Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Flat Filtering Cascade Control of Fourth Order Systems ⋆

M. Ramírez-Neria * A. Luviano-Juarez ** N. Lozada-Castillo ** G. Ochoa-Ortega * H. Sira-Ramirez ***

* *Departament of Universidad Politecnica del Valle de Mexico, Av. Mexiquense S/N, Tultitlan Estado de México C.P. 54910 (e-mail:mramirezn.ctrl@gmail.com, gochoa79@gmail.com)*
** *UPIITA-IPN Av. IPN 2580 Col. Barrio La Laguna Ticomán Ciudad de Mexico, Mexico,(e-mail:aluvianoj@ipn.mx, gioconda@esfm.ipn.mx)*
*** *Mechatronics Section of the Electrical Engineering Department of CINVESTAV-IPN, Apartado postal 14740, México City C.P. 07360, (e-mail:hsira@cinvestav.mx)*

Abstract: In this article, the analysis and implementation of an alternative Flat Filtering Control for a class of partially known fourth order flat systems is given. The Flat filtering control uses the cascade property of the system, which leads to a simplified control design in which high order time derivatives are algebraically simplified in terms of low order measurable states and the subsequent integral compensation. The control proposal is implemented and validated experimentally in a fourth order mechanical system (rotatory flexible joint) with accurate tracking results.

Keywords: Disturbance Rejection, Flat filtering, ADRC, Under-actuated system, Rotary Flexible Joint.

1. INTRODUCTION

Active Disturbance Rejection Control (ADRC Han (2009)) is a robust control methodology against process parameter variations that has been extensively studied during the last years. Numerous recent approaches have been developed to address many academic and industrial problems such as motion control (Zhao and Gao (2013), Sen et al. (2019), Hernández-Melgarejo et al. (2019), Touhami et al. (2019)), power electronics (Wu et al. (2017), Huangfu et al. (2019), Sun et al. (2019), Zheng and Gao (2018)), robotic tele-operation (Gutiérrez-Giles and Arteaga-Pérez (2019), Gutiérrez-Giles et al. (2019)), industrial processes (Zheng and Gao (2012), Zheng et al. (2018)), underactuated systems (Ramírez-Neria et al. (2016), Sira-Ramirez et al. (2018), Ramírez-Neria et al. (2019)), among others. These academic and industrial study cases have shown excellent performance in terms of accuracy, repeatability, energy efficiency, easy implementation and intuitiveness of each control term.

The close relation of ADRC and an accurate state and disturbance estimation has motivated the use of alternative schemes of disturbance approximation. From a practical perspective, the use of simple controllers with the least possible information of a complex system is a challenging control problem. In this sense, Generalized Proportional Integral (GPI) Control (Fliess et al. (2002)) has provided the path to obtain a technique of state approximation which, along with an iterative integral compensation has let an observer-free output based control. The link between ADRC and GPI control was stated though the *Flat Filtering Control*, which constitutes a reinterpretation of GPIC in the form of classical compensation networks (CCN). (see Sira-Ramirez et al. (2018), Sira-Ramírez et al. (2016)). As a result was developed a tool for output feedback control design in linear controllable systems. A controllable linear system exhibits a natural flat output (Brunovsky's output) from which the system is also trivially observable. Flat filtering is based on the fact that a GPI controller is viewed as a dynamical linear system which exhibits as a natural flat output a filtered version of the plant output signal. This property is particularly helpful in the design of efficient output feedback stabilization schemes and in solving output reference trajectory tracking tasks. Flat Filtering approach is naturally extended to efficiently handling control tasks on significantly perturbed differentially flat SISO nonlinear systems, affected by unknown endogenous nonlinearities, in the presence of exogenous disturbances and un-modeled dynamics. Recently, in Sira-Ramírez et al. (2019) is shown that, both, ADRC and flat filtering control are equivalent by means of an algebraic procedure involving a reduced order observer structure. This relation allows to use typical procedures of ADRC based control in the context of flat filtering control design. One of the alternative approaches of flatness based ADRC design deals with the simplification of the controllers through the use of the *cascade flat property* (Ramírez-Neria et al. (2014)) which allows to reduce the high order time derivatives estimation though algebraic equivalence. This scheme is typically used in a class of fourth order underactuated systems whose approximate linearization is flat (controllable). This principle can be also extended to a general class of fourth order nonlinear feedback linearizable systems (typically underactuated), which also exhibit the cascade property, reducing the complexity of the controller and leading to an alternative control design with the essence of the flat filtering approach.

In this article, the robust control of a class of fourth order differentially flat systems with partially known structure is tackled by means of a flat filtering control in combination

⋆ This work was partially supported by the CONACYT México and SIP-IPN under research grants SIP20201675, SIP20201830.

with the cascade property, leading to a simplified family of controllers of GPI class.

The article is organized as follows: Section 2 presents the class of systems of study and the general control design. Section III the mathematical model of a flexible link arm and controller design details is presented. Section IV presents the description of a laboratory prototype, as well as the corresponding experimental results are presented. A brief discussion of the results and the conclusions are given in Section V.

