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A Nonlinear Control Design Strategy for Piloting Fixed Wing Aircrafts

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Abstract: This paper proposes a novel nonlinear feedback control strategy for velocity and attitude control of fixed wing aircrafts. The key feature of the control design strategy is the introduction of a virtual control input in order to deal with the underactuation property of such vehicles and to indirectly control the orientaion of the aircraft. As such, the proposed strategy consists of three control loops each realising a specific task. Simulation results on Jetstream-3102 aircraft show very good performance in terms of convergence towards the desired reference trajectories and in terms of robustness with respect to modeling uncertainties.

Keywords: Attitude and speed controller, nonlinear control, aerodynamic flight, autopilot.

1. INTRODUCTION

Defining proper control strategies for aircraft control is essential to avoid fatal accidents. The most common fatal accidents are: loss of control inflight, controlled flight into terrain and runway excursion during approach and landing. For this reason, it is imperative to adopt a proper control strategy of attitude and speed of aircrafts Stevens *et al.* (2015). In general, when designing a controller, the control system designer usually based its design on the mathematical model of the aircraft and used appropriate mathematical tools to demonstrate the convergence or robustness properties of the controller. In this regards, many control design strategies has been proposed for attitude and speed in the presence of both modelled and unmodelled uncertainties and by using different methods such as linear quadratic control Stevens *et al.* (2015), feedback linearization technique Meyer *et al.* (1984), Lane and Stengel (1988), eigenstructure assignment Smith (1990), Farineau (1989), H_∞ robust control Safonov (1980), (n.d.), dynamic inversion Luo *et al.* (2011), backstepping Härkegård and Glad (2001) and sliding mode control, to mention a few. Traditional flight control systems use PID control with scheduled gains. It is well-known that this approach is very important and convenient for conventional aircrafts of the second and third generation. However, gain scheduling suffers from the inherent deficiency of relying on time-invariant linear models based on small perturbations of the full nonlinear aircraft model at a particular point in the flight envelope. In addition, the dynamic properties deteriorate when the scheduling parameters, such as speed and pitch angle, change rapidly over the small time intervals (see eg Luo *et al.* (2011)). However, it is not always judicious to choose one control design method over another based solely on the mathematical description of the system as it might not make practical sense. In effect, in Stevens *et al.* (2015) it is shown that it is important to consider the aircraft as an energy system, whereby the energy gain or loss is distributed into the kinetic and the potential energy of the aircraft. This translates into the fact that it is not possible to control the thrust independently to the rudder, elevator and the aileron deflections. Also, it is important to realise that aircraft systems are underactuated systems. This means that there are less actuators than the degrees of freedom, meaning that it is not always possible to control some variables of the aircraft directly using the available actuators. Indeed, the aircraft is a 12 order system with 4 actuators. The 12 state variables are:

- $X = (x, y, z)^T \in R^3$ which is the aircraft position expressed in the earth fixed reference frame R_E;
- $W = (u, v, w)^T \in R^3$ is the inertial speed vector expressed in the body reference frame R_B;
- $\Phi = (\phi, \theta, \psi)^T \in R^3$ which are the Euler angles describing the orientation of the aircraft relative to R_E;
- $\Omega = (p, q, r)^T \in R^3$ the angular velocity of the aircraft expressed in the body fixed reference frame

The 4 control variables are:

- $U = (\delta_a, \delta_e, \delta_r)^T \in R^3$ where $\delta_a, \delta_e, \delta_r$, are the aileron, elevator and rudder deflections respectively and
- F_T which is the thrust force due to the propulsion system.

