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# A computationally efficient approach for robust gain-scheduled output-feedback LQR design for large-scale systems

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**Abstract:** This paper proposes a novel and simple control design procedure for sub-optimal robust gain-scheduled (GS) output-feedback linear quadratic regulator (LQR) design for large-scale uncertain linear parameter-varying (LPV) systems. First, we introduce a simple and practical technique to convexify the controller design problem in the scheduled parameters. Then, we propose a computationally efficient iterative Newton-based approach for gain-scheduled output-feedback LQR design. Next, we propose a simple modification to the proposed algorithm to design robust GS controllers. Finally, the proposed algorithm is applied for air management and fueling strategy of diesel engines, where the designed robust GS proportional-integral-derivative (PID) controller is validated on a benchmark model using real-world road profile data.

*Keywords:* Linear quadratic regulator; gain-scheduled control; robust control; linear parameter-varying systems; diesel engine; air-path system.

## 1. INTRODUCTION

Finding an optimal static output-feedback (SOF) control law within the linear quadratic regulator (LQR) framework is still one of the most important open questions in control engineering, despite the availability of many approaches and numerical algorithms (Syrmos et al., 1997; Sadabadi and Peaucelle, 2016). This is mainly due to the lack of testable necessary and sufficient conditions for output-feedback stabilizability. Furthermore, the majority of the algorithms are dependent on the used bilinear/linear matrix inequality (BMI/LMI) solvers, and could work well for small/medium-sized problems, but may fail to converge to a solution or become computationally too heavy as the problem size increases (Peretz, 2018).

Recently, in Ilka et al. (2019) we have shown that within the LQR framework, it is possible to find SOFs in a reasonable time even for large-scale systems. Furthermore, it has been proved that the proposed novel iterative algorithm has a guaranteed convergence to an output-feedback solution from any stabilizing state-feedback solution with necessary and sufficient conditions. The proposed algorithm is computationally much more tractable then algorithms in the literature, including approaches based on LMIs, BMIs, nonlinear programming and ray-shooting methods (Ilka et al., 2019). The introduction of the linear parameter-varying (LPV) systems (Shamma, 2012) has opened new possibilities in LQR design. Several gain-scheduled (GS) and LPV-based LQR design techniques appeared (Veselý and Ilka, 2013, 2015a; Ilka and Veselý, 2017; Veselý and Ilka, 2017). In general, the GS controller design problem for LPV systems becomes an infinite-dimensional problem, since the closed-loop system becomes non-convex in the scheduled parameters, see for example Veselý and Ilka (2017). The non-convexity has been dealt differently, usually by restricting the closed-loop LPV structure, system or controller to avoid cross term effects on the scheduling parameters (Veselý and Ilka, 2015b, 2017).

In this paper we show that by using linear time-invariant process value (PV) and controller output (CO) filters, it is possible to convexify the closed-loop system in the scheduled parameters, and hence simplify the output-feedback GS-LQR design problem. In addition, PV and CO filters are commonly used in practical process control (Douglas, 2006). While PV filters smooth the signal feeding the controller, CO filters smooth the noise (or chatter) in the CO signal sent to the final control element. Even if PV signal noise does not appear to cause performance problems, a CO filter can offer potential benefits as it reduces fluctuations in the controller output and this reduces wear in a mechanical final control element (Douglas et al., 2015). In this paper we propose a computationally efficient iterative Newton-based approach for GS output-feedback LQR design and a simple modification for robust GS controller design. Next, we propose a simple algorithm to check the global stability of the closed-loop system. Finally, the proposed algorithm is applied for air management and fueling strategy of a diesel engine. The mathematical notation of

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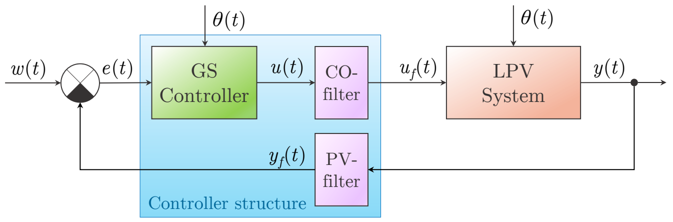


Fig. 1. Closed-loop system with GS controller, PV and CO filters.

the paper is as follows. Given a matrix $C \in \mathbb{R}^{n_y \times n_x}$, its pseudoinverse is denoted by $C^+$. For vectors $a$, $b \in \mathbb{R}^{n_a}$, $a \circ b$ denotes the Hadamard product. Given a symmetric matrix $P = P^T \in \mathbb{R}^{n_x \times n_x}$, the inequality $P > 0$ ($P \geq 0$) denotes the positive definiteness (semi definiteness) of the matrix. The identity and zero matrices are denoted by $I$ and 0. The real part of a complex number $z$ is denoted by $\Re(z)$. Finally, matrices, if not explicitly stated, are assumed to have compatible dimensions.

