}\end{aligned} \tag{10}$$

The predicted set $\overline{\mathbb{X}}_{i,k+1|k}$ has to contain all possible state values of potentially undetected true targets. Since all true targets evolve according to (2), $\overline{\mathbb{X}}_{i,k+1|k}$ is evaluated as

$$\overline{\mathbb{X}}_{i,k+1|k} = \mathbf{f}^\mathrm{t}_k\left(\overline{\mathbb{X}}_{i,k}, [\mathbf{v}_k]\right). \tag{11}$$

3.2 Correction step from measurements

Assume that at time $k+1$, UAV i evaluates $\mathcal{D}_{i,k+1}$ from the observation of $\mathbb{F}_i(\mathbf{x}^\mathrm{u}_{i,k+1})$ and, for each $j \in \mathcal{D}_{i,k+1}$, has access to $\mathbf{y}_{i,j,k+1}$ obtained using (6). Consequently

$$\mathbb{I}_{i,k+1|k+1} = \mathbb{I}_{i,k} \cup \left\{\mathcal{D}_{i,k+1}, \{\mathbf{y}_{i,j,k+1}\}_{j\in\mathcal{D}_{i,k+1}}\right\}. \tag{12}$$

Three cases have to be considered.

If $j \in \mathcal{D}_{i,k+1} \cap \mathcal{L}_{i,k+1|k}$, Target j has already been detected and may belong to $\mathbb{X}_{i,j,k+1|k}$. Nevertheless, due to decoys, there is no guarantee that $\mathbf{x}^\mathrm{t}_{j,k+1} \in \mathbb{X}_{i,j,k+1|k}$. In fact, one is only ensured that $\mathbf{x}^\mathrm{t}_{j,k+1} \in \mathbb{X}_{i,j,k+1|k} \cup \overline{\mathbb{X}}_{i,k+1|k}$. Now, since $j \in \mathcal{D}_{i,k+1}$, Target j is observed again at time $k+1$ in $\mathbb{F}_i(\mathbf{x}^\mathrm{u}_{i,k+1})$. First, if $\mathbf{y}_{i,j,k+1}$ corresponds to a measurement due to a true target, described by (6), either $\mathbf{x}^\mathrm{t}_{j,k+1} \in \mathbb{X}_{i,j,k+1|k} \cap \mathbb{F}_i(\mathbf{x}^\mathrm{u}_{i,k+1})$ (see Fig. 2a), and the measurement $\mathbf{y}_{i,j,k+1}$ confirms that $\mathbf{x}^\mathrm{t}_{j,k+1} \in \mathbb{X}_{i,j,k+1|k}$, or $\mathbf{x}^\mathrm{t}_{j,k+1} \in \overline{\mathbb{X}}_{i,k+1|k} \cap \mathbb{F}_i(\mathbf{x}^\mathrm{u}_{i,k+1})$ (see Fig. 2b), evidencing that measurements that have led to $\mathbb{X}_{i,j,k+1|k}$ are due to a decoy. Second, if $\mathbf{y}_{i,j,k+1}$ corresponds to a decoy and is described by (7), then $\mathbf{x}^\mathrm{t}_{j,k}$ may only belong to $\mathbb{X}_{i,j,k+1|k} \setminus \mathbb{F}_i(\mathbf{x}^\mathrm{u}_{i,k+1})$ or to $\overline{\mathbb{X}}_{i,k+1|k} \setminus \mathbb{F}_i(\mathbf{x}^\mathrm{u}_{i,k+1})$ (see Fig. 2c). Since one is unable to determine whether $\mathbf{y}_{i,j,k+1}$ is described by (6) or by (7), both cases have to be taken into account. Consequently, one is ensured that $\mathbf{x}^\mathrm{t}_{j,k}$ belongs either to the set

$$\mathbb{X}_{i,j,k+1|k+1} = \mathbb{S}_1 \cup \mathbb{S}_2 \cup \mathbb{S}_3 \tag{13}$$

or to

$$\overline{\mathbb{X}}_{i,k+1|k+1} = \overline{\mathbb{X}}_{i,k+1|k} \setminus \mathbb{F}_i\left(\mathbf{x}^\mathrm{u}_{i,k+1}\right), \tag{14}$$

where

$$\mathbb{S}_1 = \left\{\mathbf{x} \in \mathbb{X}_{i,j,k+1|k} \mid \mathbf{h}_{k+1}\left(\mathbf{x}^\mathrm{u}_{i,k+1}, \mathbf{x}\right) \in \mathbf{y}_{i,j,k+1} - [\mathbf{w}]\right\}$$
$$\mathbb{S}_2 = \left\{\mathbf{x} \in \overline{\mathbb{X}}_{i,k+1|k} \mid \mathbf{h}_{k+1}\left(\mathbf{x}^\mathrm{u}_{i,k+1}, \mathbf{x}\right) \in \mathbf{y}_{i,j,k+1} - [\mathbf{w}]\right\},$$

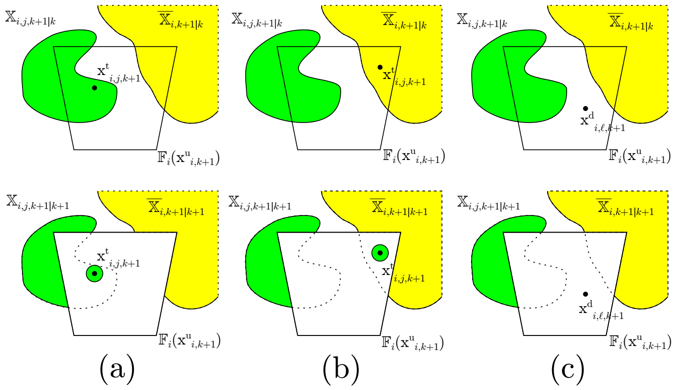

Fig. 2. Evaluation of $\mathbb{X}_{i,j,k+1|k+1}$ and $\overline{\mathbb{X}}_{i,k+1|k+1}$ using $\mathbb{F}_i(\mathbf{x}^{\mathrm{u}}_{i,k+1})$ and $\mathbf{x}^{\mathrm{t}}_{i,j,k+1}$ for an already detected target; (a) the measurement corresponds to a true target; (b) the measurement corresponds to a true target but previous measurements are related to decoy; (c) the measurement is due to a decoy; top and bottom subfigures are respectively before and after accounting for measurements

and

$$\mathbb{S}_3 = \left(\mathbb{X}_{i,j,k+1|k} \setminus \mathbb{F}_i\left(\mathbf{x}^{\mathrm{u}}_{i,k+1}\right)\right).$$

In (13), $\mathbb{S}_1$ and $\mathbb{S}_2$ are associated to a measurement $\mathbf{y}_{i,j,k+1}$ corresponding to a true target. $\mathbb{X}_{i,j,k+1|k}$ and $\overline{\mathbb{X}}_{i,k+1|k}$ are updated by keeping only the target state values that are consistent with $\mathbf{y}_{i,j,k+1}$, $\mathbf{h}_{k+1}$, and the measurement noise bounds (see Fig. 2a and b). The hypothesis that $\mathbf{y}_{i,j,k+1}$ is due to a decoy is taken into account in $\mathbb{S}_3$ and (14) to update the set of target states still to be explored (see Fig. 2c).

