

Sensorless Control of Permanent Magnet Synchronous Motors based on Finite-Time Robust Flux Observer [★]

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Abstract: A sensorless control algorithm is developed based on novel finite-time robust flux observer for the non-salient permanent magnet synchronous motor (PMSM). Total flux equality and motor model are used to find the linear regression-like model with respect to the flux. Applying dynamic regressor extension and mixing method, we obtain two independent scalar equations and construct a finite-time flux observer. The flux estimate is used to reconstruct the rotor position and velocity with well-known trigonometric relation and phase-locked loop observer, respectively. To complete a sensorless control design, we pass these estimates to standard field orient control. The efficiency and robustness of the proposed approach are demonstrated through the set of numerical simulations.

Keywords: Sensorless, nonlinear observer, flux observer, nonlinear control systems, synchronous motor

1. INTRODUCTION

Field orientation control (FOC) method for permanent magnet synchronous motors (PMSMs) is widely used because it provides the most optimal torque production and consequently high-performance control response. The control objective is defined via reference values of direct and quadratic axis currents in the frame rotating synchronously with the rotor flux. This change of the coordinates requires precise rotor position.

The position can be measured using a sensor or estimated. In some applications (cranes, elevators, vacuum pumps), the very installation of a shaft sensor is either inconvenient or not possible at all. In cost—and reliability—sensitive heating, ventilation and air conditioning systems, the use of a capable CPU with an improved observer design technique may boost the performance. Another benefit of

sensorless control is the mechanical robustness due to the absence of cables, connectors and peripheral modules.

There are two classes of the sensorless control methods: passive without probing signal and active, where a high-frequency signal is injected to monitor the rotor saliency change along with rotor motion.

The popular passive method in practice is based on estimating the back-electromotive force (EMF), see Lee and Ha (2012), where different algorithms were considered. Value of EMF is decreasing with speed, and on low speed, the results are not reliable, where signal injection methods have better performance, Li et al. (2007). However, active methods can not be applied on high speed, where maximum voltage is utilized, and can produce the torque oscillation.

Other passive methods are based on nonlinear observers and usually are much more complicated. However, recent progress in CPU and power semiconductor performance enables the implementation of such observers and related controllers in many industrial systems. Low power wind turbines frequently use synchronous permanent magnet machines as generators. For cost and reliability reasons,

[★] The work was written with the support of the Ministry of Science and Higher Education of the Russian Federation, project unique identifier RFMEFI57818X0271 “Adaptive Sensorless Control for Synchronous Electric Drives in Intelligent Robotics and Transport Systems”.

they are often designed with no shaft sensors Bolognani et al. (2005). In such cases, a reliable, high precision robust observer is required to control steering and to extract energy at low wind conditions.

In this paper, a nonlinear sensorless controller derived from the observer introduced by Pyrkin et al. (2019) is extended and tested. A key feature of the observer is independence from speed, as it requires only knowledge of the stator resistance and inductance while mechanical parameters and the magnetic flux constant are not needed. The last parameter is necessary, for instance, in Ortega et al. (2011) and Genduso et al. (2010). In this regard, the nonlinear observer from Pyrkin et al. (2019) as well as proposed extensions is robust towards magnetic flux generated by permanent magnets. Robustness of the nonlinear observer towards different gain settings and initial conditions is also shown in Pyrkin et al. (2019). In contrast to Bobtsov et al. (2015), there is not open-loop integration of current and voltage signals. It makes the controller robust to a constant bias in these signals. The proposed approach uses the stator voltage, which is not measured directly but estimated, as is done in most modern industrial drives.

This paper is organized as follows. The problem statement is described in Section 2. The main result is presented in Section 3, where flux robust nonlinear observers are constructed. In Section 4 we mention position, speed observer, and control scheme, which can be used with the proposed position observer to implement sensorless control. The computer simulation results of the proposed algorithm are described in Section 5 confirming the efficiency and robustness of the approach.