2. DYNAMICS OF FOURTH ORDER FLAT PERTURBED SYSTEMS

Let us denote the flat output as y_f. Consider a class of disturbed fourth order differentially flat systems, whose dynamics is described by the following equation:

$$y_f^{(4)} = \alpha\left(y_f, \dot{y}_f, \ddot{y}_f, y_f^{(3)}\right) u(t) + \tilde{\Phi}\left(t, y_f, \dot{y}_f, \ddot{y}_f, y_f^{(3)}, \varphi(t)\right) \quad (1)$$

The disturbance function $\tilde{\Phi}(\cdot)$, lumps all the endogenous forces such as internal, non modeled or nonlinear dynamics. The exogenous forces are represented by $\varphi(t)$. The disturbance depend on the time, the flat output and its time derivatives. It is assumed that only flat output y_f variable is available for measurement. The control again $\alpha(\cdot)$ is a nonlinear unknown gain depending of the set of the phase variables associated with the flat output y_f. The nonlinear gain it is represented as follows

$$\alpha\left(y_f, \dot{y}_f, \ddot{y}_f, y_f^{(3)}\right) = K + \tilde{\alpha}\left(y_f, \dot{y}_f, \ddot{y}_f, y_f^{(3)}\right) \quad (2)$$

Where K is a known constant gain and $\tilde{\alpha}\left(y_f, \dot{y}_f, \ddot{y}_f, y_f^{(3)}\right)$ represents the high order terms (HOT). Following the methodology of the model free control (Fliess and Join (2013)), and using well established procedures in ADRC control, the original model (1) is simplified as follows

$$y_f^{(4)} = Ku(t) + \Phi(t) \quad (3)$$

where $\Phi(t)$ is the total disturbance (see Han (2009)) described as follows:

$$\Phi(t) = \tilde{\alpha}\left(y_f, \dot{y}_f, \ddot{y}_f, y_f^{(3)}\right) u(t) + \tilde{\Phi}\left(t, y_f, \dot{y}_f, \ddot{y}_f, y_f^{(3)}, \varphi(t)\right) \quad (4)$$

the total disturbance, in addition to exogenous and endogenous disturbances, it depends now of the control input itself. Let us define the nominal reference system which depends on the desired output $y^*(t)$ and its dynamics (Sira-Ramirez and Agrawal (2004))

$$y^{*(4)} = Ku^*(t) \quad (5)$$

Then, the tracking error is defined as

$$e_y = y_f - y^*(t) \quad (6)$$

whose dynamics is given by the following expression

$$e_y^{(4)} = Ke_u(t) + \Phi(t) \quad (7)$$

where $e_u(t) = u(t) - u^*(t)$. The cascade representation of (7) is shown in Fig. 1. The integral reconstructors based on neglecting the total disturbance term $\Phi(t)$ in the simplified model (7) are implemented as follows [1]

[1] It is adopted, henceforth, the following notation for multiple integrations on a given time function $\phi(t)$ $\int_0^t \int_0^{\sigma_1} \cdots \int_0^{\sigma_{i-1}} \phi(\sigma_i) d\sigma_i \cdots d\sigma_1 = \left(\int^{(i)} \phi(t)\right)$, $\left(\int^{(0)} \phi(t)\right) = \phi(t)$

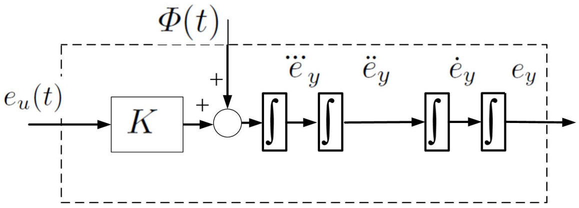

Fig. 1. Cascade block representation

$$\widehat{e^{(3)}}_y = K \int e_u \quad (8)$$

$$\hat{\ddot{e}}_y = K \int^{(2)} e_u \quad (9)$$

$$\hat{\dot{e}}_y = K \int^{(3)} e_u \quad (10)$$

The intregal reconstructors are shown in the Fig. 2.

