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The Dynamical Model of Flying-Qubit Control Systems

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Abstract: The control of flying qubits is crucial for the interconnection of quantum information processing units in the future applications. Physically, this class of problems can be modeled by the radiation of optical fields from a standing qubit (natural or artificial atoms). The photon statistics of the output field emitted from a quantum system coupled to multiple waveguides is complicated when the exciton number is not conserved, especially in presence of coherent driving that is crucial for control and optimization. In this paper, we use quantum stochastic differential equation (QSDE) to describe the photon generation process, and derive the dynamical jumps induced by photon emission. Numerical simulations show that this model can be applied to analyze the manipulation process of single qubits.

Keywords: quantum control, optimal control

1. INTRODUCTION

The emergence of quantum computing technology has been attracting intense researches all over the world Bennett and H. (1995). As the elementary unit of quantum information processing, quantum bits (qubits) are to be precisely controlled and measured so as to accomplish computational tasks that are superior over classical computers.

As is well-known in the famous Divincenzo's criteria Divincenzo (1995), the large-scale quantum computation requires both standing qubits (realized by natural or artificial atoms Kane (1998); Kielpinski et al. (2002); Blais (2004)) and flying qubits. The latter delivers quantum information among quantum computing units (e.g., between QPU and quantum storage). The natural carrier of flying qubits is the photon states of radiation fields emitted from or absorbed by the standing qubits.

It is recognized that control technology is crucial to the industrialization of quantum computers that has been believed to be possible in a foreseeable future. So far, most studies are casted to the control of standing qubits, but very few studies are seen for flying qubits. Finding a general solution to this problem is difficult. There are some methods by tracing out the fyling qubits based on the canonical Lindblad master equation Carmichael (1991), which is under the assumptions that (1) the states of the atom and the waveguide are separated at any time and (2) the driving field is much weaker than atom's frequency. In 1985, Cardinger and Collett solved the output photon state Gardiner and Collett (1985), which can be calculated through correlation functions. Recently, the solutions are obtained in various different approaches. For example, the calculations of few-photon scattering states can be done with low-dimensional coherently-driven quantum emitters Fischer et al. (2018b,a); Hanschke et al. (2018); Trivedi et al. (2018) via stochastic master equations using coarse-grained field vector Shi et al. (2015); Baragiola et al. (2012) or continuous Matrix Product States Verstraete and Cirac (2010); Cuevas et al. (2018).

In this paper we will explore this problem based on the Quantum Stochastic Differential Equation (QSDE) driven by time-dependent coherent fields, which models the control of emission process of flying qubits in atom-waveguide systems. We find that, although the model is essentially infinite-dimensional, the solutions to the output photon states can be obtained from that of a dissipative finite-dimensional differential equation. This makes it possible to build up a control model for the optimization of flying qubits in waveguide. The remainder of this paper is organized as follows.

2. RADIATION PROCESS IN ONE-DIMENSIONAL WAVEGUIDES

Suppose that we have a general open quantum system in which the Hamiltonian involves coherent controls:

$$\hat{H}(t) = \hat{H}_0 + \sum_{k=1}^{m} u_k(t)\hat{H}_k, \tag{1}$$

where $\hat{H}(t)$ is the internal Hamiltonian of the atom (photon generator) that may involve time-varying coherent controls $u_k(t)$'s that are posed through H_k's.

As is shown in Fig. 1, the flying qubits are randomly emitted to the (multiple) waveguides coupled to the emitter. The corresponding QSDE then reads

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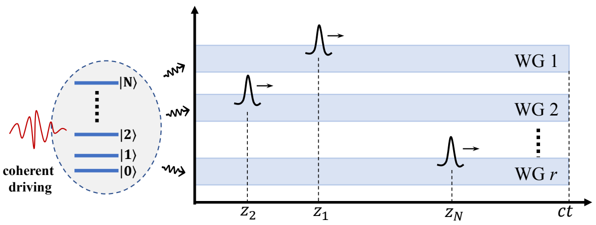