## 2. PRELIMINARIES AND PROBLEM FORMULATION

Consider an LPV system with GS controller, PV and CO filters, as shown in Fig. 1, where $u(t) \in \mathbb{R}^{n_u}$ is the controller output vector, $y(t) \in \mathbb{R}^{n_y}$ is the measurable output vector, $u_f(t) \in \mathbb{R}^{n_u}$ is the filtered controller output vector, $y_f(t) \in \mathbb{R}^{n_y}$ is the filtered measurable output vector, $w(t) \in \mathbb{R}^{n_y}$ is the reference signal vector, $e(t) = w(t) - y_f(t)$ is the control error vector, and $\theta(t) \in \Theta^{n_\theta}$ is the vector of scheduled parameters.

The continuous-time linear parameter-varying system in this paper is defined as

$$\begin{aligned} \dot{x}(t) &= A(\theta(t))x(t) + B(\theta(t))u_f(t), \\ z(t) &= C_z(\theta(t))x(t) + D_z(\theta(t))u_f(t), \\ y(t) &= C(\theta(t))x(t) + D(\theta(t))u_f(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $z(t) \in \mathbb{R}^{n_z}$ is the performance output vector, and the matrix functions $A(\theta(t))$, $B(\theta(t))$, $C_z(\theta(t))$, $D_z(\theta(t))$, $C(\theta(t))$, and $D(\theta(t))$ belong to a convex set, a polytope with $n_\theta$ vertices that can be formally defined as

$$\begin{aligned} M(\theta) \in \{A(\theta), B(\theta), C_z(\theta), D_z(\theta), C(\theta), D(\theta)\} = \\ \sum_{i=1}^{n_\theta} M_i \theta_i(t), \sum_{i=1}^{n_\theta} \theta_i(t) = 1,\ \theta_i(t) \geq 0, \end{aligned} \tag{2}$$

where $M_i \in \{A_i, B_i, C_{z_i}, D_{z_i}, C_i, D_i\}$ are constant matrices of corresponding dimensions, and $\theta_i(t)$, $i = 1, 2, \ldots, n_\theta$ are constant or time-varying known scheduled parameters.

The gain-scheduled output-feedback control (for $w(t) = 0$) is defined as

$$u(t) = -F(\theta(t))y_f(t), \tag{3}$$

where $F(\theta(t)) \in \mathbb{R}^{n_u \times n_y}$ is the control gain matrix parametrized as (2). Control law (3) is defined in a static output-feedback (SOF) form. However, many different controller structures like proportional-integral (PI), realizable proportional-integral-derivative (PID$_f$), realizable proportional-deriva-tive (PD$_f$), realizable derivative (D$_f$), even full/reduced order dynamic output-feedback controllers (DOF), dynamic output-feedback with integral and realizable derivative part (DOFID$_f$), or dynamic output-feedback with realizable derivative part (DOFD$_f$), can be transformed to this SOF form by augmenting the system with additional state variables (Ilka, 2018).

In this paper we assume that the PV and CO filters are designed a priori and are defined as follows

$$\begin{aligned} \dot{x}_{f_{PV}}(t) &= A_{f_{PV}} x_{f_{PV}}(t) + B_{f_{PV}} y(t), \\ y_f(t) &= C_{f_{PV}} x_{f_{PV}}(t), \end{aligned} \tag{4}$$

$$\begin{aligned} \dot{x}_{f_{CO}}(t) &= A_{f_{CO}} x_{f_{CO}}(t) + B_{f_{CO}} u(t), \\ u_f(t) &= C_{f_{CO}} x_{f_{CO}}(t), \end{aligned} \tag{5}$$

where $x_{f_{PV}}(t)$ and $x_{f_{CO}}(t)$ are the state vectors of the PV and CO filters, and $A_{f_{PV}}$, $B_{f_{PV}}$, $C_{f_{PV}}$, $A_{f_{CO}}$, $B_{f_{CO}}$, and $C_{f_{CO}}$ are known constant matrices with appropriate dimensions.