If $j \in \mathcal{D}_{i,k+1}$ and $j \notin \mathcal{L}_{i,k+1|k}$, a new target has been detected. Before detection, this new true target or decoy is only known to belong to $\overline{\mathbb{X}}_{i,k+1|k}$. One has also to take into account the measurement $\mathbf{y}_{i,j,k+1}$ related to this newly detected target. The set of all values of $\mathbf{x}^{\mathrm{t}}_{j,k+1}$ consistent with $\overline{\mathbb{X}}_{i,k+1|k}$, $\mathbf{y}_{i,j,k+1}$, the measurement equation (6), and the measurement noise bound $[\mathbf{w}]$ is in this case

$$\mathbb{X}_{i,j,k+1|k+1} = \left\{\mathbf{x} \in \overline{\mathbb{X}}_{i,k+1|k} \mid \mathbf{h}_{k+1}\left(\mathbf{x}^{\mathrm{u}}_{i,k+1}, \mathbf{x}\right) \in \mathbf{y}_{i,j,k+1} - [\mathbf{w}]\right\}. \quad (15)$$

When $j \notin \mathcal{D}_{i,k+1}$ and $j \in \mathcal{L}_{i,k+1|k}$, Target j, which was previously detected, is not observed in $\mathbb{F}_i(\mathbf{x}^{\mathrm{u}}_{i,k+1})$. The set of all values of $\mathbf{x}^{\mathrm{t}}_{j,k+1}$ that are consistent with $\mathbb{X}_{i,j,k+1|k}$ and that do not belong to $\mathbb{F}_i(\mathbf{x}^{\mathrm{u}}_{i,k+1})$ is then

$$\mathbb{X}_{i,j,k+1|k+1} = \mathbb{X}_{i,j,k+1|k} \setminus \mathbb{F}_i(\mathbf{x}^{\mathrm{u}}_{i,k+1}), \quad (16)$$

where $\mathbb{B} \setminus \mathbb{A} = \{x \in \mathbb{B} \mid x \notin \mathbb{A}\}$.

Finally, one has to incorporate in the set of potentially detected targets the indexes of newly detected targets and to remove all target indexes j whose set $\mathbb{X}_{i,j,k+1|k+1}$ is empty

$$\mathcal{L}_{i,k+1|k+1} = \left\{j \in \mathcal{L}_{i,k+1|k} \cup \mathcal{D}_{i,k+1} \mid \mathbb{X}_{i,j,k+1|k+1} \neq \emptyset\right\}.$$

When a true target of index j and one or several decoys confused with the true target of index j are simultaneously in $\mathbb{F}_i(\mathbf{x}^{\mathrm{u}}_{i,k+1})$, the resulting measurements are processed independently. The resulting set estimate is the union of the set estimates obtained for each of these measurements.

3.3 Correction step from communications

At the end of each time step k, UAV i communicates to its neighbours the sets $\mathcal{L}_{i,k+1|k+1}$, $\mathcal{X}_{i,k+1|k+1} = \left\{\mathbb{X}_{i,j,k+1|k+1}\right\}_{j\in\mathcal{L}_{i,k+1|k+1}}$, and $\overline{\mathbb{X}}_{i,k+1|k+1}$ and receives the corresponding sets from its neighbours. Using this additional information, UAV i updates its estimates. Thus,

$$\mathbb{I}_{i,k+1|k+1} = \mathbb{I}_{i,k+1|k+1} \bigcup_{j\in\mathcal{N}_{i,k+1}} \left\{\mathcal{L}_{j,k+1|k+1}, \mathcal{X}_{j,k+1|k+1}, \overline{\mathbb{X}}_{j,k+1|k+1}\right\}. \quad (17)$$

Among the neighbours $\mathcal{N}_{i,k+1}$ of UAV i, let $\mathcal{N}^{j}_{i,k+1}$ be the subset of neighbours which *believe* they have detected target j up to time $k+1$ and let $\overline{\mathcal{N}}^{j}_{i,k+1}$ be the subset of neighbours which *are sure* not to have detected target j up to time $k+1$. For each target j, already potentially detected by UAV i, one has either $\mathbf{x}^{\mathrm{t}}_{j,k+1} \in \mathbb{X}_{i,j,k+1|k+1}$ or $\mathbf{x}^{\mathrm{t}}_{j,k+1} \in \overline{\mathbb{X}}_{i,k+1|k+1}$. Introduce

$$\widetilde{\mathbb{X}}_{i,j,k+1|k+1} = \mathbb{X}_0 \setminus \left(\mathbb{X}_{i,j,k+1|k+1} \cup \overline{\mathbb{X}}_{i,k+1|k+1}\right)$$

as the set *proved not to contain* the state of target j. One has $\mathbf{x}^{\mathrm{t}}_{j,k+1} \notin \widetilde{\mathbb{X}}_{i,j,k+1|k+1}$.

Moreover, for all UAVs $\ell \in \mathcal{N}^{j}_{i,k+1}$, one has either $\mathbf{x}^{\mathrm{t}}_{j,k+1} \in \mathbb{X}_{\ell,j,k+1|k+1}$ or $\mathbf{x}^{\mathrm{t}}_{j,k+1} \in \overline{\mathbb{X}}_{\ell,k+1|k+1}$ and $\mathbf{x}^{\mathrm{t}}_{j,k+1} \notin \widetilde{\mathbb{X}}_{\ell,j,k+1|k+1}$. Finally, for UAVs $\ell \in \overline{\mathcal{N}}^{j}_{i,k+1}$, *i.e.*, which have not yet detected target j, one is sure to have $\mathbf{x}^{\mathrm{t}}_{j,k+1} \in \overline{\mathbb{X}}_{\ell,k+1|k+1}$ and $\mathbf{x}^{\mathrm{t}}_{j,k+1} \notin \widetilde{\mathbb{X}}_{\ell,j,k+1|k+1} = \mathbb{X}_0 \setminus \overline{\mathbb{X}}_{\ell,k+1|k+1}$. Consequently, upon reception of information from its neighbours, $\mathbb{X}_{i,j,k+1}$ is evaluated by UAV i as

$$\mathbb{X}_{i,j,k+1} = \bigcup_{\ell\in\mathcal{N}^{j}_{i,k+1}\cup\{i\}} \mathbb{X}_{\ell,j,k+1|k+1} \setminus \bigcup_{\ell\in\mathcal{N}_{i,k+1}\cup\{i\}} \widetilde{\mathbb{X}}_{\ell,k+1|k+1}, \quad (18)$$

i.e., as the union of all possible state values accounting for the measurements that have been obtained once target j has been detected $\mathbb{X}_{\ell,j,k+1|k+1}$, $\ell \in \mathcal{N}_{i,k+1} \cup \{i\}$ deprived by the union of all sets which have been proved not to contain target j at time $k+1$.