Fig. 1. A snapshot of multiphoton emission from an N-level atom to r waveguides. The photons are randomly emitted to each waveguides and can be detected in the waveguide.

$$\mathrm{d}\hat{U}(t) = \Bigg[-\left(i\hat{H}(t) + \frac{1}{2}\sum_{j=1}^{r} \hat{L}_j^\dagger(t)\hat{L}_j(t) \right)\mathrm{d}t + \sum_{j=1}^{r}\left(\mathrm{d}\hat{A}_j^\dagger(t)\hat{L}_j(t) - \hat{L}_j^\dagger(t)\mathrm{d}\hat{A}_j(t) \right)\Bigg]\hat{U}(t),$$

where $\hat{L}_j(t)$ represents the coupling operator of the atom to the jth waveguide, which may also vary with time. $\mathrm{d}\hat{A}_j^\dagger(t)$ is the quantum noise operator of the jth waveguide.

Suppose that the system's Hilbert space is $\mathcal{H} = \mathrm{span}\{|0\rangle, |1\rangle, \cdots, |N\rangle\}$ (N could be infinity). Let $|\Psi(t)\rangle$ be the entire state of the atom-waveguide system, and assume the composite system is initially at $|\Psi(0)\rangle = |\Omega\rangle \otimes |\psi(0)\rangle$, i.e., the waveguide is empty. As the definition of the increments $B(t), B^\dagger(t)$ that point to the future, $[U(t), dB(t)] = [U(t), dB^\dagger(t)] = 0$ that will result in $dB(t)|\Psi(t)\rangle = 0$. Then the QSDE of $|\Psi(t)\rangle$ can be simplified as

$$\mathrm{d}|\Psi(t)\rangle = \left[-i\hat{H}_{\mathrm{eff}}(t)\mathrm{d}t + \sum_{j=1}^{r}\mathrm{d}\hat{A}_j^\dagger(t)\hat{L}_j(t) \right]|\Psi(t)\rangle, \quad (2)$$

where $H_{\mathrm{eff}}(t) = H(t) - \frac{i}{2}\sum_{j=1}^{r}\hat{L}_j^\dagger(t)\hat{L}_j(t)$.

To resolve this equation, according to the temporal mode basis we expand $|\Psi(t)\rangle$ as follows:

$$|\Psi(t)\rangle = |\Omega\rangle \otimes |\psi^0(t)\rangle + \sum_{\ell=1}^{\infty}\sum_{j_1,\cdots,j_\ell}\int_0^t\int_0^{z_1}\cdots\int_0^{z_{\ell-1}} \mathrm{d}\hat{A}_{j_1}^\dagger(z_1)\cdots\mathrm{d}\hat{A}_{j_\ell}^\dagger(z_\ell)|\Omega\rangle \otimes |\psi_{z_1,\cdots,z_\ell}^{j_1,\cdots,j_\ell}(t)\rangle, \quad (3)$$

where the (unnormalized) state vectors associated with each multi-photon states are defined as follows:

$$|\psi(t)\rangle = \sum_{k=0}^{N}\xi^0(t,k)|k\rangle, \quad (4)$$

$$|\psi_{z_1,\cdots,z_\ell}^{j_1,\cdots,j_\ell}(t)\rangle = \sum_{k=0}^{N}\xi_{z_1,\cdots,z_\ell}^{j_1,\cdots,j_\ell}(t,k)|k\rangle, \quad (5)$$

in which the function $\xi_{z_1,\cdots,z_\ell}^{j_1,\cdots,j_\ell}(t,k)$ indicates the probability amplitude density of observing ℓ photons at positions $z_1 > \cdots > z_\ell$ (assuming $c = 1$) in the j_1th,$\cdots$,j_ℓth waveguides, respectively, at time t and when the system is at state $|k\rangle$.