The main problem studied in this paper is the gain-scheduled output-feedback linear quadratic regulator (GS-OFLQR) design problem for the system (1), with PV and CO filters, which minimizes the quadratic cost function defined as

$$\begin{aligned} J = \frac{1}{2} \int_0^\infty \Big( & \tilde{x}(t)^T Q(\theta(t)) \tilde{x}(t) + u(t)^T R(\theta(t)) u(t) \\ & + 2\tilde{x}(t)^T N(\theta(t)) u(t) \Big) dt, \end{aligned} \tag{6}$$

where $\tilde{x}(t)^T = [x(t)^T, x_{f_{PV}}^T, x_{f_{CO}}^T]$, and $Q(\theta(t))$, $R(\theta(t))$, and $N(\theta(t))$ are parametrized in the same way as system matrices in (2), such that $R(\theta(t)) > 0$ and $Q(\theta(t)) - N(\theta(t))R(\theta(t))^{-1}N(\theta(t))^T \geq 0$.

## 3. GS-OFLQR DESIGN

For controller design, the system (1), control law (3), PV (4) and CO (5) filters, can be transformed to the following form

$$\begin{aligned} \dot{\tilde{x}}(t) &= \tilde{A}(\theta(t))\tilde{x}(t) + \tilde{B}u(t), \\ y_f(t) &= \tilde{C}\tilde{x}, \end{aligned} \tag{7}$$

where

$$\tilde{A}(\theta(t)) = \begin{bmatrix} A(\theta(t)), & 0, & B(\theta(t))C_{f_{CO}} \\ B_{f_{PV}} C(\theta(t)), & A_{f_{PV}}, & B_{f_{PV}} D(\theta(t)) C_{f_{CO}} \\ 0, & 0, & A_{f_{CO}} \end{bmatrix},$$

$$\tilde{B} = [0,\ 0,\ B_{f_{CO}}]^T,\ \tilde{C} = [0,\ I,\ 0].$$

Consequently, the closed-loop system is

$$\dot{\tilde{x}}(t) = \big(\tilde{A}(\theta(t)) - \tilde{B}F(\theta(t))\tilde{C}\big)\tilde{x}(t) = A_{cl}(\theta(t))\tilde{x}(t). \tag{8}$$

Let us recall some related terminology.

*Definition 1.* A square matrix $A \in \mathbb{R}^{n_x \times n_x}$ is said to be *stable* if and only if for every eigenvalues $\lambda_i$ of $A$, $\Re(\lambda_i) \leq 0$.

The next theorem gives sufficient stability conditions for the closed-loop system (8).

*Theorem 1.* The closed-loop system (8) is quadratically stable for all $\theta \in \Omega$ if there exist a positive definite matrix $P \in \mathbb{R}^{\tilde{n}_x \times \tilde{n}_x}$ and matrices $L_j$, $j = 1, 2, \ldots, 6$ of corresponding dimensions, such that the following LMIs hold

$$M_i = \begin{bmatrix} M_{i_{11}}, & M_{i_{12}}, & M_{i_{13}} \\ M_{i_{12}}^T, & M_{i_{22}}, & M_{i_{23}} \\ M_{i_{13}}^T, & M_{i_{23}}^T, & M_{i_{33}} \end{bmatrix} \prec 0,\ i = 1, 2, \ldots, n_\theta, \tag{9}$$

where

$$
\begin{aligned}
M_{i_{11}} &= L_1^T + L_1, \quad M_{i_{12}} = P - L_1^T \tilde{A}_i + L_2 - L_4^T F_i \tilde{C}, \\
M_{i_{13}} &= L_3 + L_4^T - L_1^T \tilde{B}, \\
M_{i_{22}} &= -L_2^T \tilde{A}_i - \tilde{A}_i^T L_2 - L_5^T F_i \tilde{C} - \tilde{C}^T F_i^T L_5, \\
M_{i_{23}} &= L_5^T - L_2^T \tilde{B} - \tilde{A}_i^T L_3 - C^T F_i^T L_6, \\
M_{i_{33}} &= L_6^T + L_6 - L_3^T \tilde{B} - \tilde{B}^T L_3.
\end{aligned}
$$