Using similar arguments, for each target j which has not yet been detected by UAV i, $\mathbb{X}_{i,j,k+1}$ is evaluated as

$$\mathbb{X}_{i,j,k+1} = \bigcup_{\ell\in\mathcal{N}^{j}_{i,k+1}} \mathbb{X}_{\ell,j,k+1|k+1} \setminus \bigcup_{\ell\in\mathcal{N}_{i,k+1}\cup\{i\}} \widetilde{\mathbb{X}}_{\ell,k+1|k+1}. \quad (19)$$

Again, all target indexes j for which $\mathbb{X}_{i,j,k+1}$ is empty have to be removed from $\mathcal{L}_{i,k+1}$

$$\mathcal{L}_{i,k+1} = \left\{j \in \bigcup\nolimits_{\mathcal{N}_{i,k+1}\cup\{i\}} \mathcal{L}_{\ell,k+1|k+1} \mid \mathbb{X}_{i,j,k+1} = \emptyset\right\},$$

and

$$\mathcal{X}_{i,k+1} = \{\mathbb{X}_{i,j,k+1}\}_{j\in\mathcal{L}_{i,k+1}}.$$

Finally, the subset of the state space still to be explored is the intersection of the unexplored part of the state space by UAV i and its neighbours

$$\overline{\mathbb{X}}_{i,k+1} = \overline{\mathbb{X}}_{i,k+1|k+1} \bigcap_{\ell\in\mathcal{N}_i} \overline{\mathbb{X}}_{\ell,k+1|k+1}. \quad (20)$$

Fig. 3 illustrates the sets resulting from (18), (19) and (20) for two cases. The size of $\mathbb{X}_{i,j,k+1}$ may be smaller than $\mathbb{X}_{i,j,k+1|k+1}$ as it is the case in Fig. 3a), when some subsets of $\mathbb{X}_{i,j,k+1|k+1}$ have been proved by another UAV not to

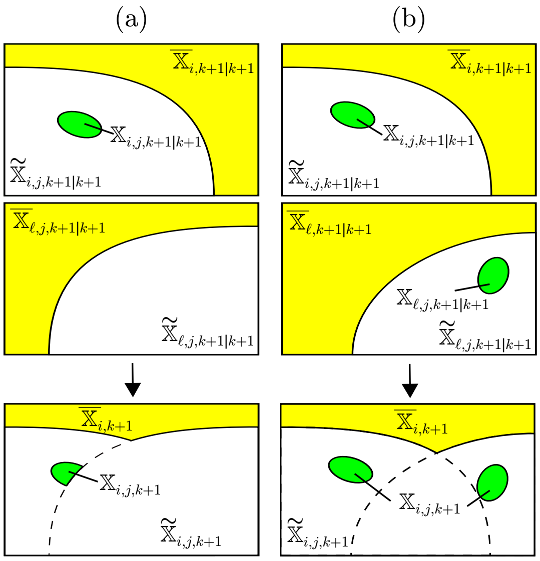

Fig. 3. Set estimates evaluated by UAVs i and ℓ before communication (two top subfigures) and after communication and update (bottom subfigures); (a) $\mathbb{X}_{i,j,k+1}$ is smaller than $\mathbb{X}_{i,j,k+1|k+1}$ since some subsets of $\overline{\mathbb{X}}_{i,j,k+1|k+1}$ have been proved by UAV ℓ not to contain a target; (b) $\mathbb{X}_{i,j,k+1}$ is larger than $\mathbb{X}_{i,j,k+1|k+1}$, since both UAVs have to account for the two different hypotheses on the state estimate of target j.

contain a target. It may also be larger, as is the case in Fig. 3b), where UAV ℓ has obtained measurements leading to another hypothesis on the state estimate of target j. Communications result in set $\overline{\mathbb{X}}_{i,k+1}$ which size is always reduced compared to that of $\overline{\mathbb{X}}_{i,k+1|k+1}$.

Algorithm 1 summarizes the prediction and correction steps from both measurements and communications.

4. COOPERATIVE CONTROL DESIGN

At time k, once the communications have been performed with its neighbours and the updated set estimates have been evaluated, each UAV i has access to $\mathcal{L}_{i,k}$, $\mathcal{X}_{i,k}$, and $\overline{\mathbb{X}}_{i,k}$. Ideally, the UAVs should then determine, in a distributed way, the sequence of control inputs which minimizes the estimation uncertainty at time $k+h$

$$\Phi_{k+h} = \frac{1}{N_{\mathrm{u}}} \sum_{i=1}^{N_{\mathrm{u}}} \Phi\left(\mathcal{X}_{i,k+h}, \overline{\mathbb{X}}_{i,k+h}\right), \tag{21}$$

where $h \geqslant 1$ is the prediction horizon. One relies on the distributed Model Predictive Control (MPC) formalism introduced, *e.g.*, in Morari and Lee (1999); Rochefort et al. (2014) and considers several additional simplifications.

One will consider a sequential approach, in which a subset $\mathcal{N}^{\mathrm{P}}_{i,k} \subset \mathcal{N}_{i,k}$ of neighbours of UAV i is assumed to have already evaluated their own control inputs and transmitted it as well as their state value at time k to UAV i. Let $(\mathbf{u}_{\ell,k+1}, \ldots, \mathbf{u}_{\ell,k+h})$ be the sequence of control inputs already evaluated by UAV $\ell \in \mathcal{N}^{\mathrm{P}}_{i,k}$. To evaluate the sequence of control inputs for UAV i, the communication graph is assumed to remain constant over the prediction horizon. Moreover, since UAV i has only access to the control inputs evaluated by its neighbours in $\mathcal{N}^{\mathrm{P}}_{i,k}$, the

Algorithm 1. Cooperative robust set-membership target state estimator
CoopRSMTSE$(\mathcal{L}_{i,k}, \mathcal{X}_{i,k}, \overline{\mathbb{X}}_{i,k})$

