

Modeling and discrepancy based control of underactuated large gantry cranes

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Abstract: An important structural dynamics problem of the large gantry cranes are horizontal elastic oscillations mainly excited by the trolley motion. They reduce the crane operation performance and lead to faster material wear of the crane construction. In this article a distributed parameter model of large gantry cranes applying Hamilton's principle is presented. In order to stabilize the system dynamics the use of a generalized error measure, called discrepancy, is proposed. Applying the associated stability theory, i.e. stability with respect to two discrepancies, a nonlinear stabilizing control for the underactuated gantry crane is derived. The proposed control strategy has been verified by simulations.

Keywords: Distributed parameter system; discrepancy based control; gantry crane; payload oscillations; position control; structural dynamics.

1. INTRODUCTION

Currently, the control of underactuated crane systems is an active field of research. In Abdel-rahman et al. (2003) and Ramli et al. (2017) authors overviewed a variety of models and control techniques for different types of cranes. Most of the contributions focus on the damping of load swaying due to the crane positioning applying different feedback and feedforward control approaches, e.g. energy-based control Sun and Fang (2012), Sun et al. (2018), Won and Hoang (2018), sliding mode control Zhou et al. (2017), Xiao et al. (2018), Wang et al. (2018), Lu et al. (2017) and input shaping techniques Abdullahi et al. (2018), Ramli et al. (2018). In the majority of contributions the structural system dynamics has been neglected. However, continuous increase of crane dimensions and utilizing lightweight profiles led to limited stiffness of the structure. Thus, this assumption is not valid for large cranes and a coupling between elastic structural vibrations and the trolley movements has to be taken into account.

In the last years, the structural dynamics problem has been stated for different types of cranes (Zrnić et al. (2006); Rauscher and Sawodny (2017); Schlott et al. (2016); Kimmerle et al. (2018); Miloradovic and Vujanac (2016); Sowa et al. (2018)) including large gantry cranes (Golovin and Palis (2019); Gašić et al. (2013); Yazid et al. (2011); Kreuzer et al. (2012); Ryu and Kong (2012)). Here, two main structural dynamical problems can be stated: vertical girder vibrations due to the trolley travel and load hoisting, and horizontal low frequency oscillations in the trolley motion direction. In this contribution, the focus is on the horizontal oscillations as they are particularly unfavorable due to the large amplitudes and their weakly damped behaviour. They provide additional mechanical stresses leading to faster construction wear and decrease

crane operation performance. In addition, applying feedback control, neglecting the structural dynamics, may result in the excitation of resonance frequencies and even unstable closed loop dynamics.

In the literature different approaches to solve the structural dynamics problem for large gantry cranes have been proposed, e.g. structure stiffening by construction optimization Zrnić et al. (2005), passive and active dampers via additional weight as counter-mass Recktenwald (2011) and linear robust active damping approach as an extension for the trolley motion control system Golovin and Palis (2019). The later has been verified on a laboratory gantry crane.

This contribution is concerned with the modeling and nonlinear control of underactuated large gantry cranes with limited stiffness. In order to derive the distributed parameter crane model Hamilton's principle has been utilized. The main control objectives for underactuated large gantry cranes are payload positioning without swaying and a simultaneous reduction of structural oscillations in the trolley travel direction. Here, in order to stabilize the system dynamics the use of a generalized error measure, called discrepancy, is proposed (Palis and Kienle (2012, 2014)). Applying the associated Lyapunov stability theory a nonlinear stabilizing control has been derived. The resulting control law has been verified in a simulation study, where the distributed dynamics have been discretized using finite differences.

Section 2 presents the derivation of the coupled model of the gantry crane including its structural dynamics. The concept of stability with respect to two discrepancies as well as the corresponding discrepancy based control design are introduced in section 3. Section 4 concludes the article with results from a simulation study.