*Proof 1.* Consider the Lyapunov function candidate as

$$V(t) = \tilde{x}(t)^T P \tilde{x}(t). \tag{10}$$

The first derivative of the Lyapunov function (10) is then

$$
\begin{aligned}
\dot{V}(t) &= \dot{\tilde{x}}(t)^T P \tilde{x}(t) + \tilde{x}(t)^T P \dot{\tilde{x}}(t) \\
&= v(t)^T \begin{bmatrix} 0, & P, & 0 \\ P, & 0, & 0 \\ 0, & 0, & 0 \end{bmatrix} v(t),
\end{aligned} \tag{11}
$$

where $v(t)^T = [\dot{\tilde{x}}(t)^T,\ \tilde{x}(t)^T,\ u(t)^T]$. To separate the Lyapunov matrix $P$ from the system's matrices the auxiliary matrices $L_j$, $j = 1, 2, \ldots, 6$ of corresponding dimensions are used in the following form

$$
\begin{aligned}
2(L_1 \dot{\tilde{x}}(t) + &L_2 \tilde{x}(t) + L_3 u(t))^T \\
&(\dot{\tilde{x}}(t) - \tilde{A}(\theta(t))\tilde{x}(t) - \tilde{B}u(t)) = 0,
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
2(L_4 \dot{\tilde{x}}(t) + L_5 \tilde{x}(t) + L_6 u(t))^T& \\
(u(t) - F(\theta(t))\tilde{C}\tilde{x}(t)) &= 0,
\end{aligned} \tag{13}
$$

Summarizing equations (12) and (13) with the time derivative of the Lyapunov function (11) we can write

$$v(t)^T M(\theta(t)) v(t) \leq -\epsilon v(t)^T v(t), \ \epsilon \geq 0, \tag{14}$$

which implies that the closed-loop system is stable for some $\epsilon \geq 0$ if $P \succ 0$.

From inequality (14) for $\epsilon \to 0$ we can obtain

$$M(\theta(t)) \prec 0, \tag{15}$$

where $M(\theta(t))$ is symmetric and convex in $\theta$, therefore it is enough to check the definiteness at the vertices of $\theta$, i.e. we get the LMIs (9) which completes the proof.

In this paper, we propose a systematic procedure by splitting the whole optimization problem into $n_\theta$ sub-optimization problems, i.e. to design local controllers at the vertices of $\theta$, for which we propose to use our recent modified Newton's method introduced in Ilka et al. (2019). The modified Newton's method for SOF controller design, adopted from Ilka et al. (2019), for the $i$-th vertex of $\theta(t)$ is summarized in Algorithm 1. For convergence proof and more detailed explanation please see Ilka et al. (2019). Then the results of these sub-optimization problems are united to get the sub-optimal gain-scheduled output-feedback controller. The overall stability is then tested with the introduced stability criteria. The algorithm for gain-scheduled output-feedback LQR design can be summarized as Algorithm 2.

*Remark 1.* If system (7) is stabilizable and $(\hat{A}, \hat{C}_q)$, where $\hat{Q} = \hat{C}_q^T \hat{C}_q$ is a full-rank factorisation of $\hat{Q}$, is detectable, then the standard state-feedback LQR solution of (7) for some $\hat{Q} \geq 0$ always gives a Lyapunov matrix $P_1$ for which $\hat{A} - \hat{S}P_1$ is stable in Algorithm 1.

**Algorithm 1:** Modified Newton's method for SOF controller design for the $i$-th vertex of $\theta(t)$.

1 Define $\hat{A} = \tilde{A}_i - \tilde{B}R_i^{-1}N_i^T$, $\hat{Q} = Q_i - N_i R_i^{-1} N_i^T$ and $\hat{S} = \tilde{B}R_i^{-1}\tilde{B}^T$, and choose an initial guess $P_1 = P_1^T$ such that $\hat{A} - \hat{S}P_1$ is stable (such $P_1$ can be obtained via the standard state-feedback LQR design, see Remark 1).
2 **for** $j = 1 : \mathrm{max\_iteration}$ **do**
3    $F_{ij} = R_i^{-1}(\tilde{B}^T P_j + N_i^T)\tilde{C}^+$,
4    $G_j = F_{ij}\tilde{C} - R_i^{-1}(\tilde{B}^T P_j + N_i^T)$,
5    $\mathcal{R}(P_j) = \hat{Q} + G_j^T R_i G_j + \hat{A}^T P_j + P_j \hat{A} - P_j \hat{S} P_j$,
6    **if** $\mathrm{trace}(\mathcal{R}(P_j)^T \mathcal{R}(P_j)) > \epsilon$ **then**
7      $X_j \leftarrow (\hat{A} - \hat{S}P_j)^T X_j + X_j(\hat{A} - \hat{S}P_j) = -\mathcal{R}(P_j)$,
8      $P_{j+1} = P_j + X_j$,
9    **else**
10      **break;**
11    **end**
12 **end**