The differential of $|\Psi_t\rangle$ on the left hand can be expanded as

$$\mathrm{d}|\Psi(t)\rangle = |\Omega\rangle \otimes |\dot{\psi}_t^0\rangle + \mathrm{d}t\sum_{\ell=1}^{\infty}\sum_{j_1,\cdots,j_\ell}\int_0^t\int_0^{z_1}\cdots\int_0^{z_{\ell-1}} \mathrm{d}A_{z_1}^\dagger\cdots\mathrm{d}\hat{A}_{z_\ell}^\dagger|\Omega\rangle \otimes |\dot{\psi}_{z_1,\cdots,z_\ell}^{j_1\cdots j_\ell}(t)\rangle + \mathrm{d}\hat{A}_t^\dagger\sum_{\ell=1}^{\infty}\sum_{j_1,\cdots,j_\ell}\int_0^t\int_0^{z_2}\cdots\int_0^{z_{\ell-1}} \mathrm{d}\hat{A}_{z_2}^\dagger\cdots\mathrm{d}\hat{A}_{z_\ell}^\dagger|\Omega\rangle \otimes |\psi_{t,z_2,\cdots,z_\ell}^{j_1,j_2,\cdots,j_\ell}(t)\rangle,$$

where the "dot" represents partial derivative with respect to time t. Replacing Eq. (2) on the right hand and compare the atomic states associated with each multiphoton state $\mathrm{d}\hat{A}_{z_1}^{j_1\dagger}\cdots\mathrm{d}\hat{A}_{z_\ell}^{j_\ell\dagger}|\Omega\rangle$, we obtain the following group of equations on the wave vectors:

$$|\dot{\psi}_{z_1,\cdots,z_\ell}^{j_1,\cdots,j_\ell}(t)\rangle = -iH_{\mathrm{eff}}(t)|\psi_{z_1,\cdots,z_\ell}^{j_1,\cdots,j_\ell}(t)\rangle, \quad (6)$$

for all $\ell = 0, 1, \cdots$ and $1 \le j_1, \cdots, j_\ell \le r$, and the boundary conditions:

$$|\psi_{t,z_2,\cdots,z_\ell}^{j_1,j_2,\cdots,j_\ell}(t)\rangle = \hat{L}_{j_1}(t)|\psi_{z_2,\cdots,z_\ell}^{j_2,\cdots,j_\ell}(t)\rangle, \quad (7)$$

i.e., the order of the tensor can be reduced by contracting z_1 when $z_1 = t$.

Notice that Eq. (6) is uniform for all $j_1, \cdots, j_\ell$, and hence we introduce the $N \times N$ dissipative propagator

$$\dot{\hat{P}}(t) = -iH_{\mathrm{eff}}(t)\hat{P}(t), \quad (8)$$

where $\hat{P}(0) = \mathbb{I}_N$, which can be always numerically solved for given time-varying $\hat{H}(t)$ and $\hat{L}_j(t)$. We denote the Green function (or transition operator) $\hat{G}(t,\tau) = \hat{P}(t)\hat{P}^{-1}(\tau)$ from τ to t, then

$$|\psi_{z_1,\cdots,z_\ell}^{j_1,\cdots,j_\ell}(t)\rangle = \hat{G}_{t,\tau}|\psi_{z_1,\cdots,z_\ell}^{j_1,\cdots,j_\ell}(\tau)\rangle, \quad (9)$$

for $\ell = 0, 1, \cdots$, and any $t \ge \tau$.