The dynamics of the angular velocity Ω and the inertial speed vector W are directly affected by the control U and F_T, while the position X and the orientation Φ are not. Instead, X and Φ are coupled with W and Ω respectively. On the other hand, since 4 inputs variables can only

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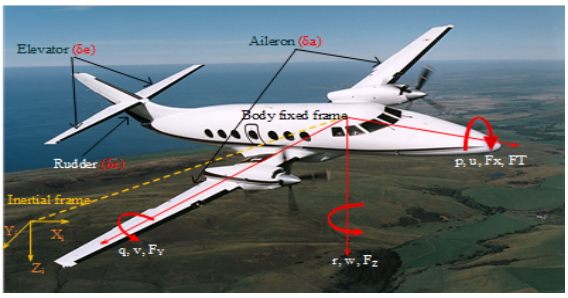

Fig. 1. Jetstream-3102 aircraft.

control 4 states variables, one must find judicious ways to affect and indirectly control the rest of the state variables. In this paper, we propose a novel strategy for velocity and attitude control of an aircraft based on the above considerations. For this, we first reduce the model of the aircraft by considering the norm of the inertial speed $V = \|W\|^2 = W^T W$ rather than its individual components. In that case, the model is reduced to a systems of 10 variables. The input variable F_T is used to track a reference speed trajectory V_{ref}. Next a virtual control Ω_v is introduced to indirectly control the orientation Φ towards a desired reference trajectory Φ_{ref}. This is a crucial point in the design strategy since Φ is not directly affected by any real actuators as mentioned above. The input U is then used to steer Ω towards a desired reference trajectory Ω_{ref}. Another key point in the design strategy is that the reference angular velocity Ω_{ref} is chosen in such a way that it permit the virtual control input to track the desired orientation. The derived controller U is dependent on the control F_T ensuring indirectly a natural distribution between the kinetic and potential energy of the aircraft. As such, 3 control loops are designed each realising a specific control objective. Furthermore, the three control loop are design so as to make sure that the aircraft does not stall. This is measured by ensuring that $\|\dot{X}\|$ remain bounded. The performance of the proposed control design is evaluated using a model of a fixed wing aircraft (Jetstream 3102). This paper is organized as follows: in the next section, the dynamic model of a fixed wing aircraft attitude is presented. In Section 3, the autopilot design methodology is presented. In Section 4, the performance of the proposed design methodology is shown via simulation using a Jetstream-3102 aircraft. Finally, some conclusions are drawn in Section 5.

Notations: $C_\theta = \cos(\theta)$, $S_\theta = \sin(\theta)$; $T_\theta = \tan(\theta)$.

2. AIRCRAFT AERODYNAMICAL MODEL

In what follows, a brief description of the main features of the aircraft is introduced as well as its dynamic model. The system structure is presented and a model reduction is performed in order to facilitate the development of a speed and attitude control design. The considered aircraft is a British Aerospace (Jetstream-3102), which is a fixed wing, twin turboprop aircraft as illustrated in Figure 1. This type of aircraft has as control inputs the throttle setting command (δ_{th}), and the deflection angles of the three control surfaces: elevator (δ_e), ailerons (δ_a), and rudder (δ_r) (see Figure 1). We consider this particular aircraft due to the fact that all its aerodynamic coefficients are available in the literature. For more details, the reader may refer to Jameson (2013),Cooke (2006). The wing surface area $s = 280ft^2$, the wingspan $b = 46ft$, the mean aerodynamic chord $\bar{c} = 6.5ft$, and the mass $m = 6890lbs$ of the aircraft are considered to be constant. The forces F and moments M_G acting on the aircraft at the center of gravity are issued from three major sources: gravity (F_G), engine thrust (F_E) and aerodynamic forces (F_A); that is

$$F = F_G + F_E + F_A \tag{1}$$

$$M_G = M_E + M_A \tag{2}$$

The gravitational force F_G is directed along the normal of the earth plane and is considered constant over the attitude envelope. More precisely $F_G = mg\zeta$, where $\zeta = (-S_\theta \; S_\phi C_\theta \; C_\phi C_\theta)^T$ and g is the acceleration due to gravity. We assume that the engines are positioned in such a way that the thrust force due to the propulsion system F_E acts parallel to the x-axis of the aircraft body, that is $F_E = (F_T, 0, 0)^T$. Using Newton-Euler convention, in the body-fixed reference frame the force and aerodynamic moments is given as:

$$F = m\frac{dW}{dt} + \Omega \times W \tag{3}$$

$$M_G = \frac{d(I_G\Omega)}{dt} + \Omega \times I_G\Omega \tag{4}$$

where I_G is the moment of inertia, $\Omega = (p, q, r)^T$ is the angular velocity of the aircraft and $W = (u, v, w)^T$ is the inertial speed vector of the aircraft center of gravity of the aircraft. The aerodynamic force $F_A = (F_x, F_y, F_z)^T = p_a s(C_x, C_y, C_z)^T$ where $p_a = \frac{1}{2}\rho V$ is the aerodynamic pressure with ρ being the ambient air density and $V = W^T W$ aircraft velocity and C_x, C_y, C_z are the aerodynamic coefficients given as Jameson (2013), Cooke (2006):

$$\begin{cases} C_x = C_{x,0} + C_{x,1}\alpha + C_{x,2}\alpha^2 + C_{x,3}q\dfrac{\bar{c}}{\sqrt{V}} \\ \quad +C_{x,4}\delta_a + C_{x,5}\delta_e + C_{x,6}\delta_r + C_{x,7}F_T \\ C_y = C_{y,0} + C_{y,1}\beta + C_{y,2}\beta^2 + C_{y,3}p\dfrac{b}{2\sqrt{V}} \\ \quad +C_{y,4}\delta_a + C_{y,5}\delta_e + C_{y,6}\delta_r + C_{y,7}F_T \\ C_z = C_{z,0} + C_{z,1}\alpha + C_{z,2}\alpha^2 + C_{z,3}q\dfrac{\bar{c}}{\sqrt{V}} \\ \quad +C_{z,4}\delta_a + C_{z,5}\delta_e + C_{z,6}\delta_r + C_{z,7}F_T \end{cases} \tag{5}$$

Note that the above coefficients are given up to a second order Taylor approximation in the side-slip angles and up to a first order Taylor approximation in the control inputs. Now as the influence of the of δ_a and δ_r are minimal in the x-direction, we assume that $C_{x,4} = C_{x,6} = 0$. By a similar reasoning and taking into account physical structural consideration, we have $C_{y,2} = C_{y,5} = C_{z,2} = C_{z,4} = C_{z,6} = 0$. Hence, the above expression simplifies into:

$$\begin{cases} C_x = C_{x,0} + C_{x,1}\alpha + C_{x,2}\alpha^2 + C_{x,3}q\dfrac{\bar{c}}{\sqrt{V}} \\ \quad +C_{x,5}\delta_e + C_{x,7}F_T \\ C_y = C_{y,0} + C_{y,1}\beta + C_{y,3}p\dfrac{b}{2\sqrt{V}} + C_{y,4}\delta_a \\ \quad +C_{y,6}\delta_r + C_{y,7}F_T \\ C_z = C_{z,0} + C_{z,1}\alpha + C_{z,3}q\dfrac{\bar{c}}{2\sqrt{V}} + C_{z,5}\delta_e + C_{z,7}F_T \end{cases} \tag{6}$$

From equations (1), (2) and (3), the dynamics of the inertial speed, W, is given by

$$\dot{W} = \mathbf{R}_1(\Omega)W + g\zeta + \frac{1}{m}F_A + \frac{1}{m}B_0F_T \tag{7}$$

$$\dot{W} = \mathbf{R}_1(\Omega)W + \Psi + \mathbf{B}_2U + \frac{1}{m}B_3F_T \tag{8}$$

Where $\Psi = g\zeta + \tilde{\Psi}$, with $U = (\delta_a, \delta_e, \delta_r)^T$, $\Psi = g\zeta + \tilde{\Psi}$

$$\tilde{\Psi} = \frac{\rho s V}{2m}\begin{bmatrix} C_{x,0} + C_{x,1}\alpha + C_{x,2}\alpha^2 + C_{x,3}q\dfrac{\bar{c}}{\sqrt{V}} \\ C_{y,0} + C_{y,1}\beta + C_{y,3}\dfrac{pb}{2\sqrt{V}} \\ C_{z,0} + C_{z,1}\alpha + C_{z,2}q\dfrac{\bar{c}}{\sqrt{V}} \end{bmatrix}$$

$$\mathbf{R}_1(\Omega) = \begin{pmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{pmatrix}, B_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{B}_2 = \frac{1}{2}\rho s V\begin{pmatrix} 0 & C_{x,4} & 0 \\ C_{y,3} & 0 & C_{y,4} \\ 0 & C_{z,3} & 0 \end{pmatrix}, B_3 = \frac{1}{2}\rho s V\begin{pmatrix} C_{x,5}+1 \\ C_{y,5} \\ C_{z,4} \end{pmatrix}$$