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2. GANTRY CRANE MODELING

In this contribution, a nonlinear model-based control approach for the gantry crane is proposed. It requires the establishment of the corresponding dynamic plant model. Essentially, an operational cycle of the gantry crane consists of the hoisting of the load, its horizontal movement and deposition. From a control point of view, the horizontal movement phase is the most challenging tasks. The payload should be delivered as fast as possible to the desired position without swaying. However, for large cranes such movements may excite the natural frequencies of the crane structure. These important effects should be reflected in the dynamic model. For convenience, the following assumptions for model derivations are made:

- (1) as the scope of the contribution is on the horizontal vibrations the considered frame structure of the crane consists of the upper horizontal beam which is rigid in flexure and two supported columns that have limited lateral stiffness;
- (2) the structure is assumed to be symmetrical such that the problem can be reduced and only one half of the structure has to be taken into account;
- (3) the mass density and the bending stiffness of the crane columns are distributed along the spatial coordinate, and they are assumed to be constant along the spatial coordinate;
- (4) rotary inertia, shear deformation and buckling effects can be neglected;
- (5) the hoisting process is neglected, such that the rope length can be assumed to be constant;
- (6) the trolley and the payload are connected by a massless rigid rope and the elongation of the rope is neglected;
- (7) the moment of inertia of the load can be neglected;
- (8) nonlinear friction effects can be neglected;
- (9) external disturbances on the crane and load can be neglected, e.g. wind;
- (10) the trolley is actuated via a current controlled DC-motor with gear, therefore the force $F_t(t)$ depends linearly on the motor torque $\tau(t)$, i.e. $F_t(t) = k_{tr}\tau(t)$ where k_{tr} is the transformation coefficient.

In Fig. 1 the schematic diagram of the motion of the gantry crane is depicted. Here, m_t is the mass of the crane trolley, m_p is the mass of the payload, m_c is the mass of the crane girder, F_t is the external force being applied to the trolley, $\varphi(t)$ is the sway angle, $z(t)$ is the trolley displacement, l is the rope length, $w(x, t)$ is the displacement of the crane structure in horizontal direction depending on both position x and time t , L is the length of the crane legs, ρ_c is the mass density, E is Young's modulus and I is the moment of inertia of a cross-sectional area.

In order to derive a suitable gantry crane model Hamilton's principle based on the kinetic energy $T(t)$, potential energy $U(t)$ and virtual work done by non-conservative forces $W(t)$ can be written as follows

$$\int_{t_1}^{t_2} (\delta T - \delta U + \delta W) dt = 0, \quad (1)$$

where δ represents the variational operator, t_1 and t_2 are initial and final moments in time.

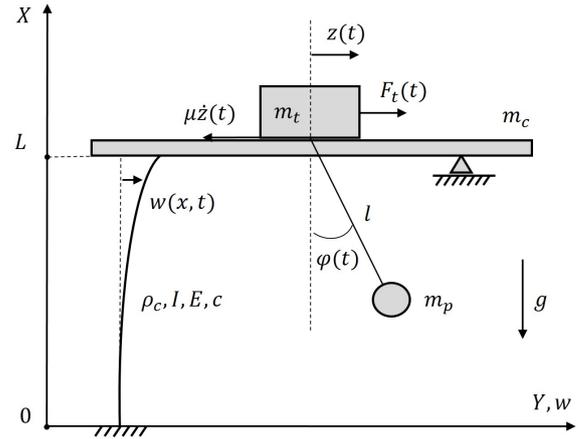


Fig. 1. Gantry crane

The vector of generalized coordinates is chosen as follows

$$q = [w(x, t) \ z(t) \ \varphi(t)]. \quad (2)$$

Then the position vectors of girder r_c , trolley r_t and payload r_p can be written as follows

$$r_c = \begin{bmatrix} w(L, t) \\ 0 \end{bmatrix}, \quad (3)$$

$$r_t = \begin{bmatrix} w(L, t) + z(t) \\ 0 \end{bmatrix}, \quad (4)$$

$$r_p = \begin{bmatrix} w(L, t) + z(t) + l \sin \varphi \\ -l \cos \varphi \end{bmatrix}. \quad (5)$$

The corresponding kinetic energy of the gantry crane can be represented as follows

$$T = \frac{1}{2} \int_0^L \rho_c \dot{w}^2 dx + \frac{1}{2} m_c \dot{w}^2(L, t) + \frac{1}{2} m_t (\dot{w}(L, t) + \dot{z})^2 + \frac{1}{2} m_p [(\dot{w}(L, t) + \dot{z} + \dot{\varphi} l \cos \varphi)^2 + (\dot{\varphi} l \sin \varphi)^2], \quad (6)$$

and the potential energy can be formulated as follows

$$U = \frac{1}{2} \int_0^L EI (w'')^2 dx - m_p g l \cos \varphi, \quad (7)$$

where the *dot* symbol denotes the derivative along the time and the *prime* symbol along the spatial coordinate.