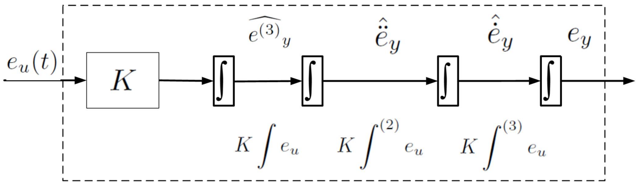

Fig. 2. Integral reconstructors

Since the total disturbance $\Phi(t)$ is unknown, the flat output time derivatives $\widehat{e^{(3)}}_y$, $\hat{\ddot{e}}_y$ and $\hat{\dot{e}}_y$ depend only of the control input error e_u and the gain of the system K. So, the time derivatives can not be estimated accurately with (8)-(10) if they are corrupted by noise. In order to overcome this problem, the *cascade flat property* is used (see Ramírez-Neria et al. (2016), Ramírez-Neria et al. (2019)) typically found on underactuated mechanical systems. The error dynamics (7) are represented as a cascade connection of two independent blocks (see Fig.3): The first one, controlled by the input error e_u whose corresponding output is given as the acceleration error. For flat systems (1) using the *cascade flat property* we found that the acceleration error $\hat{\ddot{e}}_y$ is a function which depends on the measurable flat output y_f the available states of the system and the desired trajectory $\ddot{y}_f^*(t)$. Let us define the acceleration error $\hat{\ddot{e}}_y = e_{2y}$, now e_{2y} acts as a measurable auxiliary input to the second block, which consists of a chain of two integrators associated, respectively, with the phase variables: $\hat{\dot{e}}_y$ and e_y. Since e_{2y} is measurable, the time derivatives of the tracking error (8)-(10) can be estimated as follows

$$\widehat{e^{(3)}}_y = K \int e_u \quad (11)$$

$$\hat{\ddot{e}}_y = e_{2y} = \ddot{y}_f - \ddot{y}^*(t) \quad (12)$$

$$\hat{\dot{e}}_y = \int e_{2y} \quad (13)$$

Notice that only $\widehat{e^{(3)}}_y$ is estimated using the control input as is shown in Fig. 3. We propose the application of the following feedback control law

$$e_u = -\frac{1}{K}\left(k_3 \widehat{e_y^{(3)}} + k_2 \hat{\ddot{e}}_y + k_1 \hat{\dot{e}}_y + k_0 e_y\right) \quad (14)$$

if we apply the control law (14) to the simplified error dynamics (7) may not lead to accurate tracking results since

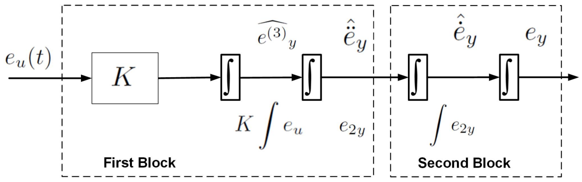

Fig. 3. Flat output property

the integral reconstructors $\widehat{e^{(3)}}_y$, $\hat{\ddot{e}}_y$ and $\hat{\dot{e}}_y$ from equations (8)-(10) are inaccurate due to the total disturbance $\Phi(t)$ is not compensated. To overcome these facts, as stated in flat filtering approach, let proceed to compensate both effects with a finite number m of iterated integrals of the tracking error (see Sira-Ramírez et al. (2016) and Sira-Ramirez et al. (2018)). The proposed controller (14) is now expressed as:

$$e_u = -\frac{1}{K}\left(k_{m+3}\widehat{e_y^{(3)}} + k_{m+2}\hat{\ddot{e}}_y + k_{m+1}\hat{\dot{e}}_y + k_m e_y \right.$$
$$\left. + k_{m-1}\int^{(1)} e_y + k_{m-2}\int^{(2)} e_y + \cdots + k_0 \int^{(m)} e_y\right) \tag{15}$$

using the advantage of cascade property, the estimated states (11)-(13) are substituted in (15)

$$e_u(t) = -\frac{1}{K}\left(k_{m+3}K\int e_u + k_{m+2}e_{2y} + k_{m+1}\int e_{2y} \right.$$
$$\left. + k_m e_y + k_{m-1}\int^{(1)} e_y + k_{m-2}\int^{(2)} e_y + \cdots + k_0\int^{(m)} e_y\right) \tag{16}$$

The proposed controller (16) can be represented in the frequency domain as follows:

$$e_u(s) = -\frac{1}{K}\left(k_{m+3}K\frac{e_u(s)}{s} + \left(k_{m+2} + \frac{k_{m+1}}{s}\right)e_{2y}(s)\right)$$
$$-\frac{1}{K}\left(k_m + \frac{k_{m-1}}{s} + \frac{k_{m-2}}{s^2} + \cdots + \frac{k_0}{s^m}\right)e_y(s) \tag{17}$$

After some algebraic manipulations, the following parallel controller structure is obtained

$$e_u(s) = -\frac{1}{K}\left(\frac{k_{m+2}s + k_{m+1}}{s + k_{m+3}}\right)e_{2y}(s) \tag{18}$$
$$-\frac{1}{K}\left(\frac{k_m s^m + k_{m-1}s^{m-1} + k_{m-2}s^{m-2} + \cdots + k_0}{s^{m-1}(s + k_{m+3})}\right)e_y(s)$$

The closed loop tracking error system, expressed in the frequency domain, leads, after using $s^2 e_y = e_{2y}(s)$ to the following closed loop dynamics for the simplified perturbed system (7)

$$\left(s^{m+4} + k_{m+3}s^{m+3} + k_{m+2}s^{m+2} + k_{m+1}s^{m+1} + k_m s^m \right.$$
$$\left. + k_{m-1}s^{m-1} + k_{m-2}s^{m-2} + \cdots + k_0\right)e_y(s) = s^m\Phi(s) \tag{19}$$