In other words,

$$F_A = m\tilde{\Psi} + m\mathbf{B}_2U + m\left(B_3 - B_0\right)F_T \tag{9}$$

The propulsive forces can also create moments if the thrust does not act through the aircraft center of gravity. We assume the engine is mounted in such a way that the thrust point lies in the body axes xz-plane and offsetted from the center of gravity by Z_{TP} in the body-axes z-direction so that $M_E = (0, F_TZ_{TP}, 0)^T$. The moments caused by aerodynamic forces M_A and aerodynamic moments coefficients are given by:

$$M_A = \frac{1}{2}\rho s V(bC_l, \bar{c}C_m, bC_n)^T$$

where C_l, C_m and C_n are given by a first order Taylor approximation in the various variables involved the aerodynamic moments coefficients as well as taking in account the physical constraints:

$$\begin{cases} C_l = C_{l,1}\beta + \dfrac{b}{2\sqrt{V}}(C_{l,2}p + C_{l,3}r) + C_{l,4}\delta_a + C_{l,5}\delta_r \\ C_m = C_{m,0} + C_{m1}\alpha + \dfrac{\bar{c}}{2\sqrt{V}}(C_{m,2}\dot{\alpha} + qC_{m,3}) + C_{m,4}\delta_e \\ C_n = C_{n,1}\beta + \dfrac{b}{2\sqrt{V}}(rC_{n,2} + pC_{n,3}) + C_{n,4}\delta_a + C_{n,5}\delta_r \end{cases} \tag{10}$$

It is important to note that the above two Taylor approximations in the aerodynamic moment and coefficients leads to modeling errors and uncertainties on the systems parameters. Consequently, we get

$$\dot{\Omega} = \gamma(\Omega) + \frac{1}{2}\rho s V\mathbf{P}_1\Pi\left(\Omega, W\right) + \frac{1}{2}\rho s V\mathbf{P}_1\mathbf{B}_1U + P_2F_T \tag{11}$$

where:

$$\gamma(\Omega) = \begin{pmatrix} q(a_1p + a_2r) \\ a_5pr - a_6(p^2 - r^2) \\ q(a_8p - a_1r) \end{pmatrix}, \mathbf{P}_1 = \begin{pmatrix} a_3 & a_4 & 0 \\ 0 & 0 & a_7 \\ a_4 & a_9 & 0 \end{pmatrix}$$

$$\Pi = \begin{bmatrix} b\left(C_{l,1}\beta + \dfrac{b}{2\sqrt{V}}(C_{l,2}p + C_{l,3}r)\right) \\ \bar{c}\left(C_{n,1}\beta + \dfrac{b}{2\sqrt{V}}(rC_{n,2} + pC_{n,3})\right) \\ b\left(C_{m,0} + C_{m,1}\alpha + \dfrac{\bar{c}}{2\sqrt{V}}(C_{m,2}\dot{\alpha} + qC_{m,3})\right) \end{bmatrix}$$

$$P_2 = \begin{pmatrix} 0 \\ a_7Z_{TP} \\ 0 \end{pmatrix}, \mathbf{B}_1 = \begin{pmatrix} bC_{l,4} & 0 & bC_{l,5} \\ \bar{c}C_{n,4} & 0 & \bar{c}C_{n,5} \\ 0 & bC_{m,4} & 0 \end{pmatrix}$$