Repeatedly using Eq.(6) and Eq.(8), we obtain the following recursive solution:

$$\begin{aligned}
&|\psi_{z_1,\cdots,z_\ell}^{j_1,\cdots,j_\ell}(t)\rangle \\
&= \hat{G}(t,z_1)|\psi_{z_1,\cdots,z_\ell}^{j_1,\cdots,j_\ell}(z_1)\rangle \\
&= \hat{G}(t,z_1)\hat{L}_{j_1}(z_1)|\psi_{z_2,\cdots,z_\ell}^{j_2,\cdots,j_\ell}(z_1)\rangle \\
&= \cdots \\
&= \hat{G}(t,z_1)\hat{L}_{j_1}(z_1)\hat{G}(z_1,z_2)\hat{L}_{j_2}(z_2) \\
&\quad\cdots\hat{G}(z_{\ell-1},z_\ell)\hat{L}_{j_\ell}(z_\ell)\hat{G}(z_\ell,0)|\psi(0)\rangle,
\end{aligned} \quad (10)$$

which clearly describes the process of observing a photon in the j_pth waveguide at position z_p. The emission of photons (indicated by $\hat{L}_{j_p}(z_p)$ into the j_pth waveguide) are separated by nonunitary evolutions.

In terms of the propagators defined in (8), the above solution can also be written in a more compact form:

$$|\psi_{z_1,\cdots,z_\ell}^{j_1,\cdots,j_\ell}(t)\rangle = \hat{P}(t)\hat{\tilde{L}}_{j_1}(z_1)\cdots\hat{\tilde{L}}_{j_\ell}(z_\ell)|\psi(0)\rangle, \quad (11)$$

where each $\hat{\tilde{L}}_{j_p}(z_p) = \hat{P}^{-1}(z_p)\hat{L}_{j_p}(z_p)\hat{P}(z_p)$.

3. THE OUTGOING FLYING QUBITS

The radiation state of the field in the waveguides after leaving the atom can be simply obtained by pushing $t \to \infty$:

$$|\psi_{z_1,\cdots,z_\ell}^{j_1,\cdots,j_\ell}(\infty)\rangle = \hat{P}(\infty)\hat{\tilde{L}}_{j_1}(z_1)\cdots\hat{\tilde{L}}_{j_\ell}(z_\ell)|\psi(0)\rangle. \quad (12)$$

Under most circumstances, the atom will decay to the ground state $|0\rangle$ and thus $\hat{P}(\infty) = |0\rangle\langle 0|$, and only the ground-state component of $|\psi^{j_1,\cdots,j_\ell}_{z_1,\cdots,z_\ell}(\infty)\rangle$ survives, i.e.,

$$|\psi^{j_1,\cdots,j_\ell}_{z_1,\cdots,z_\ell}(\infty)\rangle = \xi^{j_1,\cdots,j_\ell}_{z_1,\cdots,z_\ell}(\infty,0)|0\rangle, \tag{13}$$

for all j_p and z_p. Therefore, the asymptotic state $|\Psi(\infty)\rangle$ can be decomposed as

$$|\Psi_\infty\rangle = \left[\xi(\infty,0) + \sum_{\ell=1}^{\infty}\sum_{j_1,\cdots,j_\ell}\int_0^t\int_0^{z_1}\cdots\int_0^{z_{\ell-1}} \xi^{j_1,\cdots,j_\ell}_{z_1,\cdots,z_\ell}(\infty,0)\mathrm{d}\hat{A}^{j_1\dagger}_{z_1}\cdots\mathrm{d}\hat{A}^{j_\ell\dagger}_{z_\ell}\right]|\Omega\rangle\otimes|0\rangle. \tag{14}$$

Now it is clear that the scalar functions $\xi^{j_1,\cdots,j_\ell}_{z_1,\cdots,z_\ell}(\infty,0)$ characterizes the shape of ℓ-photon states emitted to the waveguides.

In the case that the atom decays to some superposition state, the waveguides and the atom will be entangled and thus there are no explicit waveforms for each ℓ-photon components. The role of flying qubits is to transfer quantum information between standing qubits. It is natural to interpret the above process as the sending of flying qubits, via which the standing qubit state is transferred to the radiation field.

4. A SIMULATION EXAMPLE

Consider the case of a two-level emitter driven by coherent control fields. In the rotating-frame, the coherent part of the Hamiltonian is as follows:

$$\hat{H}(t) = \frac{u(t)}{2}\hat{\sigma}_+ + \frac{u^*(t)}{2}\hat{\sigma}_-, \tag{15}$$

where $\hat{\sigma}_\pm$ are the standard Pauli matrices. Suppose the emitter is coupled to only one waveguide with coupling operator being $\hat{L} = \sqrt{2\gamma}\hat{\sigma}_-$.