We select the set of gains $[k_{m+3}, k_{m+2}, \ldots, k_1, k_0]$ matching the close loop polynomial

$$P(s) = s^{m+4} + k_{m+3}s^{m+3} + k_{m+2}s^{m+2} + k_{m+1}s^{m+1}$$
$$+ k_m s^m + k_{m-1}s^{m-1} + k_{m-2}s^{m-2} + \cdots + k_0 \tag{20}$$

with a Hurwitz polynomial: if $m+4$ is even

$$P(s) = (s^2 + 2\zeta_c\omega_c s + \omega_c^2)^{((m+4)/2)} \tag{21}$$

if $m+4$ is odd

$$P(s) = (s + \omega_c)(s^2 + 2\zeta_c\omega_c s + \omega_c^2)^{((m+3)/2)} \tag{22}$$

In order to implement the proposed controller in the time domain, from (18) it is defined a filtered version of the tracking error $e_{yf}(s)$ and the acceleration $e_{2yf}(s)$.

$$e_{yf}(s) = \frac{e_y(s)}{s^{m-1}(s + k_{m+3})} \tag{23}$$

$$e_{2yf}(s) = \frac{e_{2y}(s)}{s + k_{m+3}} \tag{24}$$

we define the following states $e_0 = e_{yf}$, $e_1 = \dot{e}_{yf}$, $e_2 = \ddot{e}_{yf}, \ldots, e_{m-1} = e_f^{(m-1)}$. The filtered error dynamics (23) and (24) can be represented in state space as follows:

$$\dot{e}_i = e_{i+1}, \quad i = 0, 1, \ldots, m-2$$
$$\dot{e}_{m-1} = e_y - k_{m+3}e_{m-1}$$
$$\dot{e}_m = e_{2y} - k_{m+3}e_m \tag{25}$$

Finally the flat filtering control is synthesized using (18) and (25) as follows

$$u(t) = -\frac{1}{K}\left(k_{m+2}e_{2y} + (k_{m+1} - k_{m+2}k_{m+3})e_m + k_m e_y\right.$$
$$\left. + (k_{m-1} - k_m k_{m+3})e_{m-1} + k_{m-2}e_{m-2} + \cdots + k_0 e_0\right)$$
$$+ \frac{1}{K}y^{*(4)}(t) \tag{26}$$

3. ROTARY FLEXIBLE JOINT CASE OF STUDY

Consider the Rotary Flexible Joint studied in the work Ramirez-Neria et al. (2016), it consists of a free arm attached to two identical springs. The springs are mounted to an aluminum chassis which is fastened to the rotary base which is actuated by a DC motor, the Figure 4 shows a diagram of the system with the following parameters:

θ_1	Angular position of the rotating base
θ_2	Angular position of the arm
J_a	Moment of inertia of the arm
J_b	Moment of inertia of the rotating base
m	Mass of the arm
l	Arm length
k_s	Spring stiffness
τ	Torque applied to the system

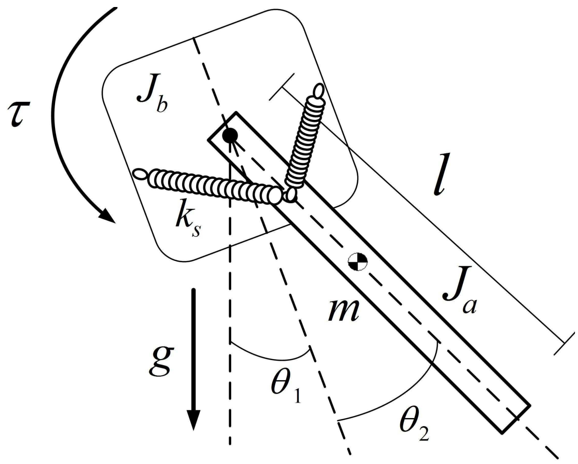

Fig. 4. Rotary Flexible Joint diagram

The angular position of the arm tip is defined by the relation

$$\vartheta = \theta_1 + \theta_2$$

In order to determine the dynamic model of the flexible-joint robot, the Euler-Lagrange approach was followed.

Using the Euler Lagrange methodology with generalized coordinates θ_1, θ_2, we obtain the dynamic of the system

$$\tau = J_b\ddot{\theta}_1 + J_a\ddot{\vartheta} + \frac{mgl}{2}\sin(\vartheta)$$
$$0 = J_a\ddot{\vartheta} + K_s\theta_2 + \frac{mgl}{2}\sin(\vartheta)$$

the motor torque is represented with τ_m and the torque applied to the system are related as: $\tau = N\tau_m$, being N the mechanical advantage of the pulley system. The voltage applied to the motor $V(t)$, and the torque generated by the motor τ_m, are obtained using the following simplified relation

$$\tau_m = \frac{k_\tau}{R_m}V(t) - \frac{k_\tau^2 N}{R_m}\dot{\theta}_1$$

where R_m denotes the armature resistance, k_τ represents the torque constant of the motor. Using last relation, the dynamics of the rotary flexible joint, are given by

$$\frac{Nk_\tau}{R_a}V(t) = (J_b+J_a)\,\ddot{\theta}_1 + J_a\ddot{\theta}_2 + \frac{k_\tau^2N^2}{R_a}\dot{\theta}_1 + \frac{mgl}{2}\sin(\vartheta) \tag{27}$$

$$0 = J_a\ddot{\vartheta} + K_s\theta_2 + \frac{mgl}{2}\sin(\vartheta) \tag{28}$$