2. PROBLEM FORMULATION

The classical, two-phase $\alpha\beta$ model of the unsaturated, *non-salient* PMSM described by Krause (1986); Nam (2010), is considered. In the stationary $\alpha\beta$ frame, the following holds for surface-mounted PMSM (SPMSM)

$$\begin{aligned}\dot{\lambda}_{\alpha\beta} &= v_{\alpha\beta} - Ri_{\alpha\beta}, \\ J\dot{\omega} &= -B\omega + \tau_e - \tau_L, \\ \dot{\theta} &= \omega,\end{aligned}\quad (1)$$

where $\lambda_{\alpha\beta} \in \mathbb{R}^2$ is the total flux, $i_{\alpha\beta} \in \mathbb{R}^2$ are the currents, $v_{\alpha\beta} \in \mathbb{R}^2$ are the voltages, $R > 0$ is the stator windings resistance, $J > 0$ is the rotor inertia, $\theta \in \mathbb{S} := [0, 2\pi)$ is the rotor phase, $\omega \in \mathbb{R}$ is the mechanical angular velocity, $B \geq 0$ is the viscous friction coefficient, $\tau_L \in \mathbb{R}$ is the—possibly time-varying—load torque, τ_e is the torque of electrical origin.

For surface-mounted PMSM's the total flux satisfies

$$\lambda_{\alpha\beta} = Li_{\alpha\beta} + \lambda_m \begin{bmatrix} \cos(n_p\theta) \\ \sin(n_p\theta) \end{bmatrix}, \quad (2)$$

where $L > 0$ is the stator inductance and $n_p \in \mathbb{N}$ is the number of pole pairs.

The most often problem is reconstruct the rotor position $\theta(t)$ using measurements of the current $i_{\alpha\beta} =: i$ and voltage $v_{\alpha\beta} =: v$, and the most popular way, based on (2), is to use the following algorithm

$$\hat{\theta} = \frac{1}{n_p} \arctan \left\{ \frac{\lambda_\alpha - Li_\alpha}{\lambda_\beta - Li_\beta} \right\}, \quad (3)$$

which requires the knowledge of the magnetic flux $\lambda_{\alpha\beta}$.

The goal of this work is design the sensorless control for SPMSM. The solution is based on nonlinear observer for system (1) and FOC.

The robust finite-time observer of the magnetic flux $\lambda := \lambda_{\alpha\beta}$ using measurements of the current i and voltage v should provide estimate, such that for fixed $t_1 > 0$ the following equality holds:

$$\hat{\lambda}^{FTO}(t) = \lambda(t), \quad \forall t \geq t_1. \quad (4)$$

Then, the rotor position estimate $\hat{\theta}$ is calculated using (3) and velocity estimate is reconstructed with PLL-type observer Nam (2010).

As usual in parameter estimation and observation problems, the following forward completeness and boundedness assumptions are imposed.

Assumption 1. The control signal $v_{\alpha\beta}$ and the unknown external load torque τ_L are such that the trajectories of the PMSM model (1) exist for all $t > 0$ and are bounded.

3. FLUX OBSERVER

In this section we show a finite-time flux observer. Firstly, the flux observer synthesis is presented. That approach use DREM algorithm that provide a better transient in contrast to standard gradient approach. After that using the finite-time observer we improve the convergence rate. As positions and velocities observers are proposed to be used as described in Bobtsov et al. (2015) and Nam (2010), we give their description in the form of statements without evidence.

Using the squaring operation and the basic trigonometric identity under (2) one can get

$$\lambda^\top \lambda - 2L\lambda^\top i + L^2 i^\top i - \lambda_m^2 = 0. \quad (5)$$

Apply a filter $F(p) = \frac{\alpha p}{p + \alpha}$, $p := \frac{d}{dt}$, with some $\alpha > 0$.