Input: $\mathcal{L}_{i,k}$, $\mathcal{X}_{i,k}$, and $\overline{\mathbb{X}}_{i,k}$
Output: $\mathcal{L}_{i,k+1}$, $\mathcal{X}_{i,k+1}$, and $\overline{\mathbb{X}}_{i,k+1}$
Prediction step
$\mathcal{L}_{i,k+1|k} = \mathcal{L}_{i,k}$
$\mathbb{X}_{i,j,k+1|k} = \mathbf{f}^{\mathrm{t}}_k\left(\mathbb{X}_{i,j,k}, [\mathbf{v}_k]\right)$, for all $j \in \mathcal{L}_{i,k+1|k}$
$\overline{\mathbb{X}}_{i,k+1|k} = \mathbf{f}^{\mathrm{t}}_k\left(\overline{\mathbb{X}}_{i,k}, [\mathbf{v}_k]\right)$
Correction step from measurements
$\mathcal{L}_{i,k+1|k+1} = \mathcal{L}_{i,k+1|k} \cup \mathcal{D}_{i,k+1}$
$\overline{\mathbb{X}}_{i,k+1|k+1} = \overline{\mathbb{X}}_{i,k+1|k} \setminus \mathbb{F}_i(\mathbf{x}^{\mathrm{u}}_{i,k+1})$
For all $j \in \mathcal{L}_{i,k+1|k+1}$
If $j \in \mathcal{D}_{i,k+1} \cap \mathcal{L}_{i,k+1|k}$
$\mathbb{X}_{i,j,k+1|k+1} = \mathbb{X}_{i,j,k+1|k} \setminus \mathbb{F}_i\left(\mathbf{x}^{\mathrm{u}}_{i,k+1}\right) \cup \Big\{\mathbf{x} \in \mathbb{X}_{i,j,k+1|k} \cup \overline{\mathbb{X}}_{i,k+1|k} \mid \mathbf{h}_{k+1}\left(\mathbf{x}^{\mathrm{u}}_{i,k+1}, \mathbf{x}\right) \in \mathbf{y}_{i,j,k+1} - [\mathbf{w}_{k+1}]\Big\}.$
Else if $j \in \mathcal{D}_{i,k+1}$ and $j \notin \mathcal{L}_{i,k+1|k}$
$\mathbb{X}_{i,j,k+1|k+1} = \overline{\mathbb{X}}_{i,k+1|k} \cap \mathbf{h}^{-1}_{i,k+1}\left(\mathbf{y}_{i,j,k+1} - [\mathbf{w}_{k+1}]\right)$
Else $(j \notin \mathcal{D}_{i,k+1}$ and $j \in \mathcal{L}_{i,k+1|k})$
$\mathbb{X}_{i,j,k+1|k+1} = \mathbb{X}_{i,j,k+1|k} \setminus \mathbb{F}_i\left(\mathbf{x}^{\mathrm{u}}_{i,k+1}\right)$
$\mathcal{L}_{i,k+1|k+1} = \left\{j \in \mathcal{L}_{i,k+1|k} \cup \mathcal{D}_{i,k+1} \mid \mathbb{X}_{i,j,k+1|k+1} \neq \emptyset\right\}$
Correction step from communications
$\mathcal{L}_{i,k+1} = \mathcal{L}_{i,k+1|k+1} \bigcup_{\ell \in \mathcal{N}_{i,k+1}} \mathcal{L}_{\ell,k+1|k+1}$
For all $j \in \mathcal{L}_{i,k+1}$
If $j \in \mathcal{L}_{i,k+1|k+1}$
$\mathbb{X}_{i,j,k+1} = \bigcup_{\ell \in \mathcal{N}^j_{i,k+1} \cup \{i\}} \mathbb{X}_{\ell,j,k+1|k+1} \setminus \bigcup_{\ell \in \mathcal{N}_{i,k+1} \cup \{i\}} \widetilde{\mathbb{X}}_{\ell,k+1|k+1}$
Else
$\mathbb{X}_{i,j,k+1} = \bigcup_{\ell \in \mathcal{N}^j_{i,k+1}} \mathbb{X}_{\ell,j,k+1|k+1} \setminus \bigcup_{\ell \in \mathcal{N}_{i,k+1} \cup \{i\}} \widetilde{\mathbb{X}}_{\ell,k+1|k+1}$
$\mathcal{L}_{i,k+1} = \left\{j \in \bigcup_{\mathcal{N}_{i,k+1} \cup \{i\}} \mathcal{L}_{\ell,k+1|k+1} \mid \mathbb{X}_{i,j,k+1} = \emptyset\right\}$
$\mathcal{X}_{i,k+1} = \left\{\mathbb{X}_{i,j,k+1}\right\}_{j \in \mathcal{L}_{i,k+1}}$

information, that will be provided by these agents via communication at the time steps $k+\kappa$, $\kappa = 1, \ldots, h$, will be considered in the construction of predicted values of $\mathcal{L}_{i,k}$, $\mathcal{X}_{i,k+\kappa}$, and $\overline{\mathbb{X}}_{i,k+\kappa}$, $\kappa = 1, \ldots, h$ to evaluate $\Phi\left(\mathcal{X}_{i,k+h}, \overline{\mathbb{X}}_{i,k+h}\right)$. To infer the estimates evaluated by its neighbours in $\mathcal{N}^{\mathrm{P}}_{i,k}$, UAV i will further neglect all information that its neighbours may receive from their own neighbours, which do not belong to $\mathcal{N}^{\mathrm{P}}_{i,k}$. Finally, at time k, all estimates are assumed to be equal, *i.e*, $\mathcal{X}_{\ell,k} = \mathcal{X}_{i,k}$ and $\overline{\mathbb{X}}_{\ell,k} = \overline{\mathbb{X}}_{i,k}$, $\ell \in \mathcal{N}^{P}_{i,k}$.

4.1 One step-ahead prediction horizon

Our aim is now to determine the impact of $\{\mathbf{u}_{\ell,k+1}\}_{\ell \in \mathcal{N}^{\mathrm{P}}_{i,k}}$ and of $\mathbf{u}_{i,k+1}$ on $\mathcal{X}_{i,k+1}$ and $\overline{\mathbb{X}}_{i,k+1}$. For that purpose, one will adapt the approach introduced in Reynaud et al. (2018) to the case where decoys may be present.