In order to obtain a state space representation, the accelerations $\ddot{\theta}_1$ and $\ddot{\theta}_2$ can be fully determined

$$\ddot{\theta}_1 = a_1\theta_2 - a_2\dot{\theta}_1 + bV(t) \tag{29}$$
$$\ddot{\theta}_2 = -a_3\theta_2 - a_4\sin(\theta_1(t) + \theta_2) + a_2\dot{\theta}_1 - bV(t) \tag{30}$$

where $a_1 = \frac{k_\tau^2N^2}{J_bR_m}$, $a_2 = \frac{k_s}{J_b}$, $a_3 = \frac{k_s(J_a+J_b)}{J_aJ_b}$, $a_4 = \frac{mgl}{2J_a}$, $b = \frac{k_\tau N}{J_bR_m}$. Now, performing the variable change $x := [x_1, x_2, x_3, x_4]^T = [\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2]^T$ then system (29)-(30) can be rewritten as

$$\dot{x}(t) = f(x) + g(x)V(t), \tag{31}$$

where

$$f(x) = \begin{pmatrix} x_2 \\ -a_2x_2 + a_2x_3 \\ x_4 \\ a_1x_2 - a_3x_3 - a_4\sin(x_1+x_3) \end{pmatrix}, \quad g(x) = \begin{pmatrix} 0 \\ b \\ 0 \\ -b \end{pmatrix}.$$

Now, the control law design via feedback linearization is presented. Let define the output of the system as follows

$$h(x) = x_1 + x_3, \tag{32}$$

We compute $L_fh(x)$ the Lie derivatives of $h(x)$ with respect to $f(x)$ are given as

$$\dot{h}(x) = L_fh(x) \tag{33}$$
$$\ddot{h}(x) = L_f^2h(x) \tag{34}$$
$$h^{(3)}(x) = L_f^3h(x) \tag{35}$$
$$h^{(4)}(x) = L_f^4h(x) + L_gL_f^3h(x)V(t) \tag{36}$$

where

$$\begin{aligned}
L_fh(x) &= x_2 + x_4 \\
L_f^2h(x) &= (a_2 - a_3)x_3 - a_4\sin(x_1 + x_3) \\
L_f^3h(x) &= -a_4(x_2 + x_4)\cos(x_1 + x_3) + (a_2 - a_3)x_4 \\
L_f^4h(x) &= +a_4(a_3 - a_2)x_3\cos(x_1 + x_3) \\
&\quad + (a_2 - a_3)(a_1x_2 - a_3x_3) + \left[(x_2 + x_4)^2\right. \\
&\quad \left. -(a_2 - a_3) + a_4\cos(x_1 + x_3)\right]a_4\sin(x_1 + x_3) \\
L_gL_f^3h(x) &= -(a_2 - a_3)b.
\end{aligned}$$

Since $L_gL_f^3h(x) \neq 0$ in the whole state space, the relative degree of the system is four. Notice that the flat output coincides with $y_f = h(x)$, so we have

$$y_f = x_1 + x_3 \tag{37}$$
$$\dot{y}_f = x_2 + x_4 \tag{38}$$
$$\ddot{y}_f = (a_2 - a_3)x_3 - a_4\sin(x_1 + x_3) \tag{39}$$
$$y_f^{(3)} = -a_4(x_2 + x_4)\cos(x_1 + x_3) + (a_2 - a_3)x_4 \tag{40}$$
$$y_f^{(4)} = KV(t) + \Phi(t) \tag{41}$$

with $K = L_gL_f^3h(t)$ and $\Phi(t) = L_f^4h(x)$ notice that the acceleration $\ddot{y}_f$ is measurable and the *cascade flat property* can be used to simplify the control design. Given the flat output trajectory error defined in equation (6)

$$e_y = y_f - y^*(t)$$

and the tracking error perturbed dynamics (7) for the Flexible link arm system (37)-(41)

$$e_y^{(4)} = Ke_V(t) + \Phi(t) \tag{42}$$

with $e_V(t) = V(t) - V^*(t)$. We propose the following flat filtering controller using (25) and (26) with $m = 5$

$$\begin{aligned}
\dot{e}_i &= e_{i+1}, \quad i = 0,1,2,3 \\
\dot{e}_4 &= e_y - k_8e_4 \\
\dot{e}_5 &= e_{2y} - k_8e_5
\end{aligned} \tag{43}$$

$$\begin{aligned}
V(t) = &-\frac{1}{K}\left(k_7e_{2y} + (k_6 - k_7k_8)e_5 + k_5e_y\right. \\
&\left.+(k_4 - k_5k_8)e_4 + k_3e_3 + k_2e_2 + k_1e_1 + k_0e_0\right) \\
&+\frac{1}{K}y^{*(4)}(t)
\end{aligned} \tag{44}$$

the set of gains is chosen matching close loop characteristic polynomial with a Hurwitz polynomial

$$P(s) = s^9 + k_8s^8 + k_7s^7 + k_6s^6 + k_5s^5 + k_4s^4 + \cdots + k_0$$
$$P(s) = (s + \omega_c)(s^2 + 2\zeta_c\omega_cs + \omega_c^2)^4 \tag{45}$$