Replacing the flux derivative with the first equation of system (1) yields

$$\begin{aligned}2 \frac{\alpha}{p + \alpha} [\lambda^\top (v - Ri - Lpi)] + \frac{\alpha}{p + \alpha} [L^2 pi^\top i - 2Lv^\top i] \\ + 2RL \frac{\alpha}{p + \alpha} [i^\top i] = 0.\end{aligned}\quad (6)$$

Notice that the unknown flux is under filtering operation in the first term of (6). In order to extract it we use the Swapping Lemma (Sastry and Bodson, 2011, Lemma 3.6.5):

$$\begin{aligned}\lambda^\top \frac{\alpha}{p + \alpha} [2v - 2Ri - 2Lpi] \\ - \frac{1}{p + \alpha} \left[(v - Ri)^\top \frac{\alpha}{p + \alpha} [2v - 2Ri - 2Lpi] \right] \\ + \frac{\alpha}{p + \alpha} [L^2 pi^\top i - 2Lv^\top i] \\ + 2RL \frac{\alpha}{p + \alpha} [i^\top i] = 0,\end{aligned}\quad (7)$$

where we have replaced $\dot{\lambda}^\top = (v - Ri)^\top$ as before.

The last equation can be rewritten in a linear regression form

$$z(\alpha) = g(\alpha)^\top \lambda, \quad (8)$$

where z and q are known,

$$z(\alpha) := -\frac{1}{p+\alpha} \left[(v - Ri)^\top \frac{\alpha}{p+\alpha} [2v - 2Ri - 2Lpi] \right] + \frac{\alpha}{p+\alpha} [L^2 pi^\top i - 2Lv^\top i] + 2RL \frac{\alpha}{p+\alpha} [i^\top i] = 0,$$

$$g(\alpha) := \frac{\alpha}{p+\alpha} [2v - 2Ri] - \frac{\alpha p}{p+\alpha} [2Li].$$

Next, following the DREM algorithm Aranovskiy et al. (2017) combing two regressor equations (8) with different parameters $\alpha_1 > 0$ and $\alpha_2 > 0$ we form the extended regression model

$$Y = Q\lambda, \quad (9)$$

where

$$Y := \begin{pmatrix} z(\alpha_1) \\ z(\alpha_2) \end{pmatrix}, \quad (10)$$

$$Q := \begin{pmatrix} g(\alpha_1)^\top \\ g(\alpha_2)^\top \end{pmatrix}. \quad (11)$$

Taking in account that the flux is a time-varying signal we get

$$\dot{\hat{\lambda}}_\alpha = v_\alpha - Ri_\alpha + \gamma_\alpha \Delta (\xi_\alpha - \Delta \hat{\lambda}_\alpha), \quad (12)$$

$$\dot{\hat{\lambda}}_\beta = v_\beta - Ri_\beta + \gamma_\beta \Delta (\xi_\beta - \Delta \hat{\lambda}_\beta), \quad (13)$$

where Δ is the determinant of Q :

$$\Delta = \det\{Q\}, \quad (14)$$

$\xi_{\alpha,\beta} = \text{adj}\{Q\}Y$ is computed using adjoint matrix of Q and $\gamma_{\alpha,\beta} > 0$, are design parameters.

Proposition 1. Consider the PMSM model described by (1) verifying Assumption 1. Let the observer defined as

$$\hat{\lambda}_{\alpha\beta}^{FTO}(t) = \frac{\hat{\lambda}(t) - \hat{\lambda}(0)w_1(t) - w_2(t)}{1 - w_1(t)}, \quad (15)$$

where $\hat{\lambda}(t) = [\hat{\lambda}_\alpha(t); \hat{\lambda}_\beta(t)]$, with $\hat{\lambda}_\alpha(t)$ and $\hat{\lambda}_\beta(t)$ are given by (12)–(13),

$$\dot{w}_1(t) = -\gamma \Delta^2(t) w_1(t),$$

$$\dot{w}_2(t) = -\gamma \Delta^2(t) w_2(t) + w_1(t) H(i_{\alpha\beta}, u_{\alpha\beta}),$$

$\Delta^2(t)$ is defined by (14), H is given by (17), $w_1(0) = 1$, $w_2(0) = 0$.

If $\int_0^{t_1} \Delta^2(s) ds > 0$ for some $t_1 > 0$, then $\hat{\lambda}^{FTO}(t) = \lambda(t)$, $\forall t \geq t_1$.