According to Fig. 1 the work done by non-conservative forces, namely actuating force, trolley viscous friction force and structural linear damping force, can be written as

$$\delta W = (F_t - \mu \dot{z}) \delta z - \int_0^L c \dot{w} \delta w dx, \quad (8)$$

where μ is the viscous friction coefficient and c is the linear structural damping.

Substituting energies (6), (7) and (8) into Hamilton's principle (1), taking corresponding variations and applying integration by parts with respect to t and x , the following equation is obtained:

$$\begin{aligned}
0 = & - \int_{t_1}^{t_2} \int_0^L [\rho_c \ddot{w} + EI w'''' + c \dot{w}] \delta w \, dx \, dt \\
& - \int_{t_1}^{t_2} [EI w''''(0, t)] \delta w(0, t) \, dt \\
& + \int_{t_1}^{t_2} [EI w''(0, t)] \delta w'(0, t) \, dt \\
& - \int_{t_1}^{t_2} [EI w''(L, t)] \delta w'(L, t) \, dt \\
& - \int_{t_1}^{t_2} [m_\Sigma \ddot{w}(L, t) + m_s \ddot{z} + m_p l \ddot{\varphi} \cos \varphi \\
& - m_p l \dot{\varphi}^2 \sin \varphi - EI w''''(L, t)] \delta w(L, t) \, dt \\
& - \int_{t_1}^{t_2} [m_s \ddot{w}(L, t) + m_s \ddot{z} + m_p l \ddot{\varphi} \cos \varphi \\
& - m_p l \dot{\varphi}^2 \sin \varphi - F_t + \mu \dot{z}] \delta z \, dt \\
& - m_p l \int_{t_1}^{t_2} [l \ddot{\varphi} + \ddot{w}(L, t) \cos \varphi + \ddot{z} \cos \varphi \\
& + g \sin \varphi] \delta \varphi \, dt, \tag{9}
\end{aligned}$$

where $m_\Sigma = m_p + m_t + m_c$ and $m_s = m_p + m_t$.

Here, as variations are arbitrary, eq. (9) holds only if the integrands vanish. Thus, taking into account the geometrical boundary conditions $w(0, t) = w_x(0, t) = 0$ the equations of motion follow:

$$0 = \rho_c \ddot{w} + EI w'''' + c \dot{w}, \tag{10}$$

$$0 = w(0, t) = w'(0, t) = w''(L, t), \tag{11}$$

$$\begin{aligned}
0 = & m_\Sigma \ddot{w}(L, t) + m_s \ddot{z} + m_p l \ddot{\varphi} \cos \varphi \\
& - m_p l \dot{\varphi}^2 \sin \varphi - EI w''''(L, t), \tag{12}
\end{aligned}$$

$$\begin{aligned}
0 = & m_s \ddot{w}(L, t) + m_s \ddot{z} + m_p l \ddot{\varphi} \cos \varphi \\
& - m_p l \dot{\varphi}^2 \sin \varphi - F_t + \mu \dot{z}, \tag{13}
\end{aligned}$$

$$0 = l \ddot{\varphi} + \ddot{w}(L, t) \cos \varphi + \ddot{z} \cos \varphi + g \sin \varphi. \tag{14}$$

3. CONTROL DESIGN

This section focuses on stability with respect to two discrepancies and the associated control design. The derived equations of motion consist of a partial differential equation (PDE) (10) with corresponding boundary conditions (b.c.) (11) representing the structural dynamics of the crane and a system of nonlinear ordinary differential equations (ODE) (12-14), which act on the boundary of the PDE and represent the coupled motion of the girder, trolley and payload.