We first look at the spontaneous emission that can be directly derived from our model. In this case, the system starts from the excited state $|\psi(0)\rangle = |1\rangle$ and the driving field $u(t) = 0$, under which the effective Hamiltonian becomes

$$\hat{H}_{\text{eff}}(\tau) = -i\gamma\hat{\sigma}_+\hat{\sigma}_-. \tag{16}$$

It is easy to calculate the transition operator

$$\hat{P}(t) = \begin{bmatrix} e^{-\gamma t} & 0 \\ 0 & 1 \end{bmatrix}, \tag{17}$$

under the basis

$$|0\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad |1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{18}$$

The asymptotic transition operator

$$\hat{P}(\infty) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = |0\rangle\langle 0| \tag{19}$$

indicates that the atomic state eventually decays to the ground state.

According to Eq. (12), we can calculate the outgoing single-photon wavepacket

$$\xi^1_z(\infty,0) = \sqrt{2\gamma}e^{-\gamma z}, \tag{20}$$

and $\xi^{1,\cdots,1}_{z_1,\cdots,z_\ell}(\infty,0) = 0$ for all multi-photon processes. The probability of emitting single-photon is

$$N_1 = \int_0^\infty |\xi^1_z(\infty,0)|^2 dz = 1, \tag{21}$$

which agrees with the well-known results of spontaneous emission.

Now let us turn to the realistic case of generating flying qubits. The emitter is initially prepared at the ground state $|\psi(0)\rangle = |0\rangle$ and flipped to $|1\rangle$ by a π-pulse, after which the photon is emitted. If the duration of the π-pulse is sufficient short (under high-power driving fields), the emitter is instantaneously pumped and then decays, and this is equivalent to the above spontaneous emission process.

When the pulse width is not sufficiently short, the radiated field will not be at a perfect single-photon state, and the single-photon part can be precisely solved by our derived model. Let $u(t) = \Omega e^{i\phi}$ for $0 < t < T$, and $u(t) = 0$ for the rest of time, where Ω is the power of the Rabi driving field. Since the phase ϕ dosen't affect the statistical distribution of emitted photons, we chose $\phi = 0$ in the following calculations. The effective Hamiltonian

$$\hat{H}_{\text{eff}}(t) = \hat{H}(t) - i\gamma\hat{\sigma}_+\hat{\sigma}_-, \tag{22}$$

in which the non-vanishing $\hat{H}(t)$ during $0 < t < T$ injects energy to the atom. According to the strength of the power Ω, we discuss the emitted single-photon state in the following three classes.

In the strong-driving regime, i.e., when $\Omega > \gamma$, the energy's injection is faster than its dissipation rate. Let $\omega = \sqrt{\Omega^2 - \gamma^2}$, we have

$$\hat{P}(t) = e^{-\frac{\gamma t}{2}}\begin{bmatrix} \cos\frac{\omega t}{2} - \frac{\gamma}{\omega}\sin\frac{\omega t}{2} & \frac{-i\Omega}{\omega}\sin\frac{\omega t}{2} \\ \frac{-i\Omega}{\omega}\sin\frac{\omega t}{2} & \cos\frac{\omega t}{2} + \frac{\gamma}{\omega}\sin\frac{\omega t}{2} \end{bmatrix}, \tag{23}$$

for $0 < t < T$ and

$$\hat{P}(t) = \begin{bmatrix} e^{-\gamma(t-\frac{T}{2})} & \\ & e^{-\frac{\gamma T}{2})} \end{bmatrix} \times \begin{bmatrix} \cos\frac{\omega T}{2} - \frac{\gamma}{\omega}\sin\frac{\omega T}{2} & \frac{-i\Omega}{\omega}\sin\frac{\omega T}{2} \\ \frac{-i\Omega}{\omega}\sin\frac{\omega T}{2} & \cos\frac{\omega T}{2} + \frac{\gamma}{\omega}\sin\frac{\omega T}{2} \end{bmatrix}, \tag{24}$$

for $t > T$.