Proof. Combining equations (12) and (13) we get

$$\dot{\hat{\lambda}}(t) = H(i_{\alpha\beta}(t), u_{\alpha\beta}(t)) + \gamma \Delta(t) (\xi - \Delta(t) \hat{\lambda}), \quad (16)$$

where

$$H(i_{\alpha\beta}(t), u_{\alpha\beta}(t)) = v_{\alpha\beta} - Ri_{\alpha\beta}. \quad (17)$$

Next, we get the well-known error equation for (16)

$$\dot{\tilde{\lambda}}(t) = -\gamma \Delta^2(t) \tilde{\lambda}(0), \quad (18)$$

where

$$\tilde{\lambda}(t) = \lambda(t) - \hat{\lambda}(t) = e^{-\gamma \int_0^t \Delta^2(s) ds} (\lambda(0) - \hat{\lambda}(0)). \quad (19)$$

Notice that

$$\lambda(0) = \lambda(t) - \int_0^t H(i_{\alpha\beta}(s), u_{\alpha\beta}(s)) ds. \quad (20)$$

Introduce the auxiliary signal $w_1(t)$

$$\dot{w}_1(t) = -\gamma \Delta^2(t) w_1(t), \quad w_1(0) = 1. \quad (21)$$

Hence,

$$w_1(t) = e^{-\gamma \int_0^t \Delta^2(s) ds}, \quad (22)$$

and we have

$$\lambda(t) - \hat{\lambda}(t) = w_1(t) \left(\lambda(t) - \hat{\lambda}(0) - \int_0^t H(i_{\alpha\beta}(s), u_{\alpha\beta}(s)) ds \right). \quad (23)$$

Define the second auxiliary signal

$$w_2(t) = w_1(t) \int_0^t H(i_{\alpha\beta}(s), u_{\alpha\beta}(s)) ds, \quad (24)$$

and found what $w_2(t)$ is the output of the system

$$\dot{w}_2(t) = -\gamma \Delta^2 w_2(t) + w_1(t) H(i_{\alpha\beta}(t), u_{\alpha\beta}(t)), \quad (25)$$

$$w_2(0) = 0.$$

Flux λ can be explicitly found from (23) for $t \geq t_1$, when $\int_0^{t_1} \Delta^2(s) ds > 0$ and $w_1(t) < 1$:

$$\lambda_{\alpha\beta}(t) = \frac{\hat{\lambda}(t) - \hat{\lambda}(0)w_1(t) - w_2(t)}{1 - w_1(t)}, \quad (26)$$

which is the desired conclusion.

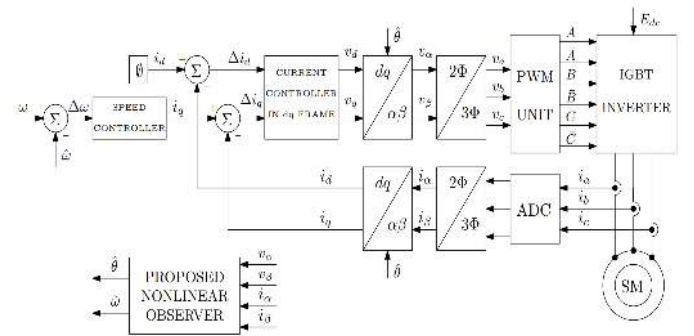


Fig. 1. The implementation of the sensorless control of a permanent magnet synchronous motor with the proposed finite-time observer

4. SENSORLESS FIELD-ORIENT CONTROL

To construct sensorless field-orient control we also need estimates of the position and angular velocity of the rotor.