The dynamics of the aircraft position, $X = (x, y, z)^T$, is given by:

$$\dot{X} = \mathbf{R}_0(\Phi)W \tag{12}$$

with

$$\mathbf{R}_0(\Phi) = \begin{pmatrix} C_\theta C_\psi & S_\phi S_\theta C_\psi - C_\phi S_\psi & C_\phi S_\theta C_\psi + S_\phi S_\psi \\ C_\theta S_\psi & S_\phi S_\theta S_\psi + C_\phi C_\psi & C_\phi S_\theta S_\psi - S_\phi C_\psi \\ -S_\theta & S_\phi C_\theta & C_\theta \end{pmatrix}$$

In the earth fixed reference frame, the rotational velocity is described by the variables $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$. However, in the body-fixed frame, the rotational velocity is described by roll p, pitch q and yaw rates, r. The relation between those two sets of variables are given by:

$$\dot{\Phi} = \mathbf{\Gamma}\left(\Phi\right)\Omega \tag{13}$$

and

$$\mathbf{\Gamma}\left(\Phi\right) = \begin{pmatrix} 1 & T_\theta S_\phi & T_\theta C_\phi \\ 0 & C_\phi & -S_\phi \\ 0 & \dfrac{S_\phi}{C_\theta} & \dfrac{C_\phi}{C_\theta} \end{pmatrix}$$

In summary, the dynamical behavior of the aircraft model using Newton–Euler convention, is given by:

$$\begin{cases} \dot{X} = \mathbf{R}_0W \\ \dot{W} = \mathbf{R}_1W + \mathbf{\Psi} + B_2U + \dfrac{1}{m}B_3F_T \\ \dot{\Omega} = \gamma + \dfrac{1}{2}\rho s V\mathbf{P}_1\Pi + \dfrac{1}{2}\rho s V\mathbf{P}_1\mathbf{B}_1U + P_2F_T \\ \dot{\Phi} = \mathbf{\Gamma}\Omega \end{cases} \tag{14}$$

where we have dropped the arguments for simplicity of notations.

3. AUTOPILOT DESIGN METHODOLOGY

The main aim of the present work is to design an autopilot in order to track a desired attitude and velocity in spite of modeling errors and/or uncertainties on parameters that can affect the aircraft model. For this, one has first to make some observation about the aircraft systems' structure. From the above equations (14), one can see that the system possesses the structure as illustrated in Figure 2, where for simplicity, we have denoted:

$$\begin{cases} g(.) = \mathbf{R}_1W + \mathbf{\Psi} + B_2U + \dfrac{1}{m}B_3F_T \\ f(.) = \gamma + \dfrac{1}{2}\rho s V\mathbf{P}_1\Pi + \dfrac{1}{2}\rho s V\mathbf{P}_1\mathbf{B}_1U + P_2F_T \end{cases} \tag{15}$$

As mentioned in the introduction, it can be observed that the dynamics of angular velocity Ω and inertial speed W are directly affected by the control inputs U and thrust force F_T while the other two variables X and Φ are not. The dynamics of the Φ and X are indirectly affected by the actuators U and F_T through their tight coupling with Ω and W. Now, it is well-known that the 4 inputs variables U and F_T can only control 4 states variables. Therefore,we need to find judicious ways to indirectly control the rest of the state variables. For this we start by reducing the model of the aircraft given by (14) by taking into account the practical consideration of piloting. In effect, the pilot does not control the individual components of the velocity but rather its magnitude or norm $V = W^TW = \|W\|^2$. The dynamics of V is given by:

$$\dot{V} = 2W^T\dot{W} = 2W^T\left(\mathbf{R}_1W + \Psi + \mathbf{B}_2U + \frac{1}{m}B_3F_T\right)$$