**Algorithm 2:** Gain-scheduled OFLQR design.

1 **for** $i = 1 : n_\theta$ **do**
2    $F_i \leftarrow$ by Algorithm 1 for $\tilde{A}_i$, $\tilde{B}$, $\tilde{C}$, $Q_i$, $R_i$ and $N_i$,
3 **end**
4 $F(\theta(t)) = \sum_{i=1}^{n_\theta} F_i \theta_i$,
5 $A_{cl}(\theta(t)) = \tilde{A}(\theta(t)) - \tilde{B}F(\theta(t))\tilde{C}$,
6 **if** $A_{cl}(\theta(t))$ *is stable by Theorem 1* **then**
7    The controller design was successful.
8 **else**
9    The algorithm failed. Try to increase the robustness properties of the local controllers and go to step 1.
10 **end**

### 3.1 Robust GS-OFLQR design

The idea of transforming the robust control problem to an optimal control problem is not new. The former idea has already been explored for robust state-feedback controller design in Feng and Brandt (1998), and Feng (2000) for matched and mis-matched uncertainties. Furthermore, in Tripathy et al. (2016) this idea has been further improved by allowing polytopic uncertainty for a restrictive state-feedback case. Since in Ilka et al. (2019) the SOF controller design problem has been transformed to a problem of solving a Riccati-like quadratic matrix equation, the introduced techniques for standard state-feedback design, i.e. for standard matrix Riccati equation in Feng and Brandt (1998), Feng (2000) and Tripathy et al. (2016), can be implemented directly. Furthermore, since the algorithm from Ilka et al. (2019) belong to the LQR framework, all the well-known modifications and/or extensions of the standard LQR design can be applied here as well. Therefore, one can apply the techniques in Misra (1996) for different eigenvalue placements (pole-placement techniques in LQ). Hence, for example an identification error on the state vector as

$$x(t) = x_r(t) \pm \Delta_x \circ x_r(t), \tag{16}$$

can be compensated by shifting the poles by $\Delta_x$ by modifying the system (7) as

$$\dot{\tilde{x}}(t) = \big(\tilde{A}(\theta(t)) + \text{diag}(\Delta_x)\big)\tilde{x}(t) + \tilde{B}\tilde{u}(t)$$
$$\tilde{y}(t) = \tilde{C}\tilde{x}. \tag{17}$$

### *3.2 Guidelines for robust GS-OFLQR design*

Besides affine, piece-wise affine or direct LPV modeling, Algorithm 2 is especially effective for grid-based LPV models. Furthermore, grid-based LPV identification and modeling is supported by the Control System Toolbox, Matlab/Simulink as well (The Mathworks, Inc., 2017). The following steps can serve as general guidelines for grid-based LPV systems

(1) Analyze the nonlinear system and select the measurable or estimable outputs which can be used as scheduled parameters.
(2) Identify the nonlinear plant along the trajectory of these scheduled parameters, (e.g. by using the Control System Toolbox in Matlab).
(3) Validate the identified grid-based LPV model, and determine the uncertainties (as matched or mis-matched uncertainties, or as maximal percentage errors for states/outputs).
(4) Use the Bryson rules (Bryson and Ho, 1975) to initialize the weighting matrices.
(5) Design a robust GS-OFLQR by Algorithm 2 with compensation for the identification error based on Feng and Brandt (1998), Feng (2000), Tripathy et al. (2016) or Misra (1996).
(6) Check the global stability of the closed-loop system with Theorem 1, or with any other approach available in the literature.
(7) Fine-tune the controller using the weighting matrices.

## 4. AIR MANAGEMENT AND FUELING STRATEGY

One of the important problems within the automotive industry is the air management and fueling strategy (AMFS) of an internal combustion engine. In this paper we consider a diesel engine with variable geometry turbine (VGT) and exhaust gas recirculation (EGR), as illustrated in Fig. 2.

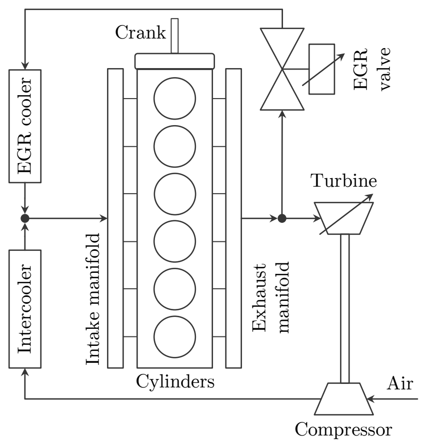


Fig. 2. Air path system of diesel engine. The engine includes an exhaust gas recirculation (EGR) and a variable geometry turbine (VGT).