Using (3) and (15) we estimate the unmeasured angle as described in Bobtsov et al. (2015) also in finite-time:

$$\hat{\theta}^{FTO} = \frac{1}{n_p} \arctan \left\{ \frac{\hat{\lambda}_{\alpha}^{FTO} - Li_{\alpha}}{\hat{\lambda}_{\beta}^{FTO} - Li_{\beta}} \right\}. \quad (27)$$

The velocity is estimated with the help of a standard PLL-type speed estimator, see Nam (2010):

$$\dot{\chi}_1 = K_p (\hat{\theta}^{FTO} - \chi_1) + K_i \chi_2,$$

$$\dot{\chi}_2 = \hat{\theta}^{FTO} - \chi_1, \quad (28)$$

$$\hat{\omega} = K_p (\hat{\theta}^{FTO} - \chi_1) + K_i \chi_2,$$

where $K_p > 0$ and $K_i > 0$ are proportional and integral gains, respectively.

We use the observer outputs to control the PMSM, the angle $\hat{\theta}^{FTO}$ for transformation of the Park and the velocity $\hat{\omega}$ for sensorless field-orient control. The full structure of the sensorless control of PMSM with the proposed nonlinear position observer is shown in Fig. 1.

5. SIMULATION RESULTS

The series of simulations have been performed in MATLAB Simulink. The model of the drive uses parameters of the BMP0701F, which are presented in Table 1. In all simulation the motor is controlled by the proposed sensorless controller: the forward and inverse Park transformations are calculated using the position estimate, the speed controller in FOC uses the output of speed observer.

The initial conditions are equal to zero, $\gamma_\alpha = \gamma_\beta = 0.02$, $\alpha_1 = 50$, $\alpha_2 = 400$. Integral and proportional PI coefficients for field-oriented control and speed observer are presented in Table 1.

Table 1. Parameters of Simulation

PMSM motor	
Inductance $L(mH)$	40.03
Resistance $R(\Omega)$	8.875
Drive inertia $j(kg \cdot m^2)$	$59 \cdot 10^{-6}$
Pairs of poles $n_p(-)$	5
Magnetic flux $\lambda_m(Wb)$	0.2086
Velocity Observer	
K_p	175
K_i	50
Sensorless Field-orient Control	
$K_p(\rightarrow \omega)$	1
$K_i(\rightarrow \omega)$	0.25
$K_p(\rightarrow i_d)$	1
$K_i(\rightarrow i_d)$	0.1
$K_p(\rightarrow i_q)$	1
$K_i(\rightarrow i_q)$	0.1
Third case with incorrect parameters	
$L_{observer}(mH)$	60
$R_{observer}(\Omega)$	5.32

The reference signal for the angular velocity is a step signal of the following form:

$$\omega_*(t) = \begin{cases} 20 & 0 < t \leq 0.2, \\ 30 & 0.2 < t \leq 0.4, \\ 40 & 0.4 < t \leq 0.6, \\ 50 & 0.6 < t \leq 0.8, \\ 60 & 0.8 < t \leq 1. \end{cases}$$

The external load torque is depicted in Fig. 3.

In the simulations, the noise and parameter uncertainty effects are considered, which gives us three cases:

- (1) without noise, with correct parameters,
- (2) with noise, with correct parameters,
- (3) with noise, with incorrect parameters.

In the second and third cases, the measurements are corrupted by the additive noise. It is simulated as a uniformly distributed process ranging within $[-2.5, 2.5]$ V for voltages, and $[-0.2, 0.2]$ A for current. The example of the noised measurements is depicted in Fig. 2. In the third case, incorrect model parameter values of the stator resistance and inductance are used in the observer, see Table 1.

In all cases, the variables with subscript “1” relates to sensorless control without using the finite-time observers. Variables of FTO modification have subscript “2”.

The norm of flux estimation errors, velocity and position estimates, external load torque are depicted in Fig. 4–6. The performance of the flux observer with and without finite-time modification is high and in the noised cases is similar. The incorrect parameters do not affect the observer and controller.

6. CONCLUSION

In this paper we describe the sensorless controller for the SPMSM. Using novel parametrization we obtain equation in linear regression form. Applying dynamic regressor extension and mixing approach Aranovskiy et al. (2017) gives two scalar equations for α, β flux components, which are used to construct auxiliary flux observer. Outputs of this observer are using to obtain finite-time flux estimate.