Since $\mathbf{R}_1$ is skew-symmetric, we have $W^T\mathbf{R}_1W = 0$, so that

$$\dot{V} = 2W^T\Psi + 2W^T\mathbf{B}_2U + \frac{2}{m}W^TB_3F_T.$$

Next, the pilot has to ensure that the aircraft does not stall. For this, we have to ensure that the derivative of the position X does not escape to infinity. As a result, we impose the following condition:

$$\|\dot{X}\|^2 \leq M$$

where $M > 0$. Note that since $\mathbf{R}_0$ is an orthogonal matrix (i.e. $\mathbf{R}_0^T = \mathbf{R}_0^{-1}$), we have

$$\|\dot{X}\|^2 = \dot{X}^T\dot{X} = W^TW = V.$$

Therefore, the non-stalling condition reduces to ensuiring that $V \leq M$. As the result, the above aircraft model can be reduced to an 8th order system described by:

$$\begin{cases} \|\dot{X}\|^2 = V \\ \dot{V} = 2W^T\Psi + 2W^T\mathbf{B}_2U + \frac{2}{m}W^TB_3F_T \\ \dot{\Omega} = \gamma + \frac{1}{2}\rho sV\mathbf{P}_1\Pi + \frac{1}{2}\rho sV\mathbf{P}_1\mathbf{B}_1U + P_2F_T \\ \dot{\Phi} = \mathbf{\Gamma}\Omega \end{cases} \tag{16}$$

Based on the above observations, we adopt the following design strategy, which is also illustrated in Figure2:

- First, we introduce a virtual control Ω_v to control the orientation Φ towards the desired reference trajectory Φ_{ref}. We refer this controller as *Controller 1* as depicted in Figure2.
- Next, we use the input variable F_T to design a controller to track a given speed trajectory V_{ref}. This is *Controller 2* in Figure 2.
- Finally, we employ the input U is to steer Ω towards a given reference trajectory Ω_{ref}; which is chosen in such a way that it permit the virtual control input to track the desired orientation Φ_{ref}. This is referred to as *Controller 3* in Figure2.

In what follows, we detail the development of each controllers.

3.1 Design of Controller 1

Let $\Phi_{ref} = (\phi_{ref}, \theta_{ref}, \psi_{ref})^T$ be the desired orientation. Our objective is to steer $\Phi = (\phi, \theta, \psi)^T$ to $\Phi_{ref} = (\phi_{ref}, \theta_{ref}, \psi_{ref})^T$ using a virtual control input Ω_v since Φ is not directly affected by the real actuators. We have

$$\dot{\Phi} = \mathbf{\Gamma}\Omega_v$$

We seek for a controller of the form

$$\Omega_v = -\mathbf{\Gamma}^{-1}\mathbf{K}_0(\Phi - \mathbf{K}_0^{-1}\Omega) \tag{17}$$

where $\mathbf{K}_0 = diag(k_{0,1}, k_{0,2}, k_{0,3})$ is a gain matrix with $k_{0,i} > 0, i = 1, 2, 3$. The closed-loop system is given by:

$$\dot{\Phi} = -\mathbf{K}_0(\Phi - \mathbf{K}_0^{-1}\Omega)$$

Consequently, the closed-loop system simplifies to

$$\dot{\Phi} = -\mathbf{K}_0\Phi + \Omega \tag{18}$$

It can easily be seen that the above closed loop system is stable. We want $\Phi(t) = \Phi_{ref}$ when $t \to +\infty$. In other words, we want

$$\dot{\Phi}_{ref} = -\mathbf{K}_0\Phi_{ref} + \Omega_{ref} \tag{19}$$

when $t \to +\infty$ where $\Omega_{ref} = \Omega(\infty)$. We therefore deduce that

$$\Omega_{ref} = \dot{\Phi}_{ref} + \mathbf{K}_0\Phi_{ref} \tag{20}$$

By subtracting (18) with (19), we get

$$\dot{\Phi} - \dot{\Phi}_{ref} = -\mathbf{K}_0\left(\Phi - \Phi_{ref}\right) + \left(\Omega - \Omega_{ref}\right)$$

Setting $e_\Phi = \Phi - \Phi_{ref}$, one can see that

$$\dot{e}_\Phi = -\mathbf{K}_0e_\Phi + \left(\Omega - \Omega_{ref}\right)$$

which shows that $e_\Phi(t) \to 0$ when $t \to +\infty$ if $\Omega(t) \to \Omega_{ref}$ when $t \to +\infty$. We have therefore to make sure that $\Omega(t) \to \Omega_{ref}$ when $t \to +\infty$. This will be realised by Controller 3 subsequently.