4. EXPERIMENTAL RESULTS

The experimental device is shown in Figure 5. It consists of a DC motor NISCA: model NC5475, which drives a rotating base through a synchronous belt and pulley system with a 16:1 ratio. The main arm is attached to the rotating base by two identical springs, resulting in a fexible joint. Both, the angular positions of the rotating base and the arm were measured with incremental optical encoders of 1000 pulses/revolution. The data acquisition is carried out through a data card Sensoray model 626. This board, is responsible to read the signals from the optical incremental encoders and supplies control voltage to the power amplifier. The proposed control law was implemented in the Matlab-Simulink environment. Finally, the sampling time was set to be 0.001[s] with consideration of the dominating dynamics in the mechanical system under study. The rotary flexible joint parameters, used to implement the control laws (44) are: $l = 0.5$[m], $m = 0.1633$[Kg],$J_a = 0.0136$[Kg-m^2],$J_b = 0.002405$[Kg-m^2], $k_s = 4$[N-m/rad] and $N = 16$, while the motor parameters are: $k_\tau = 0.0724$[N-m/A] and $R_m = 2.983$ [Ω]. The initial conditions for the test were $[\theta_1(0), \theta_2(0)]$ this implies that the flat output $y_f(0) = 0$, the controller gains were selected using $\zeta_c = 3.3$ and $\omega_c = 18$. Fig. 6 depicts the flat output trajectory performance, the tracking error is showed in Fig. 7 we can observed it is bounded in a region approximately of $[-0.025, 0.025]$ rad. The control voltage is presented in Fig. 8.

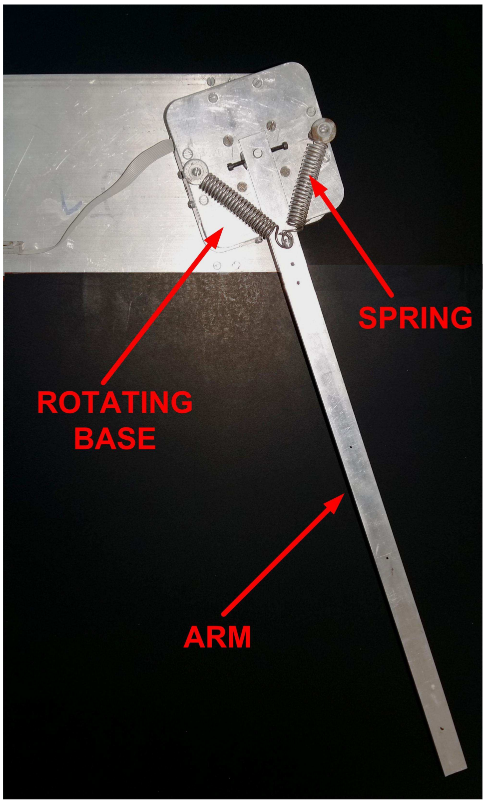

Fig. 5. Rotary flexible joint prototype

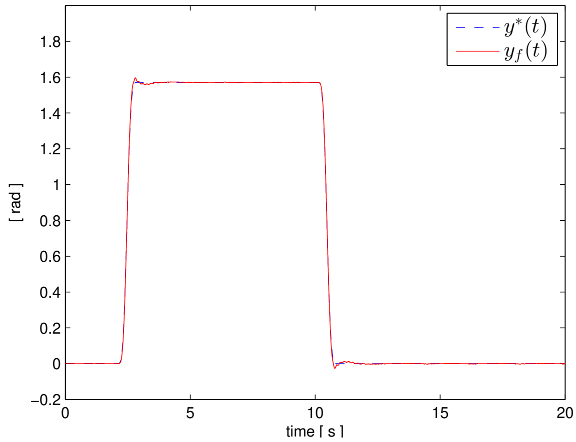

Fig. 6. Trajectory tracking flat output

4.1 External disturbance test

Fig. 9 shows the trajectory tracking performance of the proposed controller when it is affected with a set impulsive external force applied directly on the arm, the proposed controller reject the external disturbance with bounded error showed in Fig. 10, the voltage control is depicted in Fig. 11, we can notice that when the external force is applied on the arm the magnitud of the controller increase canceling out the external disturbance.