In this contribution it is assumed, that all system states related to the trolley motion, payload motion and crane oscillations at the boundary $x = L$ can be measured. The actuation is represented by a DC motor operating in current control mode, where the desired current is proportional to the control applied torque $\tau(t)$.

3.1 Stability with respect to two discrepancies

According to the works (Movchan (1960); Sirazetdinov (1967, 1987)) the most important properties and definitions about stability with respect to two discrepancies are presented in the following. Here, the process $\phi(\cdot, t)$ is a solution of a distributed parameter system and $\phi_0 = 0$ is an equilibrium of the system.

Definition 1. Discrepancy

A discrepancy is a real valued functional $\rho = \rho[\phi(\cdot, t), t]$ with the following properties

- $\rho(\phi, t) \geq 0$
- $\rho(0, t) = 0$
- for an arbitrary process $\phi(\cdot, t)$ the real valued functional $\rho[\phi(\cdot, t), t]$ is continuous with respect to t
- presenting the second discrepancy $\rho_0(\phi)$ with $\rho_0(\phi) \geq 0$ and $\rho_0(0) = 0$. Then the discrepancy $\rho[\phi(\cdot, t), t]$ is continuous at time $t = t_0$ with respect to ρ_0 at $\rho_0 = 0$, if for every $\epsilon > 0$ and $t_0 > 0$ there exists a $\beta(\epsilon, t_0) > 0$, such that from $\rho_0 \leq \beta(\epsilon, t_0)$ follows $\rho < \epsilon$

From this definition one can state that a discrepancy does not satisfy all properties of a metric, e.g. symmetry $d(x, y) = d(y, x)$ or triangular inequality $d(x, y) = d(y, z) + d(z, y)$. And more importantly, it has not to satisfy the property of definiteness, i.e. a vanishing discrepancy $\rho(\phi, t) = 0$ does not automatically mean $\phi = 0$. Thus, the discrepancy is an extension of the distance measures normally used in stability theory of DPS like L_p and L_∞ -norms.

Definition 2. Stability with respect to two discrepancies ρ and ρ_0

The equilibrium $\phi_0 = 0$ is stable in terms of Lyapunov with respect to the two discrepancies ρ and ρ_0 for all $t \geq t_0$ if for every $\epsilon > 0$ and $t_0 \geq 0$ there exists a $\beta = \beta(\epsilon, t_0)$ such that for every process $\phi(\cdot, t)$ with $\rho_0 < \beta(\epsilon, t_0)$ it follows that $\rho < \epsilon$ for all $t \geq t_0$. If in addition $\lim_{t \rightarrow \infty} \rho = 0$, then the equilibrium ϕ_0 is called asymptotically stable in terms of Lyapunov with respect to the two discrepancies ρ and ρ_0 .

A lot of nonlinear control approaches are based on the Lyapunov stability theory. In order to define a relationship between the existence of a Lyapunov functional V and stability with respect to two discrepancies the notions of positivity and positive definiteness of a functional with respect to a discrepancy should be presented.

Definition 3. Positivity with respect to a discrepancy ρ

The functional $V = V[\phi, t]$ is called positive with respect to the discrepancy ρ , if $V \geq 0$ and $V[0, t] = 0$ for all ϕ with $\rho(\phi, t) < \infty$.

Definition 4. Positive definiteness with respect to a discrepancy ρ

The functional $V = V[\phi, t]$ is positive definite with respect to the discrepancy ρ , if $V \geq 0$ and $V[0, t] = 0$ for all ϕ with $\rho(\phi, t) < \infty$, and for every $\epsilon > 0$ exists a $\beta = \beta(\epsilon) > 0$, such that $V \geq \beta(\epsilon)$ for all ϕ with $\rho[\phi, t] \geq \epsilon$.

The next two theorems state the conditions for a function V to guarantee (asymptotic) stability with respect to two discrepancies (Sirazetdinov (1987)).

Theorem 1. The process ϕ with equilibrium $\phi_0 = 0$ is stable with respect to the two discrepancies ρ and ρ_0 if and only if there exists a functional $V = V[\phi, t]$ positive definite with respect to the discrepancy ρ , continuous at time $t = t_0$ with respect to ρ_0 at $\rho_0 = 0$ and not increasing along the process ϕ , i.e. $\dot{V} \leq 0$.