Considering the control input $\mathbf{u}_{\ell,k+1}$, the predicted UAV states $\mathbf{x}^{\mathrm{u,P}}_{\ell,k+1}$ is evaluated from $\mathbf{x}^{\mathrm{u}}_{\ell,k}$ using (1) for all $\ell \in \mathcal{N}^{\mathrm{P}}_{i,k} \cup \{i\}$. The predicted value $\overline{\mathbb{X}}^{\mathrm{P}}_{\ell,k+1|k}$ of the set $\overline{\mathbb{X}}_{\ell,k+1|k}$ is simply evaluated using (11) starting from $\overline{\mathbb{X}}_{\ell,k}$ and is the same for all UAVs $\ell \in \mathcal{N}^{\mathrm{P}}_{i,k} \cup \{i\}$. Then $\overline{\mathbb{X}}_{i,k+1}$ can be easily predicted combining (14) and (20) to get

$$\overline{\mathbb{X}}^{\mathrm{P}}_{i,k+1} = \overline{\mathbb{X}}^{\mathrm{P}}_{i,k+1|k} \setminus \bigcup_{\ell \in \mathcal{N}^{\mathrm{P}}_{i,k} \cup \{i\}} \mathbb{F}_\ell\left(\mathbf{x}^{\mathrm{u,P}}_{\ell,k+1}\right). \tag{22}$$

As in Section 3.1, one is unable to determine whether new targets will be detected. Thus, the predicted value $\mathcal{L}^{\mathrm{P}}_{\ell,k+1|k}$ of $\mathcal{L}_{\ell,k+1|k}$, $\ell \in \mathcal{N}^{\mathrm{P}}_{i,k} \cup \{i\}$, is evaluated as in (9) by

$$\mathcal{L}^{\mathrm{P}}_{\ell,k+1|k} = \mathcal{L}_{i,k}. \tag{23}$$

Predicted values $\mathbb{X}^{\mathrm{P}}_{\ell,j,k+1|k}$ of $\mathbb{X}_{\ell,j,k+1|k}$, for all $j \in \mathcal{L}^{\mathrm{P}}_{\ell,k+1|k}$ are also evaluated from $\mathbb{X}_{\ell,j,k}$ using (10). The predicted values $\mathbb{X}^{\mathrm{P}}_{\ell,j,k+1|k+1}$ of $\mathbb{X}_{\ell,j,k+1|k+1}$ are much more complex to determine from $\mathbb{X}^{\mathrm{P}}_{\ell,j,k+1|k}$, since they will depend on the state of the UAVs and the actual state of Target j. Nevertheless, one may consider two cases. First, if $\mathbb{X}^{\mathrm{P}}_{\ell,j,k+1|k} \cap \mathbb{F}_\ell(\mathbf{x}^{\mathrm{u,P}}_{\ell,k+1}) = \emptyset$, then

$$\mathbb{X}^{\mathrm{P}}_{\ell,j,k+1} = \mathbb{X}^{\mathrm{P}}_{\ell,j,k+1|k}. \tag{24}$$

Here, one neglects the growth of $\mathbb{X}^{\mathrm{P}}_{\ell,j,k+1|k+1}$ compared to $\mathbb{X}^{\mathrm{P}}_{\ell,j,k+1|k}$ which may result from the observation of the true target or a decoy of index j in $\overline{\mathbb{X}}^{\mathrm{P}}_{i,k+1|k}$, see Fig. 2b. Second, if $\mathbb{X}^{\mathrm{P}}_{\ell,j,k+1|k} \cap \mathbb{F}_\ell(\mathbf{x}^{\mathrm{u,P}}_{\ell,k+1}) \neq \emptyset$, then either $\mathbf{x}^{\mathrm{t}}_{j,k+1} \notin \mathbb{F}_\ell(\mathbf{x}^{\mathrm{u,P}}_{\ell,k+1})$ and $\mathbb{X}^{\mathrm{P}}_{\ell,j,k+1} = \mathbb{X}^{\mathrm{P}}_{\ell,j,k+1|k} \setminus \mathbb{F}_\ell(\mathbf{x}^{\mathrm{u,P}}_{\ell,k+1})$ (see Fig. 2c) or $\mathbf{x}^{\mathrm{t}}_{j,k+1} \in \mathbb{F}_\ell(\mathbf{x}^{\mathrm{u,P}}_{\ell,k+1})$ and

$$\mathbb{X}^{\mathrm{P}}_{\ell,j,k+1|k+1} = \mathbb{S}^{\mathrm{P}}_1 \cup \mathbb{S}^{\mathrm{P}}_3 \tag{25}$$

where

$$\mathbb{S}^{\mathrm{P}}_1 = \left\{\mathbf{x} \in \mathbb{X}^{\mathrm{P}}{}_{\ell,j,k+1|k} \,|\, \mathbf{h}_{k+1}(\mathbf{x}^{\mathrm{u,P}}_{\ell,k+1}, \mathbf{x}) \in \mathbf{y}_{\ell,j,k+1} - [\mathbf{w}]\right\}$$

and

$$\mathbb{S}^{\mathrm{P}}_3 = \mathbb{X}^{\mathrm{P}}{}_{\ell,j,k+1|k} \setminus \mathbb{F}_\ell(\mathbf{x}^{\mathrm{u,P}}_{\ell,k+1}).$$

Here, again one neglects the effect on $\mathbb{X}^{\mathrm{P}}_{\ell,j,k+1|k+1}$ of the observation of the true target or of a decoy of index j in $\overline{\mathbb{X}}^{\mathrm{P}}_{i,k+1|k}$, associated to the set $\mathbb{S}_2$ in (13), see Fig. 2b. The main difficulty with (25) is that $\mathbf{y}_{\ell,j,k+1}$ is not available at time k. All possible values of $\mathbf{y}_{\ell,j,k+1}$ consistent with the assumptions $\mathbf{x}^{\mathrm{t}}_{j,k+1} \in \mathbb{F}_\ell(\mathbf{x}^{\mathrm{u,P}}_{\ell,k+1})$ and the measurement noise bounds $[\mathbf{w}]$ should be considered to evaluate the possible resulting sets $\mathbb{S}^{\mathrm{P}}_1$. Nevertheless, since in general $\Phi\left(\mathbb{S}^{\mathrm{P}}_1\right)$ is small compared to $\Phi\left(\mathbb{X}^{\mathrm{P}}{}_{\ell,j,k+1|k}\right)$, one assumes that (25) boils down to

$$\mathbb{X}^{\mathrm{P}}_{\ell,j,k+1|k+1} = \mathbb{X}^{\mathrm{P}}_{\ell,j,k+1|k} \setminus \mathbb{F}_\ell(\mathbf{x}^{\mathrm{u,P}}_{\ell,k+1}). \tag{26}$$

This expression is consistent with (24) in the case $\mathbb{X}^{\mathrm{P}}_{\ell,j,k+1|k} \cap \mathbb{F}_\ell(\mathbf{x}^{\mathrm{u,P}}_{\ell,k+1}) = \emptyset$.

This is clearly an approximation, since $\mathbb{X}^{\mathrm{P}}_{\ell,j,k+|k+1}$ becomes empty when $\mathbb{X}^{\mathrm{P}}_{\ell,j,k+1|k} \subset \mathbb{F}_\ell(\mathbf{x}^{\mathrm{u,P}}_{\ell,k+1})$. Nevertheless, in a model predictive control-oriented perspective, when such case occurs or even if measurements help to significantly reduce the size of $\mathbb{X}^{\mathrm{P}}_{i,j,k+1|k}$, the contribution of the set $\mathbb{X}^{\mathrm{P}}_{\ell,j,k+|k+1}$ in $\Phi\left(\mathcal{X}_{i,k+1}, \overline{\mathbb{X}}_{i,k+1}\right)$ would become negligible.