5. CONCLUSIONS AND REMARKS

In this article, a Flat Filtering controller scheme was proposed for a fourth order nonlinear feedback linearizable system which exhibits the cascade property. The effectiveness of the proposed controller was proved on a rotary flexible joint prototype exhibited an excellent behavior

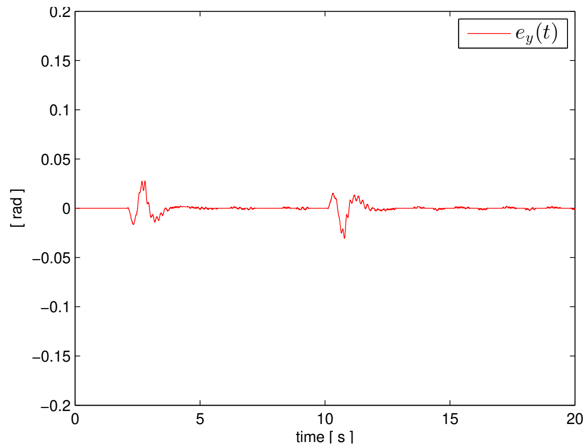

Fig. 7. Tracking error

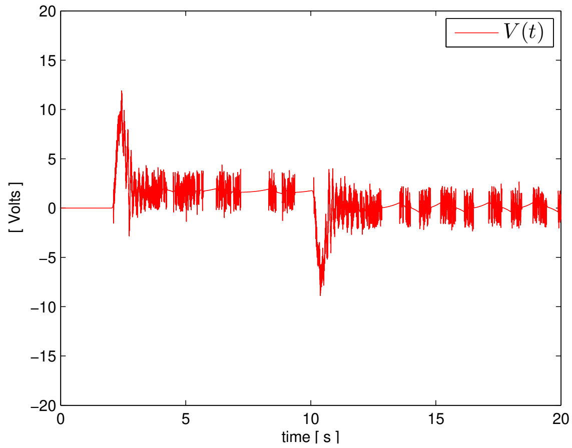

Fig. 8. Flat filtering control performance

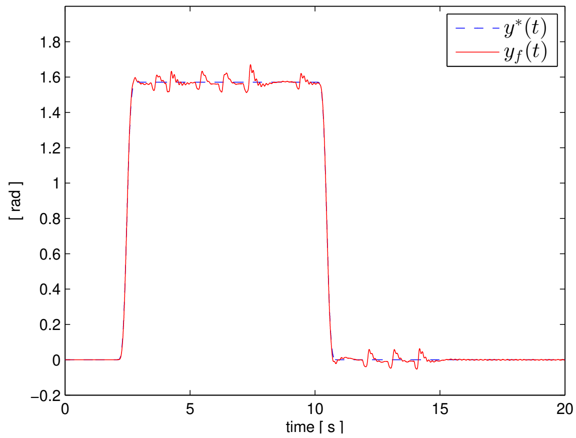

Fig. 9. Trajectory tracking flat output

and an remarkable trajectory tracking performance and robustness in presence of unknown external disturbance inputs. As a suggestion for further research, an extension of the flat filtering controller may be addressed for a suitable high order nonlinear systems.

REFERENCES

Fliess, M. and Join, C. (2013). Model-free control. *International Journal of Control*, 86(12), 2228–2252.

Fliess, M., Marquez, R., Delaleau, E., and Sira-Ramírez, H. (2002). Correcteurs proportionnels-intégraux

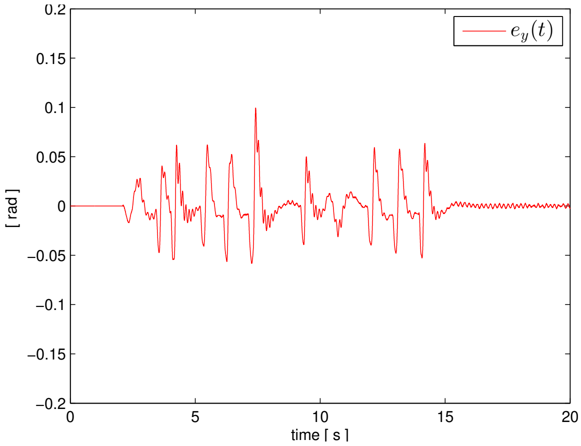

Fig. 10. Tracking error

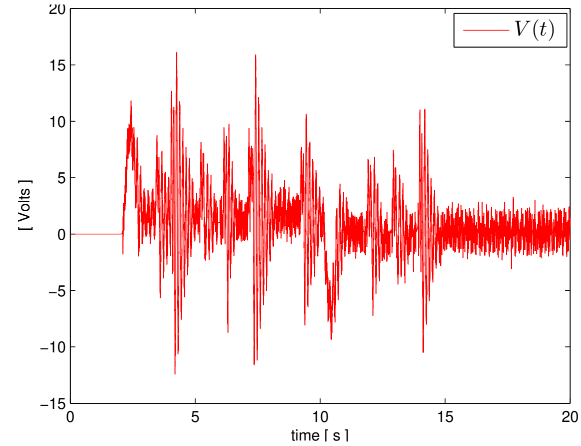

Fig. 11. Flat filtering control performance

généralisés. *ESAIM: Control, Optimisation and Calculus of Variations*, 7, 23–41.