Many approaches exist in literature for controlling such engines, whilst among the most prominent are the GS and/or the LPV control techniques (Wei and del Re, 2007; Wei et al., 2008; Liu et al., 2008; Alfieri et al., 2009; Hong et al., 2015; Buenaventura et al., 2015; Hong et al., 2017; Park et al., 2017). The local controllers for VGT and EGR are generally designed to follow prescribed set-points for intake manifold pressure, air mass flow and/or air fraction. A different approach, proposed in our recent paper Ilka et al. (2019), incorporates a multi-layer control structure (MLCS), where the upper layers are relived of the fast dynamics (intake/exhaust manifold pressures, turbine speed etc.) and generate reference torque to be tracked by the local controller. Therefore, a new low-level GS controller has been proposed with the objective to 1) follow the reference torque, while 2) minimizing fuel consumption, and 3) fulfilling constraints on inputs, states and outputs. Furthermore, the EGR control has not been considered as part of the AMFS, since for the studied system EGR is controlled by the EATS controller, which carries the main responsibility of limiting $NO_x$ emissions. In addition, in Ilka et al. (2019), the GS controller for AMFS has been designed by a heuristic time-consuming LMI-based design procedure that required closed-loop stability to be tested afterwards. This has made the controller tuning unintuitive, time-consuming and not straightforward.

With the proposed algorithm in this paper, the controller design is simplified, and a stabilizing sub-optimal GS controller for the AMFS is obtained quickly, in a straightforward way.

### *4.1 LPV modeling of AMFS*

The air path system of diesel engine with VGT can be represented in a polytopic LPV form,

$$\begin{aligned} \dot{x}(t) &= A(\theta(t))x(t) + B(\theta(t))u(t), \\ z(t) &= C_z(\theta(t))x(t) + D_z(\theta(t))u(t), \\ y(t) &= Cx(t), \end{aligned} \tag{18}$$

where $x(t) = [p_{im}(t), p_{em}(t), \omega_t(t)]^T \in \mathbb{R}^{n_x}$, where $p_{im}(t)$ (Pa) and $p_{em}(t)$ (Pa) are the input and exhaust manifold pressures, and $\omega_t(t)$ (rpm) is the turbine speed; $u(t) = [u_\delta(t), u_{vgt}(t)]^T \in \mathbb{R}^{n_u}$, where $u_\delta(t)$ (mg/stroke) is the fuel injection, and $u_{vgt}(t)$ (%) is the VGT position; $y(t) = M_e(t) \in \mathbb{R}^{n_y}$, where $M_e(t)$ (Nm) is the engine torque; and $z(t) = [W_f(t), 1/\lambda(t)]^T \in \mathbb{R}^{n_z}$ is the performance output vector, where $\lambda(t)$ (-) is the air-to-fuel ratio, and $W_f(t)$ (kg/s) is the fuel flow into the cylinders,

$$W_f(t) = n_{cyl} u_\delta(t) n_e(t), \tag{19}$$

where $n_{cyl}$ is a known constant and $n_e(t)$ (rpm) is the engine speed. The matrix functions $A(\theta(t))$, $B(\theta(t))$, $C_z(\theta(t))$, and $D_z(\theta(t))$ belong to a convex set, a polytope with $n_\theta$ vertices parametrized as in (2), where $A_i$, $B_i$, $C_{z_i}$, $D_{z_i}$, and $C$ are constant matrices of corresponding dimensions, and $\theta_i(t) \in \Theta$, $i = 1, 2, \ldots, n_{wp}$ are constant or time-varying known parameters calculated from the measurable parameters (engine torque $M_e(t)$, engine speed $n_e(t)$, and VGT position $u_{vgt}(t)$).

In this paper the LPV model is adopted from Ilka et al. (2019), which has been obtained by the *grid-based* LPV identification and modeling technique using the Control System Toolbox in Matlab. The LPV model in the form of (18) has been obtained by reducing the nine-state benchmark model, published by Eriksson et al. (2016), to the three slowest states. The reduced model is then linearized about 330 working (grid) points along the trajectory generated by engine torque $M_e(t) \in [0, 1800]$ Nm, engine speed $n_e(t) \in [400, 2000]$ rpm and VGT position $u_{vgt}(t) \in [20, 100]$ %. For more information, details and explanation about the LPV model, please see Ilka et al. (2019).