Using flux estimate the position were reconstructed and used in PPL-type speed observer Bobtsov et al. (2015); Nam (2010) to estimate rotor angular velocity.

Finally, the sensorless version of the field-orient controller has been obtained using position estimate $\hat{\theta}^{FTO}$ and velocity $\hat{\omega}$. Simulation results illustrating the high performance and robustness of the proposed approach.

The proposed method does not require signal injection for non-zero velocities and works in the wide speed range, including low speed.

7. ACKNOWLEDGEMENTS

The work was written with the support of the Ministry of Science and Higher Education of the Russian Federation, project unique identifier RFMEFI57818X0271 “Adaptive Sensorless Control for Synchronous Electric Drives in Intelligent Robotics and Transport Systems”.

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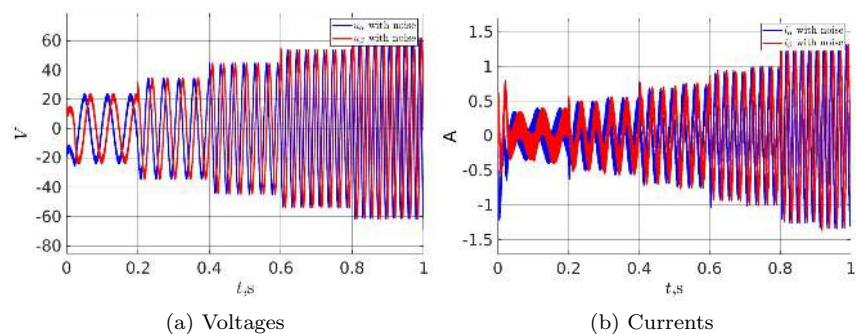