Taking into account the information provided by the communications with its neighbours, $\mathbb{X}^{\mathrm{P}}_{i,j,k+1}$ is obtained by combining all estimates $\mathbb{X}^{\mathrm{P}}_{\ell,j,k+|k+1}$ evaluated by the UAVs $\ell \in \mathcal{N}^{\mathrm{P}}_{i,k}$ with the estimate $\mathbb{X}^{\mathrm{P}}_{i,j,k+1|k+1}$ obtained by UAV i. Thus one gets

$$\mathbb{X}^{\mathrm{P}}_{i,j,k+1} = \mathbb{X}^{\mathrm{P}}_{i,j,k+1|k} \setminus \bigcup_{\ell \in \mathcal{N}^{\mathrm{P}}_{i,k} \cup \{i\}} \mathbb{F}_\ell(\mathbf{x}^{\mathrm{u,P}}_{\ell,k+1}). \tag{27}$$

Finally, determining whether new true targets or decoys will be detected is not possible. Consequently, one assumes that

$$\mathcal{L}^{\mathrm{P}}_{i,k+1} = \mathcal{L}^{\mathrm{P}}_{i,k+1|k}. \tag{28}$$

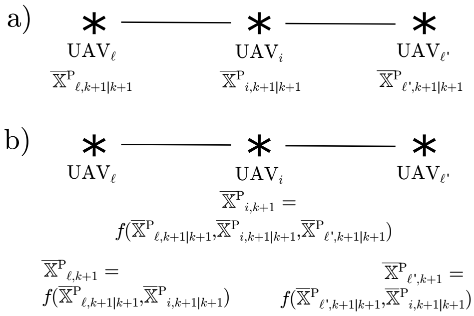

Fig. 4. Available estimates evaluated in the MPC approach; UAVs able to communicate directly are linked; a) before communication, estimates $\overline{\mathbb{X}}^{\mathrm{P}}_{\ell,k+1|k+1}$ of $\overline{\mathbb{X}}_{\ell,k+1|k+1}$, and b) after communication and processing, estimates $\overline{\mathbb{X}}^{\mathrm{P}}_{\ell,k+1}$ of $\overline{\mathbb{X}}_{\ell,k+1}$.

4.2 *h step-ahead prediction*

Assume that $\mathcal{L}^{\mathrm{P}}_{i,k+\kappa}$, $\mathcal{X}^{\mathrm{P}}_{i,k+\kappa}$, and $\overline{\mathbb{X}}^{\mathrm{P}}_{i,k+\kappa}$ have been evaluated by UAV i for some $1 \leqslant \kappa < h$. For that purpose, UAV i has evaluated $\mathbf{x}^{\mathrm{u,P}}_{\ell,k+\kappa}$ for all $\ell \in \mathcal{N}^{\mathrm{P}}_{i,k} \cup \{i\}$. The sets $\mathcal{L}^{\mathrm{P}}_{\ell,k+\kappa}$, $\mathcal{X}^{\mathrm{P}}_{\ell,k+\kappa}$, and $\overline{\mathbb{X}}^{\mathrm{P}}_{\ell,k+\kappa}$, for $\ell \in \mathcal{N}^{\mathrm{P}}_{i,k}$ are not necessarily equal to the corresponding sets evaluated by UAV i due to communication constraints. Fig. 4 illustrates this situation, where one observes that the sets used to evaluate $\overline{\mathbb{X}}^{\mathrm{P}}_{\ell,k+1}$ and $\overline{\mathbb{X}}^{\mathrm{P}}_{\ell',k+1}$ by UAVs ℓ and ℓ' are not the same. Consequently, in general, $\overline{\mathbb{X}}^{\mathrm{P}}_{\ell,k+1} \neq \overline{\mathbb{X}}^{\mathrm{P}}_{\ell',k+1}$. Nevertheless, to simplify the evaluation in the considered MPC context, one assumes that the predicted sets at time $k+\kappa$ satisfy $\mathcal{L}^{\mathrm{P}}_{\ell,k+\kappa} = \mathcal{L}^{\mathrm{P}}_{i,k+\kappa}$, $\mathcal{X}^{\mathrm{P}}_{\ell,k+\kappa} = \mathcal{X}^{\mathrm{P}}_{i,k+\kappa}$, and $\overline{\mathbb{X}}^{\mathrm{P}}_{\ell,k+\kappa} = \overline{\mathbb{X}}^{\mathrm{P}}_{i,k+\kappa}$ for all $\ell \in \mathcal{N}^{\mathrm{P}}_{i,k+\kappa} = \mathcal{N}^{\mathrm{P}}_{i,k}$.

Our aim is to determine the impact of $\{\mathbf{u}_{\ell,k+\kappa+1}\}_{\ell \in \mathcal{N}^{\mathrm{P}}_{i,k}}$ and of $\mathbf{u}_{i,k+\kappa+1}$ on the evaluation of $\mathcal{L}^{\mathrm{P}}_{i,k+\kappa+1}$, $\mathcal{X}^{\mathrm{P}}_{i,k+\kappa+1}$, and $\overline{\mathbb{X}}^{\mathrm{P}}_{i,k+\kappa+1}$ from the predicted sets at time $k+\kappa$. One is then able to evaluate recursively $\mathcal{L}^{\mathrm{P}}_{i,k+h}$, $\mathcal{X}^{\mathrm{P}}_{i,k+h}$, and $\overline{\mathbb{X}}^{\mathrm{P}}_{i,k+h}$.

First, the predicted list of detected targets is obtained extending (23) and (28) to get

$$\mathcal{L}^{\mathrm{P}}_{\ell,k+\kappa+1} = \mathcal{L}^{\mathrm{P}}_{\ell,k+\kappa+1|k+\kappa} = \mathcal{L}^{\mathrm{P}}_{\ell,k+\kappa}.$$