### 4.2 Robust GS-OFLQR design for AMFS

We propose a gain-scheduled PID$_f$ controller in the form

$$\begin{aligned} u(t) =& K_P(\theta(t))e(t) + K_I(\theta(t)) \int_0^t e(\tau)d\tau \\ &+ K_D(\theta(t))e_{D_f}(t), \end{aligned} \tag{20}$$

where $e(t) = y(t) - w(t)$, wherein $w(t)$ is the reference signal (in our case the reference torque $M_{e_{ref}}$). Furthermore, $K_P(\theta(t))$, $K_I(\theta(t))$, and $K_D(\theta(t))$ are the proportional, integral, and derivative gains parametrized as in (2), and $e_{D_f}(t)$ is derivative error filtered by

$$G_f(s) = \frac{c_f s}{s + c_f}, \tag{21}$$

where $c_f$ is a filtering coefficient. For the GS-OFLQR design the weighting matrices can be chosen as

$$\begin{aligned} Q(\theta(t)) &= Q_x(\theta(t)) + C_z(\theta(t))^T Q_z C_z(\theta(t)), \\ R(\theta(t)) &= Q_u(\theta(t)) + D_z(\theta(t))^T Q_z D_z(\theta(t)), \\ N(\theta(t)) &= Q_{xu}(\theta(t)) + C_z(\theta(t))^T Q_z D_z(\theta(t)), \end{aligned}$$

where $Q_x(\theta(t))$, $Q_u(\theta(t))$, $Q_{xu}(\theta(t))$, and $Q_z \in \mathbb{R}^{n_z \times n_z}$ can be used to tune the closed-loop system in order to balance the trade-off between the reference tracking, fuel consumption and emissions (particulates), as well as to fulfill the input/output/state constraints.

Since the reference signal $w(t)$ (reference engine torque) is bounded, we can simplify the derivation by setting $w(t) = 0$. Then, by augmenting the system (18) with additional state variables such as the integral of the measurable output $y_I(t) = \int_0^t y(\tau)d\tau$, and the filtered derivative output $y_{D_f}$ using the derivative filter (21), the control law (20) can be transformed to

$$u(t) = F(\theta)\overline{y}(t) = F(\theta(t))\overline{C}(\theta(t))\overline{x}(t), \tag{22}$$

where $\overline{y}(t)^T = [y(t)^T,\ y_I(t)^T,\ y_{D_f}(t)^T]$ is the augmented output vector and $\overline{x}(t)^T = [x(t)^T,\ y_I(t)^T,\ y_{D_f}(t)^T]$ is the augmented state vector. The augmented system is then

$$\begin{aligned} \dot{\overline{x}}(t) &= \overline{A}(\theta(t))\overline{x}(t) + \overline{B}(\theta(t))u(t), \\ \overline{y}(t) &= \overline{C}\overline{x}(t), \end{aligned} \tag{23}$$

where

$$\overline{A}(\theta(t)) = \begin{bmatrix} A(\theta(t)), & 0, & 0 \\ C, & 0, & 0 \\ B_f C, & 0, & A_f \end{bmatrix}, \overline{B}(\theta(t)) = \begin{bmatrix} B(\theta(t)) \\ 0 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} C, & 0, & 0 \\ 0, & I, & 0 \\ B_f C, & 0, & A_f \end{bmatrix}, B_f = c_f I,\ A_f = -c_f I.$$

Finally, robust GS-OFLQR PID$_f$ controller is designed for AMFS using the Algorithm 2 and (23) as the initial system with $\Delta_x^T = [0.075,\ 0.065,\ 0.0375]$, calculated from max absolute percentage errors for the states (Ilka et al., 2019).