Theorem 2. The process ϕ with equilibrium $\phi_0 = 0$ is asymptotically stable with respect to the two discrepancies ρ and ρ_0 if and only if there exists a functional $V =$

$V[\phi, t]$ positive definite with respect to the discrepancy ρ , continuous at time $t = t_0$ with respect to ρ_0 at $\rho_0 = 0$ and not increasing along the process ϕ , i.e. $\dot{V} \leq 0$, with $\lim_{t \rightarrow \infty} V = 0$.

In addition, it should be mentioned that stability with respect to two discrepancies is necessary but in general not sufficient for stability with respect to a L_p norm or L_∞ norm.

3.2 Discrepancy based control

The objective of the control system is to move the trolley according to the desired position reference signal $z_{ref}(t)$ and a simultaneous reduction of the payload swaying and crane structural oscillations. One of the options to derive a nonlinear control law for a damping strategy could be to take the overall mechanical energy $E_o = T + U$ as a candidate Lyapunov functional. However, calculating its time derivative along the system trajectory results in:

$$\dot{E}_o = \dot{z}F_t. \quad (15)$$

Here, choosing $F_t = -k\dot{z}$ as the input and \dot{z} as the output yields in the well-known energy dissipation. However, due to the underactuated nature of the system, the control law contains neither terms related to the crane structural motion nor the payload. One way to overcome this problem is to couple eq. (15) with a term depending on the payload velocity and to find reversely an appropriately shaped energy functional (Sun and Fang (2012)).

In this contribution another approach based on the discrepancy is proposed. The generalized system error can be chosen as follows

$$e = k_1\dot{w}(L, t) + k_2\dot{z} + k_3\dot{\phi}l \cos \varphi + k_4\varepsilon, \quad (16)$$

where $\varepsilon(t) = z(t) - z_{ref}(t)$ is the deviation from the reference position signal and k_1 to k_4 are the corresponding weights.

Here, the generalized error e is established in such way, that it includes the position error ε , velocity of the trolley \dot{z} and for increasing the system coupling the corresponding velocities of the girder $\dot{w}(L, t)$ and horizontal velocity of the payload $\dot{\phi}l \cos \varphi$. In order to shape the error the additional weights are introduced. Using the given generalized error results in the following discrepancy ρ

$$\rho = \frac{1}{2} (k_1\dot{w}(L, t) + k_2\dot{z} + k_3\dot{\phi}l \cos \varphi + k_4\varepsilon)^2. \quad (17)$$

The second discrepancy ρ_0 is selected to be equal to ρ at time $t = t_0 = 0$:

$$\rho_0 = \rho(t = 0) = 0. \quad (18)$$

As stated in *Theorem 2* existence of a suitable functional V is sufficient to guarantee asymptotic stability with respect to the two discrepancies ρ and ρ_0 . For this aim the corresponding Lyapunov functional can be easily represented as follows

$$V = \frac{1}{2} (k_1\dot{w}(L, t) + k_2\dot{z} + k_3\dot{\phi}l \cos \varphi + k_4\varepsilon)^2. \quad (19)$$

According to stability in terms of two discrepancies the control input should be chosen such that the time derivative \dot{V} is negative definite along the state trajectories and vanishes only for $V = 0$. Calculating the time derivative yields

$$\begin{aligned} \dot{V} = e\dot{e} = e[k_1\ddot{w}(L, t) + k_2\ddot{z} + k_3\ddot{\phi}l \cos \varphi \\ - k_3\dot{\phi}^2l \sin \varphi + k_4\dot{\varepsilon}]. \end{aligned} \quad (20)$$