Fig. 2. The noised measurements of voltage and current

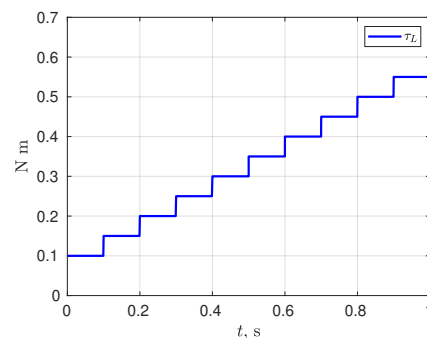


Fig. 3. The external load torque

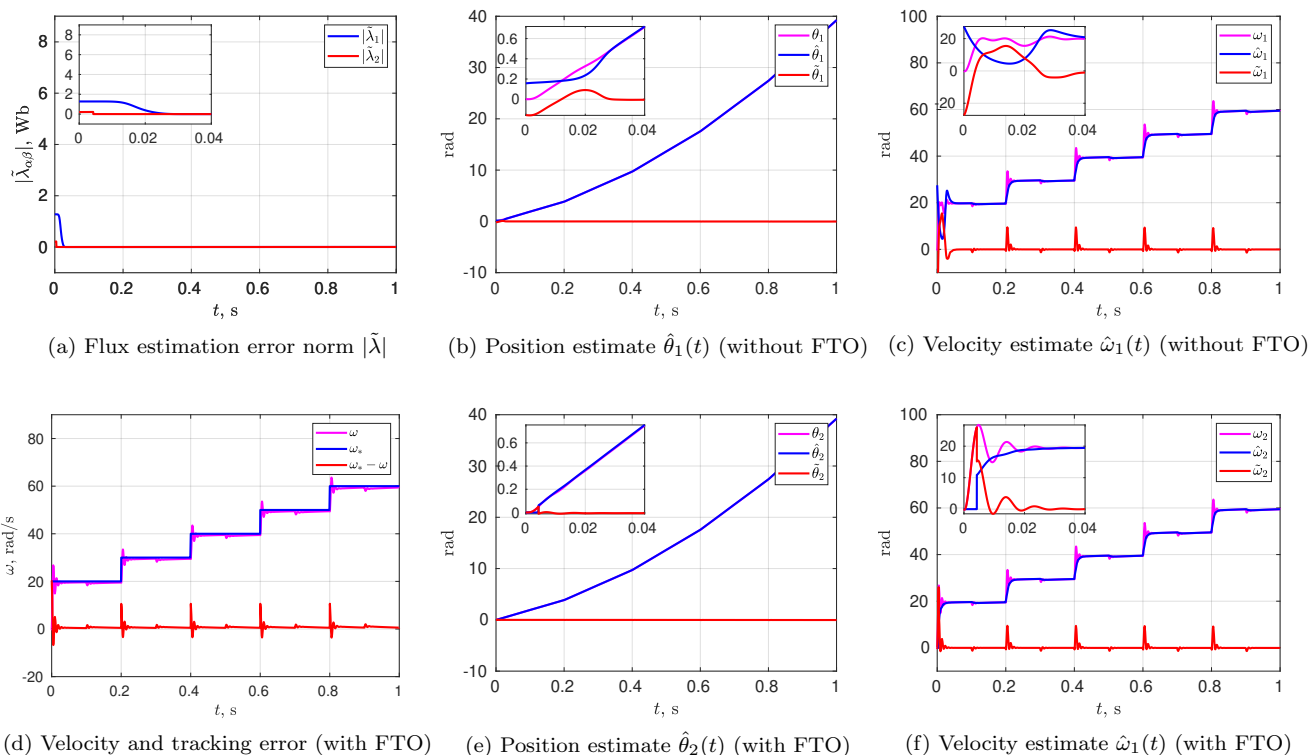


Fig. 4. Transitions for the noise-free case

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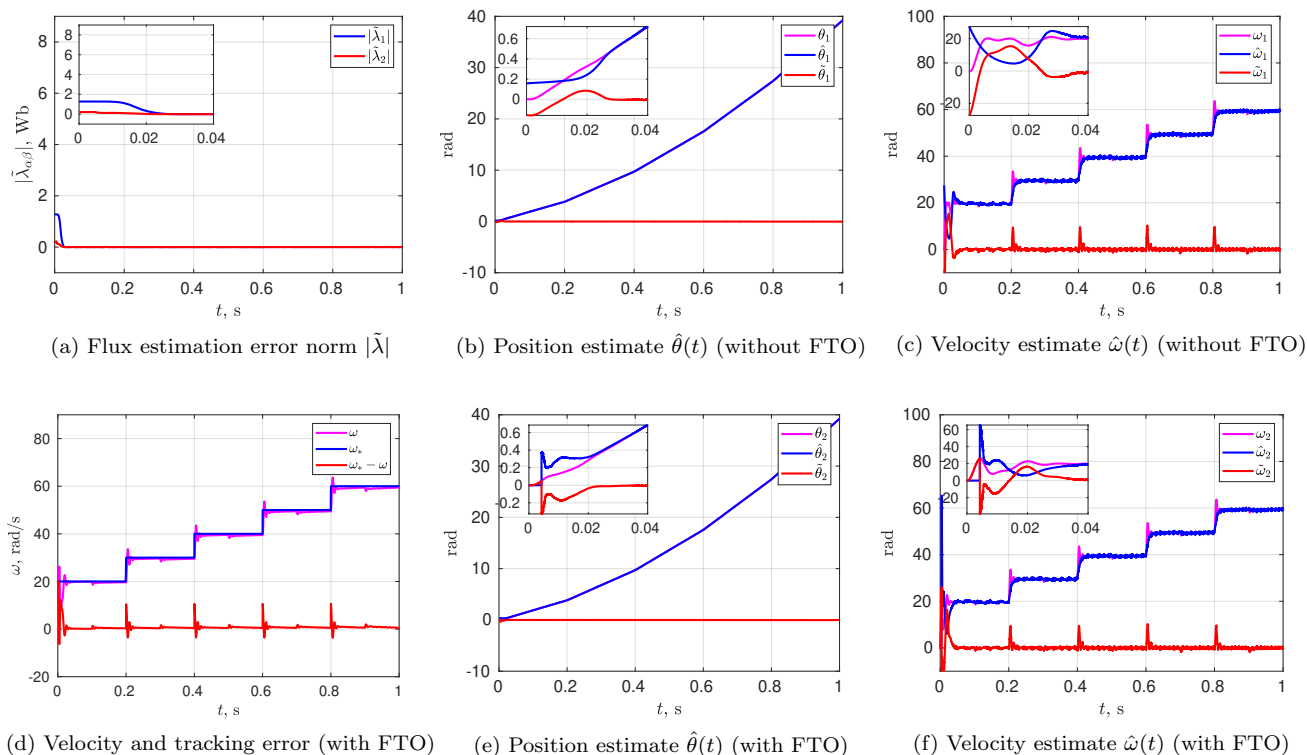