Considering the control inputs $\mathbf{u}_{\ell,k+\kappa+1}$, the predicted UAV states $\mathbf{x}^{\mathrm{u,P}}_{\ell,k+\kappa+1}$ is evaluated from $\mathbf{x}^{\mathrm{u,P}}_{\ell,k+\kappa}$ using (1) for all $\ell \in \mathcal{N}^{\mathrm{P}}_{i,k} \cup \{i\}$. Using the same assumptions as in Section 4.1, the predicted value $\overline{\mathbb{X}}^{\mathrm{P}}_{\ell,k+\kappa+1|k}$ of the set $\overline{\mathbb{X}}_{\ell,k+\kappa+1|k}$ is again evaluated using (11) starting from $\overline{\mathbb{X}}^{\mathrm{P}}_{\ell,k+\kappa}$ and is the same for all UAVs $\ell \in \mathcal{N}^{\mathrm{P}}_{i,k} \cup \{i\}$. Then, extending (22), one gets

$$\overline{\mathbb{X}}^{\mathrm{P}}_{i,k+\kappa+1} = \overline{\mathbb{X}}^{\mathrm{P}}_{i,k+\kappa+1|k+\kappa} \setminus \bigcup_{\ell \in \mathcal{N}^{\mathrm{P}}_{i,k} \cup \{i\}} \mathbb{F}_\ell\left(\mathbf{x}^{\mathrm{u,P}}_{\ell,k+\kappa+1}\right).$$

Similarly, the predicted value $\mathbb{X}^{\mathrm{P}}_{\ell,j,k+\kappa+1|k+\kappa}$ of $\mathbb{X}_{\ell,j,k+\kappa+1|k+\kappa}$, for all $j \in \mathcal{L}^{\mathrm{P}}_{\ell,k+\kappa+1|k+\kappa}$ are again evaluated from $\mathbb{X}^{\mathrm{P}}_{\ell,j,k+\kappa+1}$ using (11) and, extending (27), one gets after the communications are processed

$$\mathbb{X}^{\mathrm{P}}_{i,j,k+\kappa+1} = \mathbb{X}^{\mathrm{P}}_{i,j,k+\kappa+1|k+\kappa} \setminus \bigcup_{\ell \in \mathcal{N}^{\mathrm{P}}_{i,k} \cup \{i\}} \mathbb{F}_\ell \left(\mathbf{x}^{\mathrm{u,P}}_{\ell,k+\kappa+1} \right).$$

4.3 Practical issues

In practice, the order in which the UAVs compute their control input at each time step k has to be optimized. Assume that UAV i has access to $\mathcal{N}_{i,k}$ from previous communication. A suboptimal distributed approach can then be for UAV i to compute its control inputs only once it has received the predicted control inputs from all UAVs in $\mathcal{N}_{i,k}$ with a smaller index.

To further simplify the search, one may assume in the MPC approach that, at time k, only the control input at time $k+1$ is evaluated and that the control inputs at the next time instants remain the constant.

5. SIMULATIONS

One considers the search for $N_{\mathrm{t}} = 4$ true ground targets by $N_{\mathrm{u}} = 4$ UAVs. The target motion is modelled by

$$\begin{pmatrix} x^{\mathrm{t}}_{j,k+1,1} \\ x^{\mathrm{t}}_{j,k+1,2} \\ x^{\mathrm{t}}_{j,k+1,3} \\ x^{\mathrm{t}}_{j,k+1,4} \end{pmatrix} = \begin{pmatrix} x^{\mathrm{t}}_{j,k,1} + T x^{\mathrm{t}}_{j,k,4} \cos(v_{j,k}) \\ x^{\mathrm{t}}_{j,k,2} + T x^{\mathrm{t}}_{j,k,4} \sin(v_{j,k}) \\ x^{\mathrm{t}}_{j,k,3} \\ v_{j,k} \end{pmatrix}, \qquad (29)$$

where $(x^{\mathrm{t}}_{j,k,1}, x^{\mathrm{t}}_{j,k,2})$ represents the target coordinates in a given reference frame. The norm of speed $x^{\mathrm{t}}_{j,k,3} = 1\,\mathrm{m/s}$ is assumed constant, and $x^{\mathrm{t}}_{j,k,4}$ is the target heading angle. At each time step, $v_{j,k}$ is uniformly chosen at random in the interval $[-\pi/5, \pi/5]$.

The UAVs dynamics is modelled by

$$\begin{pmatrix} x^{\mathrm{u}}_{j,k+1,1} \\ x^{\mathrm{u}}_{j,k+1,2} \\ x^{\mathrm{u}}_{j,k+1,3} \\ x^{\mathrm{u}}_{j,k+1,4} \\ x^{\mathrm{u}}_{j,k+1,5} \\ x^{\mathrm{u}}_{j,k+1,6} \end{pmatrix} = \begin{pmatrix} x^{\mathrm{u}}_{j,k,1} + T x^{\mathrm{u}}_{j,k,4} \cos((x_{j,k,5})) \cos(u_{i,k}) \\ x^{\mathrm{u}}_{j,k,2} + T x^{\mathrm{u}}_{j,k,4} \cos((x_{j,k,5})) \sin(u_{i,k}) \\ x^{\mathrm{u}}_{j,k,3} \\ x^{\mathrm{u}}_{j,k,4} \\ x^{\mathrm{u}}_{j,k,5} \\ u_{i,k} \end{pmatrix},$$

which corresponds to a fly at a constant altitude $x^{\mathrm{u}}_{j,k,3}$ of 100 m with zero slope above the terrain at a constant speed $x^{\mathrm{u}}_{j,k,4}$ of $20\,\mathrm{ms}^{-1}$. We consider that the UAVs control input consists only in the heading angle.

The communication graph is assumed fully connected at each time instant. No delay is considered and the communication period is $T = 1\,\mathrm{s}$. Their prediction horizon for the MPC is $h = 4$. They are equipped with an optical sensor able to detect targets within its FoV, see (4). Its opening angles are equal to $\pi/4$ in both azimuth and elevation. When a target is detected at time k, one assumes that the measurement equation (6) delivers a noise corrupted measurement of its actual location $\mathbf{y}_{i,j,k}$ with an uncertainty $\mathbf{w}_{i,j,k}$ bounded in $[-5\,\mathrm{m}, 5\,\mathrm{m}]$ for both components $x^{\mathrm{t}}_{j,k,1}$ and $x^{\mathrm{t}}_{j,k,2}$. The only available knowledge is that the measurement noise belongs to this bound.