Substituting eq. (13) in (20) yields in

$$\begin{aligned} \dot{V} = e[(k_1 - k_2)\ddot{w}(L, t) + b_1l\ddot{\phi} \cos \varphi \\ - b_1l\dot{\phi}^2 \sin \varphi + k_4\dot{\varepsilon} - \frac{k_2\mu}{m_s}\dot{z} + \frac{k_2}{m_s}F_t], \end{aligned} \quad (21)$$

where

$$b_1 = \frac{k_3m_s - k_2m_p}{m_s}.$$

In order to achieve the required negative definiteness of \dot{V} the control law is chosen as follows

$$\begin{aligned} \tau = \frac{m_s}{k_2k_{tr}}[-(k_1 - k_2)\ddot{w}(L, t) - b_1l\ddot{\phi} \cos \varphi \\ + b_1l\dot{\phi}^2 \sin \varphi - k_4\dot{\varepsilon} + \frac{k_2\mu}{m_s}\dot{z} - ae], \end{aligned} \quad (22)$$

where $a > 0$ is a design parameter influencing the control performance.

The proposed control law guarantees not only stability, in the aforementioned sense, but also exponential convergence of V

$$\dot{V} = -ae^2 = -2aV. \quad (23)$$

The overall control scheme is depicted in Fig. 2.

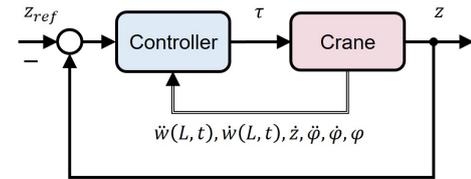


Fig. 2. Control scheme

4. RESULTS

The simulation model, including the proposed control law, has been implemented in MATLAB. For the solution of the PDE (10) the method of lines have been applied. Here, the spatial coordinate is lumped applying the finite difference method (via central difference scheme) with $N = 50$ points. For the solution of the resulting system of ODEs the *ode15s* solver has been used. All simulation parameters are summarized in Table 1.

For contrast, the results of the proposed control law can be compared with additional classical cascade scheme for drive position control where no additional damping purposes are provided. This simple control law can be written as follows

$$\tau = [k_{p,pos}(z_{ref} - z) - \dot{z}](k_{p,vel} + k_{i,vel}\frac{1}{s}), \quad (24)$$

where $k_{p,pos}$ is position controller gain, $k_{p,vel}$ and $k_{i,vel}$ are parameters of the velocity PI controller that can be adjusted according to the modulus optimum.

In order to verify both control laws (22) and (24) time responses of the positioning reference tracking have been studied. The results are depicted in Fig. 3, 4, 5 and 6. In Fig. 3 the convergence of the Lyapunov functional V is shown. From Fig. 4, it can be seen, that applying the classical cascade motion control (24) without additional

system information results in oscillatory closed loop system dynamics with large amplitudes of payload and crane swinging. Applying the designed discrepancy based control (22) yields a good damping of payload and structural motion. As can be seen from Fig. 5 and 6 not only the motion of the girder point $w(L, t)$ but also the distributed state $w(x, t)$ itself and its L_2 -norm are stabilized. This is noteworthy, as this has not been part of the design. However, it can be shown that stability in the sense of Lyapunov with respect to two discrepancies results in stability with respect to the L_p - or the L_∞ -norm if the zero dynamics associated with the discrepancy ρ is stable in the sense of the L_p - or L_∞ -norm.