According to Eq. (12), we can calculate the outgoing zero-photon wavepacket

$$\xi(\infty,0) = e^{-\frac{\gamma T}{2}}(\cos\frac{\omega T}{2} + \frac{\gamma}{\omega}\sin\frac{\omega T}{2}), \tag{25}$$

the outgoing single-photon wavepacket

$$\xi^1_z(\infty,0) = -\frac{i\sqrt{2\gamma}\Omega^2 e^{-\frac{\gamma T}{2}}}{\omega^2}\sin\frac{\omega z}{2}\cos\left(\frac{\omega T}{2} - \frac{\omega z}{2} - \varphi\right), \tag{26}$$

with $\varphi = \arctan\gamma/\omega$,for $z < T$ and

$$\xi^1_z(\infty,0) = -\frac{i\sqrt{2\gamma}\Omega e^{\frac{\gamma T}{2}}\sin\frac{\omega T}{2}}{\omega}e^{-\gamma z}, \tag{27}$$

for $z > T$.

In the balanced-driving regime, i.e., when $\Omega = \gamma$, the energy is injected equally fast with its dissipation rate.

Under this circumstance, the zero-photon wavepacket can be calculated to be

$$\xi(\infty,0)=e^{-\frac{\gamma T}{2}}(1+\frac{\gamma T}{2}), \tag{28}$$

the single-photon packet can be derived to be

$$\xi_z^1(\infty,0)=-\frac{i\sqrt{2\gamma}e^{-\frac{\gamma T}{2}}\gamma z}{2}\left(1+\frac{\gamma T}{2}-\frac{\gamma z}{2}\right), \tag{29}$$

for $z<T$ and

$$\xi_z^1(\infty,0)=-\frac{i\sqrt{2\gamma}\gamma T e^{\frac{\gamma T}{2}}}{2}e^{-\gamma z}, \tag{30}$$

for $z>T$.

In the weak-driving regime, i.e., when $\Omega<\gamma$, the energy injection is slower than its dissipation rate. Let $\omega=\sqrt{\gamma^2-\Omega^2}$, we can derive the single-photon packet (by simply replacing the triangular functions in the strong-driving case with the corresponding hyperbolic functions) as follows: the zero-photon wavepacket can be calculated to be

$$\xi(\infty,0)=e^{-\frac{\gamma T}{2}}(\cosh\frac{\omega T}{2}+\frac{\gamma}{\omega}\sinh\frac{\omega T}{2}), \tag{31}$$

the single-photon packet can be derived to be

$$\begin{aligned}\xi_z^1(\infty,0)=&\frac{\sqrt{2\gamma}\Omega^2e^{-\frac{\gamma T}{2}}}{\omega^2}\sinh\frac{\omega z}{2}\\&\cosh(\frac{\omega T}{2}-\frac{\omega z}{2}-\varphi),\end{aligned} \tag{32}$$

with $\varphi=\operatorname{artanh}\gamma/\omega$, for $z<T$ and

$$\xi_z^1(\infty,0)=\frac{\sqrt{2\gamma}\Omega e^{\frac{\gamma T}{2}}\sinh\frac{\omega T}{2}}{\omega}e^{-\gamma z}, \tag{33}$$

for $z>T$.

Figure 2 displays the profiles of the single-photon wavepackets generated by rectangular strong, balanced and weak driving π pulses, respectively $\Omega=0.4\gamma$, γ and 2γ, with $\gamma=2\pi\times 5$MHz. The single-photon wavepacket could be calculated from Eqs. (25-26), (28-29) and (31-32). It indicates that single-photon wavepacket is sinusoidal, quadratic curve and hyperbolic functions during the pulse, and there is a peak before the end of the pulse. After the pulse is over, the wavepacket exponentially decays at the rate of spontaneous radiation.