3.2 Design of Controller 2

Consider again the dynamics of V; that is

$$\dot{V} = 2W^T\left(\Psi + \mathbf{B}_2U\right) + \frac{2}{m}W^TB_3F_T$$

Equivalently, we can write

$$\dot{V} - \dot{V}_{ref} = -\dot{V}_{ref} + 2W^T\left(\Psi + \mathbf{B}_2U\right) + \frac{2}{m}W^TB_3F_T$$

where $V_{ref} = W_{ref}^TW_{ref} = \|W_{ref}\|^2$ is a desired time-varying speed. We choose the aerodynamic and thrust forces such that

$$-\dot{V}_{ref} + \frac{2}{m}W^TB_3F_T = -l_1\left(V - V_{ref}\right) - V_{ref}$$

where $l_1 > 1$. That is,

$$F_T = \frac{1}{\frac{2}{m}W^TB_3}\left(\dot{V}_{ref} - l_1\left(V - V_{ref}\right) - V_{ref}\right) \tag{21}$$

Note that

$$\begin{aligned} W^TB_3 &= \frac{1}{2}\rho Vs\begin{pmatrix} u & v & w \end{pmatrix}\begin{pmatrix} C_{x,5} + k \\ C_{y,5} \\ C_{z,4} \end{pmatrix} \\ &= \frac{1}{2}\rho Vs\left[u\left(k + C_{x,5}\right) + vC_{y,5} + wC_{z,4}\right] \neq 0. \end{aligned}$$

Then in closed loop we have:

$$\dot{V} - \dot{V}_{ref} = -l_1\left(V - V_{ref}\right) - V_{ref} + 2W^T\left(\Psi + \mathbf{B}_2U\right)$$

Setting $e_V = V - V_{ref}$, we have

$$\begin{aligned} \dot{e}_V &= -l_1e_V - V_{ref} + 2W^T\left(\Psi + \mathbf{B}_2U\right) \\ &\leq -l_1e_V - V_{ref} + \left|2W^T\left(\Psi + \mathbf{B}_2U\right)\right| \end{aligned}$$

By using the Cauchy-Schwarz inequality, we get

$$\begin{aligned} \dot{e}_V &\leq -l_1e_V - V_{ref} + \|W\|^2 + \|\Psi + \mathbf{B}_2U\|^2 \\ &\leq -l_1e_V - V_{ref} + V + \|(\Psi + \mathbf{B}_2U)\|^2 \end{aligned}$$

since $V = \|W\|^2$. Consequently,

$$\dot{e}_V \leq -\left(l_1 - 1\right)e_V + \|\Psi + \mathbf{B}_2U\|^2$$

It is therefore clear that if $\|\Psi + \mathbf{B}_2U\|^2$ is bounded, one can choose l_1 large enough so that $e_V(t) \to 0$ asymptotically when $t \to +\infty$.

The boundedness of $\|\Psi + \mathbf{B}_2U\|^2$ is ensured by Controller 3 hereafter.

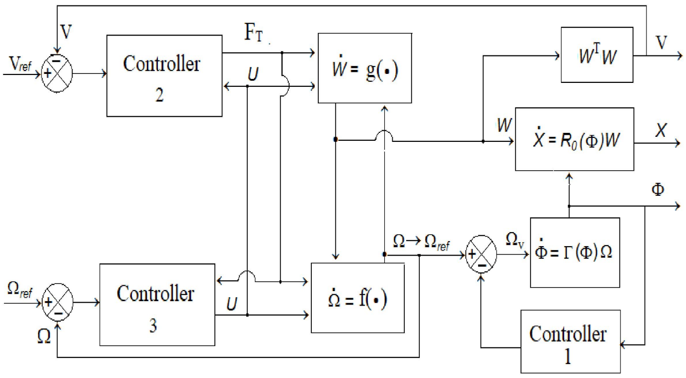

Fig. 2. Control design architecture.