$N_{\mathrm{d}} = 4$ static decoys are also considered with $\mathbf{x}^{\mathrm{d}}_{\ell,0} \in \mathbb{R}^5$, where $(x^{\mathrm{d}}_{\ell,1}, x^{\mathrm{d}}_{\ell,2})$ is chosen uniformly at random in the search area, $x^{\mathrm{d}}_{\ell,3} = 0$, $x^{\mathrm{d}}_{\ell,4} = 0$, and $x^{\mathrm{d}}_{\ell,5} = 0$. The decoys

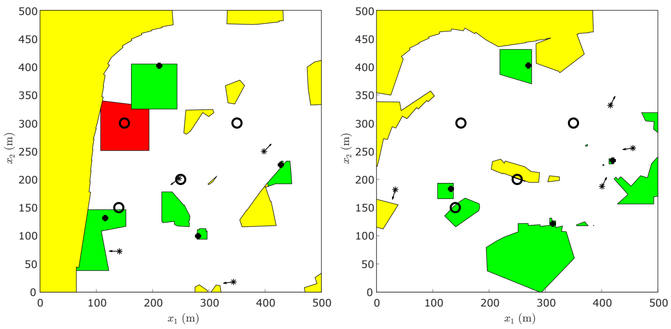

Fig. 5. 2D projection of $\overline{\mathbb{X}}_{i,k}$ (yellow), $\mathbb{X}_{i,j,k}$ (green, true targets), and $\mathbb{X}_{i,l,k}$ (red, decoys) for UAV $i = 1$ at $k = 57$ (left) and at $k = 143$ (right); the actual target positions are indicated by dots (true target) and circles (decoys); the position and heading angle of the UAVs is plotted

are confused with true targets only when they are observed from a location $(x^{\mathrm{u}}_{j,k,1}, x^{\mathrm{u}}_{j,k,2}, x^{\mathrm{u}}_{j,k,3})$ satisfying (5), where

$$g_{i,\ell}\left(\mathbf{x}^{\mathrm{u}}_{i,k}, \mathbf{x}^{\mathrm{d}}_{\ell}\right) = \left(\left(\mathbf{x}^{\mathrm{u}}_{i,k} - \mathbf{x}^{\mathrm{d}}_{\ell}\right) \cdot \mathbf{a}_\ell\right)^2 - \left(\mathbf{x}^{\mathrm{u}}_{i,k} - \mathbf{x}^{\mathrm{d}}_{\ell}\right)^2 \mathbf{a}^2_\ell \left(\cos\alpha_\ell\right)^2.$$

represents a half circular cone of $\mathbb{R}^3$ with the aperture $2\alpha_\ell$. The cone axis is parallel to $\mathbf{a}_\ell$ and its vertex is $\mathbf{x}^{\mathrm{d}}_\ell$. The aperture $2\alpha_\ell$ is uniformly distributed within $[\pi/7, \pi/5]$. The same applies to the orientation $\mathbf{a}_\ell$ of the cone for the azimuth β_ℓ bounded in $[0, 2\pi]$ and the elevation angle γ_ℓ bounded in $[\pi/7, \pi/5]$ with

$$\mathbf{a}_\ell = \left(\sin\gamma_\ell \cdot \cos\beta_\ell, \sin\gamma_\ell \cdot \sin\beta_\ell, \cos\gamma_\ell, 0, 0, 0\right)^{\mathrm{T}}.$$

The search area is a square of $500 \times 500\ \mathrm{m}^2$.

The simulations have been carried out in Matlab. For representing sets, the Matlab built-in function *polyshape* has been used. This function simplifies the handling of sets in $\mathbb{R}^2$ regarding Boolean and geometrical operations.

Fig. 5 presents the simulation results. Fig. 5 (left) shows the 2D projections of the set estimates for true targets and decoys at $k = 57$. Two true targets (green set estimates) and two decoys (red set estimates) have been detected. The set still to be explored is in yellow. The set estimates evolve due to the target motion. Since the UAVs are not able to distinguish between true targets and decoys, both sets are growing. Fig. 5 (right) presents the set estimates at $k = 143$ after some sub-areas have been monitored several times. One sees that three decoys have been uncovered and their set estimates are empty. Measurements taken from different point of views have helped to identify decoys and previous estimates got corrected.

Fig. 6 represents the evolution of the 2D projection of the sets $\overline{\mathbb{X}}_{i,k}$, $\mathbb{X}_{i,j,k}$, and $\mathbb{X}_{i,l,k}$ for UAV $i = 1$ during the elimination of a set estimate corresponding to a decoy. One sees that the red set progressively disappears and becomes empty at time $k = 127$[1].

6. CONCLUSIONS

This paper addresses the problem of cooperative target localization and tracking in presence of decoys using a fleet of UAV. A set-membership approach is introduced. The

[1] A video of the presented sequence is available at `https://drive.google.com/file/d/1F6Yi1IgD4Jl--bIL4xNHbfxNp3NNijaQ`

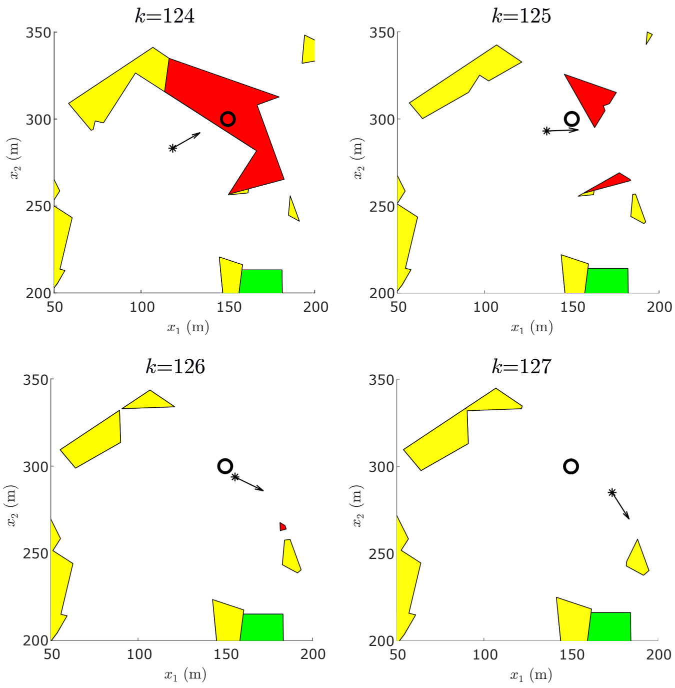

Fig. 6. Zoom on the evolution of the 2D projection of $\overline{\mathbb{X}}_{i,k}$, $\mathbb{X}_{i,j,k}$, and $\mathbb{X}_{i,l,k}$ for UAV $i = 1$ during the elimination of a set estimate corresponding to a decoy.

originality of the proposed approach is to deal with decoys, assumed to be static objects located in the monitored geographical area, which can be confused with a true target by an UAV, when present in the field of view and under specific observation conditions.

To address this problem, three different sets are introduced for each UAV: the explored set to which we know that the target does not belong, the explored set to which the target *may* belong, and the unexplored set to which the target *may* belong. Thanks to the information available by each (coming from its own sensors, or by the information shared by its UAVs neighbourhood), these three sets evolve so that, at each time step, each UAV can have a knowledge as precise as possible of the whole situation of the tracked targets.

The control input of each UAV is evaluated via a model predictive control approach. A simulation illustrates the proposed method when several UAVs cooperate to track some targets, in the presence of decoys. Further extensions of this paper include developing strategies of displacement of an UAV to see the potential targets under different point of view in order to determine whether it is a true target or a decoy. Other extensions would be to deal with moving decoys, obstacle obscuring objects or probability of non-detection for the true targets.

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