We also display the probability of single photon emission and zero-photon's in Fig. 3. The probability can be calculated by integrating the single-photon pulse, i.e.,

$$N_1=\int_0^\infty|\xi_z^1(\infty,0)|^2\mathrm{d}z, \tag{34}$$

N_1 and zero-photon's can be calculated by Eqs. (24), (27) and (30)

$$N_0=|\xi(\infty,0)|^2, \tag{35}$$

N_0. It can be seen that the single-photon probability approaches 1 when the driving power is strong, which leads to a perfect single-photon that can be employed as a flying qubit. On the contrary, the zero-photon probability approaches 1 when the driving power is feeble. Otherwise, the waveguide will have certain probability of being empty (corresponding to zero-photon case) and multi-photon state.

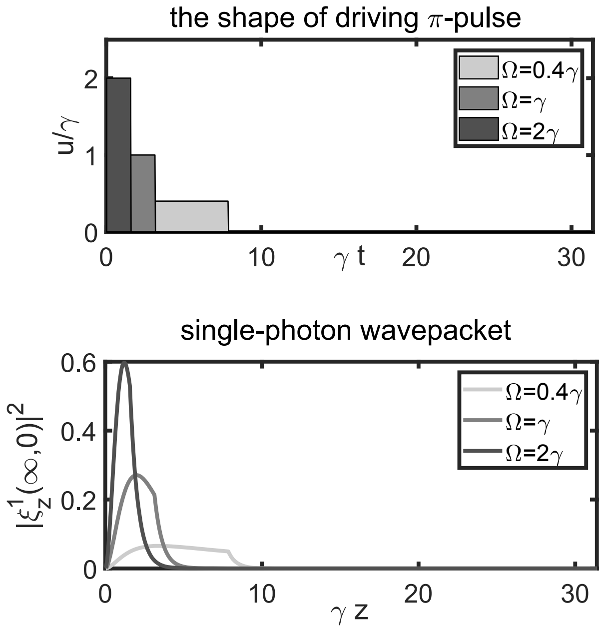

Fig. 2. The driving π pulses (above) and the corresponding emitted single-photon wavepackets (below) under weak, critical and strong drivings.

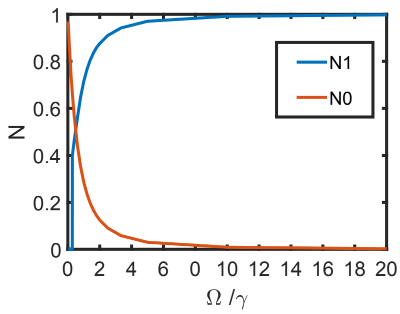

Fig. 3. The dependence of single-photon emission probability (N_1) and 0-photon's (N_0) under weak, critical and strong driving pulses.

5. DISCUSSIONS AND CONCLUSIONS

To conclude, we build up a dynamical model for the control of flying qubits via analysis of the underlying coherently driven QSDE. We show that this essentially infinite-dimensional equation can be reduced to the solution of a finite-dimensional ordinary differential equation [see Eq. (8)], from which one can calculate the single- and multi-photon wavepackets.

This work lays the foundation for analysis, observation and control of flying qubits. However, we note that the derived model should be cautiously used as the multi-photon state expansion may diverge under certain circumstances. For example, we found in the numerical example that the multi-photon expansion violates the conservation of probability in the weak-driving regime where the pulse duration is longer than the coherent time. We conjecture that the model should be valid under sufficiently short pulses. Under what condition can the expansion converge, or whether one can alternate the expansion for better convergence, are open problems.

Upon the validation of expansion, many techniques can be introduced to the control of flying qubits, such as optimal control and robust control. We expect that this model can be used to synthesize shaped single-photons and entangled photons, for the launch and absorption of flying qubits.

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