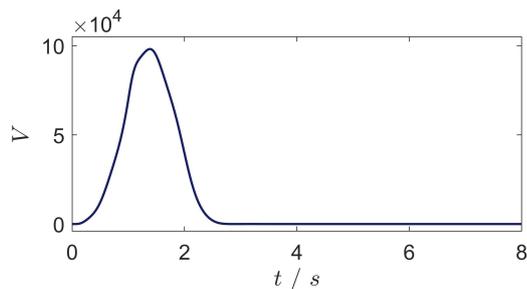


Fig. 3. Time response of the Lyapunov functional V

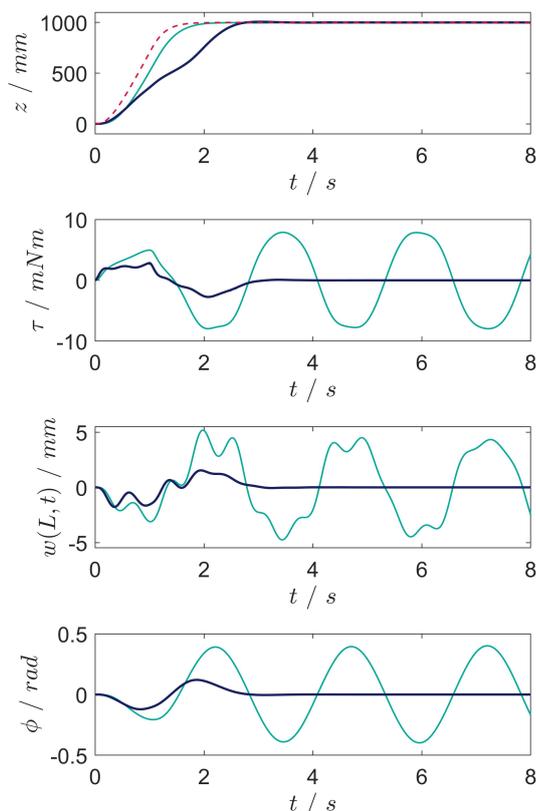


Fig. 4. Reference tracking applying cascade position control (green) and discrepancy based control (dark blue), reference position (red)

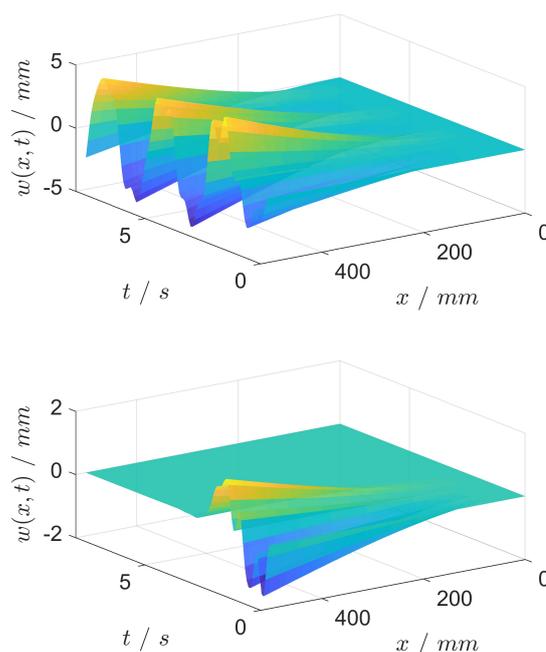


Fig. 5. Time responses of the distributed state applying cascade position control (upper) and discrepancy based control (lower)

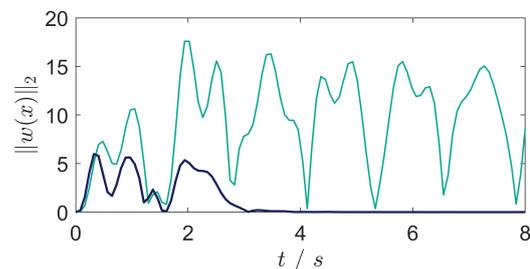


Fig. 6. L_2 -norm with respect to x of the displacement $w(x, t)$ applying cascade position control (green) and discrepancy based control (dark blue)

Table 1. Model parameters used for simulation

Parameter	Value	Parameter	Value
m_t	0.1	m_p	0.18
m_c	0.97	l	$1.5 \cdot 10^3$
μ	0.01	EI	6.9
ρ_c	3	c	5
$k_{p, pos}$	5	$k_{p, vel}$	1.4
$k_{i, vel}$	26	k_{tr}	83.33
k_1	1.3	k_2	1
k_3	0.7	k_4	0.6
a	0.035		

5. CONCLUSION

In this contribution a discrepancy based control approach for underactuated large gantry cranes is proposed. In order to derive a mathematical description of elastic gantry crane dynamics Hamilton's principle has been utilized. From a control point of view, the main objective is to achieve good load positioning and simultaneous to damp load sway and structural oscillation induced by trolley

movements. Due to the distributed nature, strong coupling and only one control handle this is a challenging task. In order to solve this problem a generalized stability theory, stability with respect to two discrepancies and the associated control approach, discrepancy based control, have been successfully applied and verified in simulations.

Future work will be concerned with the robustness analysis of the proposed approach and its practical implementation for further verifications on a laboratory flexible gantry crane.

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