Table 1. Reference tracking properties of the closed-loop system.

| *Attribute* | *Value* |
|---|---|
| Maximal settling-time: | 0.4 s |
| Overshoot: | - |
| Steady-state error: | - |
| Reference tracking MAPE: | 0.0021 % |
| Reference tracking max(APE): | 0.3584 % |

A first order filter with time constant $T_{f_{PVCO}} = 0.001$ s has been used for PV and CO. The design parameters are the filter coefficient $c_f = 100$, weighting matrices $Q_x = \mathrm{diag}([0, 0, 0, 60, 10^4, 10^2, 0, 0])$, $Q_u = \mathrm{diag}([3 \times 10^{-3}, 1])$, $Q_{xu} = 0$ and $Q_z = \mathrm{diag}([1, 5 \times 10^5])$. The weighting coefficients in $Q_x$, related to the PID$_f$ controller (proportional $Q_{x_{4,4}}$, integral $Q_{x_{5,5}}$ and derivative $Q_{x_{6,6}}$) are selected to obtain a reference tracking performance with zero steady state error, no overshoot, and settling-time less then 0.5 s. The weighting coefficients in $Q_z$ are chosen to balance the trade-off between fuel consumption and emitted particulates, by minimizing $1/\lambda$. Finally, the closed-loop stability has been checked by Theorem 1 using MOSEK LMI solver (Mosek Aps, 2019).

## 5. SIMULATION EXPERIMENT

To evaluate the obtained GS controller the road profile Söderälje-Norrköping and the benchmark model from Eriksson et al. (2016) have been used. For the simulation, the reference torque $M_{e_{ref}}(t)$, engine speed $n_e(t)$ and gears are given by the supervisory layers. Simulation results are given in Fig. 3, where beside the engine torque and speed ($M_e(t)$ and $n_e(t)$), inverse air-to-fuel ratio ($1/\lambda(t)$), turbine speed ($n_t(t)$), VGT position ($u_{vgt}(t)$), fuel injection ($u_\delta(t)$), gear, road slope and altitude are shown as well.

Fig. 4 and Table 1 show that the objectives for reference tracking are fulfilled. The total fuel mass for the whole driving cycle is $M_f = 32.7788$ kg with the proposed controller, which is less (up to 3.54 %) compared to total fuel masses obtained with fixed VGT positions fulfilling the constraint on the air-to-fuel ratio.

The obtained results are very similar to the results in Ilka et al. (2019), which was expected. The only main difference is the controller design itself. With the proposed approach the whole GS-OFLQR design was less then 3.5 s, while with the other approach more then 16 hours.

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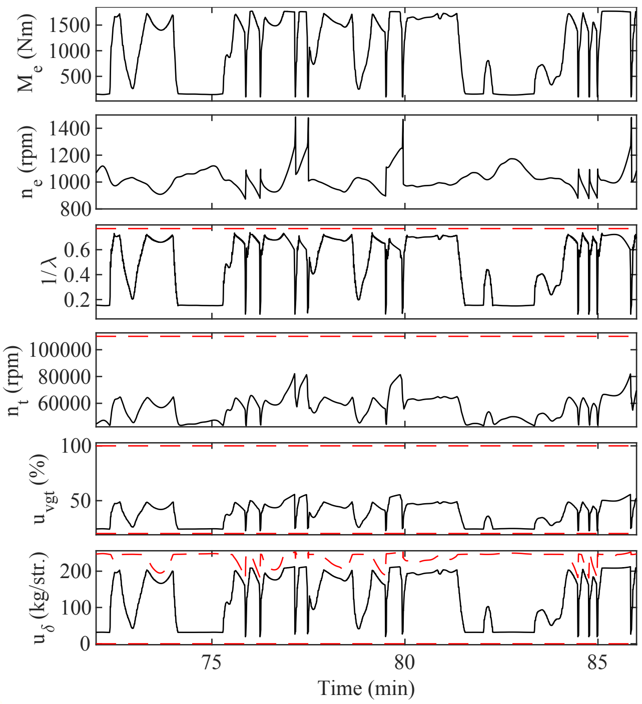


Fig. 3. Simulation results with the proposed controller (15 min slice from the 91 min driving cycle). The road profile Söderälje-Norrköping has been used to evaluate the proposed gain-scheduled controller. The red dashed lines are the constraints to be fulfilled.

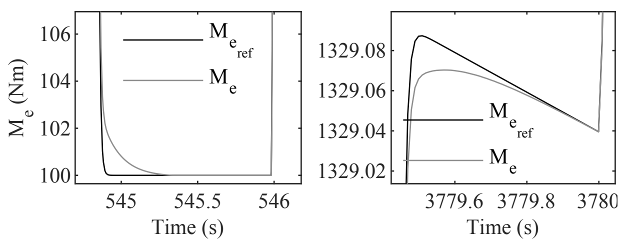


Fig. 4. Zoomed slices from the driving cycle which shows the reference tracking properties of the closed-loop system.

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