Gutiérrez-Giles, A., Arteaga-Pérez, M., and Rodríguez-Ángeles, A. (2019). Transparent master-slave teleoperation without force nor velocity measurements. In *2019 18th European Control Conference (ECC)*, 2096–2101. IEEE.

Gutiérrez-Giles, A. and Arteaga-Pérez, M.A. (2019). Transparent bilateral teleoperation interacting with unknown remote surfaces with a force/velocity observer design. *International Journal of Control*, 92(4), 840–857.

Han, J. (2009). From pid to active disturbance rejection control. *IEEE transactions on Industrial Electronics*, 56(3), 900–906.

Hernández-Melgarejo, G., Flores-Hernández, D., Luviano-Juárez, A., Castañeda, L., Chairez, I., and Di Gennaro, S. (2019). Mechatronic design and implementation of a bicycle virtual reality system. *ISA transactions*.

Huangfu, Y., Li, Q., Xu, L., Ma, R., and Gao, F. (2019). Extended state observer based flatness control for fuel cell output series interleaved boost converter. *IEEE Transactions on Industry Applications*.

Ramírez-Neria, M., Sira-Ramírez, H., Garrido-Moctezuma, R., and Luviano-Juarez, A. (2014). Linear active disturbance rejection control of underactuated systems: The case of the furuta pendulum. *ISA transactions*, 53(4), 920–928.

Ramirez-Neria, M., Ochoa-Ortega, G., Lozada-Castillo, N., Trujano-Cabrera, M.A., Campos-Lopez, J.P., and Luviano-Juárez, A. (2016). On the robust trajectory tracking task for flexible-joint robotic arm with unmodeled dynamics. *IEEE Access*, 4, 7816–7827.

Ramírez-Neria, M., Sira-Ramírez, H., Garrido-Moctezuma, R., and Luviano-Juárez, A. (2016). On the linear control of underactuated nonlinear systems via tangent flatness and active disturbance rejection control: The case of the ball and beam system. *Journal of Dynamic Systems, Measurement, and Control*, 138(10), 104501.

Ramírez-Neria, M., Sira-Ramírez, H., Garrido-Moctezuma, R., and Luviano-Juárez, A. (2019). Active disturbance rejection control of the inertia wheel pendulum through a tangent linearization approach. *International Journal of Control, Automation and Systems*, 17(1), 18–28.

Sen, C., Wenchao, X., Zhiyun, L., and HUANG, Y. (2019). On active disturbance rejection control for path following of automated guided vehicle with uncertain velocities. In *2019 American Control Conference (ACC)*, 2446–2451. IEEE.

Sira-Ramirez, H. and Agrawal, S.K. (2004). *Differentially flat systems*. Marcel Decker INC., CRC Press.

Sira-Ramírez, H., Luviano-Juárez, A., Ramirez-Neria, M., and Garrido-Moctezuma, R. (2016). Flat filtering: A classical approach to robust control of nonlinear systems. In *2016 American Control Conference (ACC)*, 3844–3849. IEEE.

Sira-Ramirez, H., Luviano-Juárez, A., Ramírez-Neria, M., and Zurita-Bustamante, E.W. (2018). *Active disturbance rejection control of dynamic systems: a flatness based approach*. Butterworth-Heinemann.

Sira-Ramírez, H., Zurita-Bustamante, E.W., and Huang, C. (2019). Equivalence among flat filters, dirty derivative-based pid controllers, adrc, and integral reconstructor-based sliding mode control. *IEEE Transactions on Control Systems Technology*.

Sun, L., Zhang, Y., Li, D., and Lee, K.Y. (2019). Tuning of active disturbance rejection control with application to power plant furnace regulation. *Control Engineering Practice*, 92, 104122.

Touhami, M., Hazzab, A., Mokhtari, F., and Sicard, P. (2019). Active disturbance rejection controller with adrc-fuzzy for mas control. *Electrotehnica, Electronica, Automatica*, 67(2), 89–97.

Wu, G., Sun, L., and Lee, K.Y. (2017). Disturbance rejection control of a fuel cell power plant in a grid-connected system. *Control Engineering Practice*, 60, 183–192.

Zhao, S. and Gao, Z. (2013). An active disturbance rejection based approach to vibration suppression in two-inertia systems. *Asian Journal of Control*, 15(2), 350–362.

Zheng, Q. and Gao, Z. (2012). An energy saving, factory-validated disturbance decoupling control design for extrusion processes. In *Proceedings of the 10th world congress on intelligent control and automation*, 2891–2896. IEEE.

Zheng, Q. and Gao, Z. (2018). Active disturbance rejection control: some recent experimental and industrial case studies. *Control Theory and Technology*, 16(4), 301–313.

Zheng, Q., Ping, Z., Soares, S., Hu, Y., and Gao, Z. (2018). An optimized active disturbance rejection approach to fan control in server. *Control Engineering Practice*, 79, 154–169.