Fig. 5. Transitions for the noise case

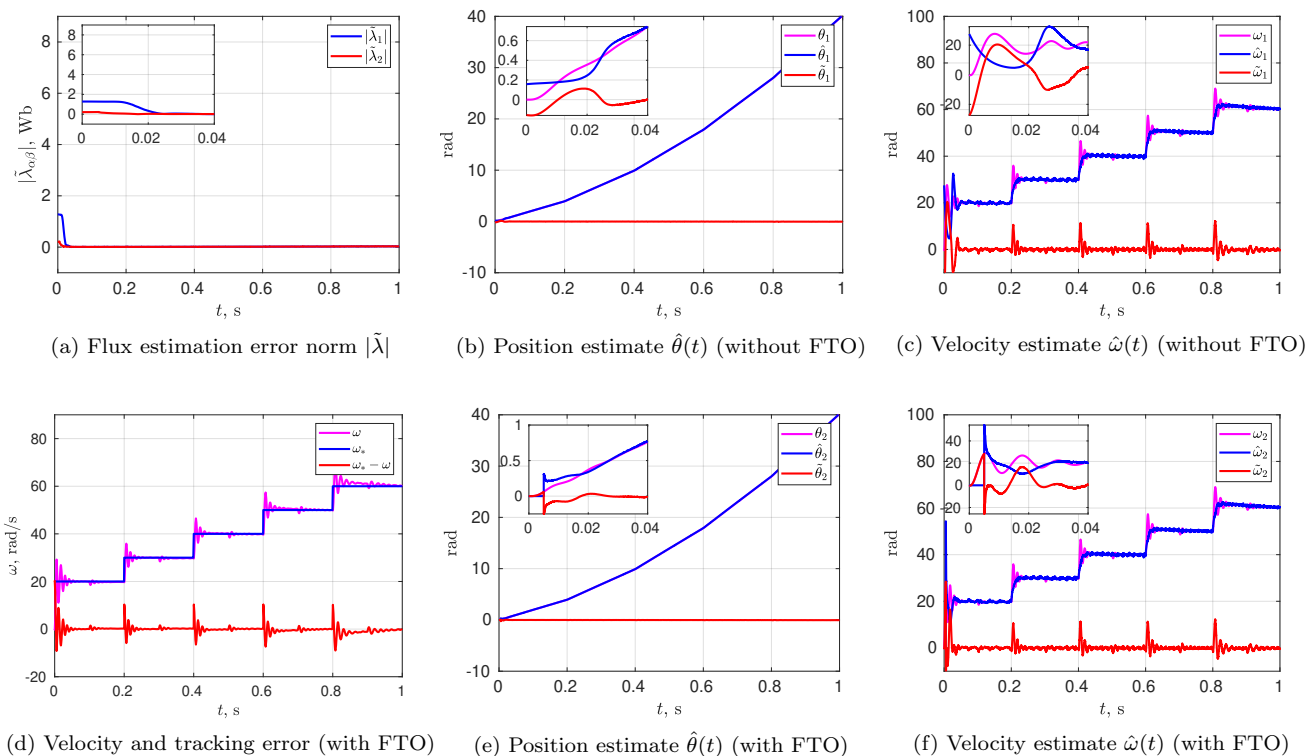


Fig. 6. Transitions for the noise case with incorrect parameters