3.3 Step 3: Design of Controller 3

The purpose of Controller 3 is to make sure that Ω tracks Ω_{ref} as $t \to +\infty$. This will be done using the control input U. Consider again the angular velocity equation from system (16):

$$\dot{\Omega} = \gamma + \frac{1}{2}\rho V s \mathbf{P}_1 \Pi + \frac{1}{2}\rho V s \mathbf{P}_1 \mathbf{B}_1 U + P_2 F_T$$

which can be equivalently written as

$$\dot{\Omega} - \dot{\Omega}_{ref} = -\dot{\Omega}_{ref} + \gamma + \frac{1}{2}\rho s V \mathbf{P}_1 \Pi + \frac{1}{2}\rho s V \mathbf{P}_1 \mathbf{B}_1 U + P_2 F_T$$

Proceeding as before, we impose

$$\begin{aligned}\dot{\Omega} - \dot{\Omega}_{ref} &= -\dot{\Omega}_{ref} + \gamma + \frac{1}{2}\rho s V \mathbf{P}_1 \Pi + \frac{1}{2}\rho s V \mathbf{P}_1 \mathbf{B}_1 U + P_2 F_T \\ &= -\eta\left(\Omega - \Omega_{ref}\right)\end{aligned}$$

with $\eta > 0$. Then,

$$\begin{aligned}U = \frac{1}{\frac{1}{2}\rho s V}\left(\mathbf{P}_1 \mathbf{B}_1\right)^{-1} \Big[\dot{\Omega}_{ref} - \eta\left(\Omega - \Omega_{ref}\right) \\ -P_2 F_T - \frac{1}{2}\rho s V \mathbf{P}_1 \Pi - \gamma\Big]\end{aligned} \tag{22}$$

In closed-loop, we have

$$\dot{\Omega} - \dot{\Omega}_{ref} = -\eta\left(\Omega - \Omega_{ref}\right)$$

Set $e_\Omega = \Omega - \Omega_{ref}$, then

$$\dot{e}_\Omega(t) = -\eta e_\Omega(t)$$

so that

$$e_\Omega(t) = e^{-\eta t} e_\Omega(0)$$

From this we can see that $e_\Omega(t) \to 0$ when $t \to +\infty$. In other words, $\Omega \to \Omega_{ref}$ when $t \to +\infty$.

Remark 1: Note that to further improve the convergence of the controller, one can add an integral term in the controller so that

$$\begin{aligned}U = \frac{1}{\frac{1}{2}\rho s V}\left(\mathbf{P}_1 \mathbf{B}_1\right)^{-1} \Big[\dot{\Omega}_{ref} - \eta\left(\Omega - \Omega_{ref}\right) \\ +\eta \int \left(\Omega - \Omega_{ref}\right) dt - P_2 F_T - \frac{1}{2}\rho s V \mathbf{P}_1 \Pi - \gamma\Big]\end{aligned} \tag{23}$$

Summary of result: To summarise, we can state that under the following control laws:

$$\Omega_v = -\mathbf{\Gamma}^{-1}\mathbf{K}_0(\Phi - \mathbf{K}_0^{-1}\Omega) \tag{24}$$

$$\begin{aligned}U = \frac{1}{\frac{1}{2}\rho s V}\left(\mathbf{P}_1 \mathbf{B}_1\right)^{-1} \Big[\dot{\Omega}_{ref} - \eta\left(\Omega - \Omega_{ref}\right) \\ -P_2 F_T - \frac{1}{2}\rho s V \mathbf{P}_1 \Pi - \gamma\Big]\end{aligned} \tag{25}$$

$$F_T = \frac{1}{\frac{2}{m} W^T B_3}\left(\dot{V}_{ref} - l_1\left(V - V_{ref}\right) - V_{ref}\right) \tag{26}$$

$$\dot{\Phi}_{ref} = -\mathbf{K}_0 \Phi_{ref} + \Omega_{ref} \tag{27}$$

where

- $l_1 > 1$, $\eta > 0$ and $\mathbf{K}_0 = diag(k_{0,1}, k_{0,2}, k_{0,3})$ is a gain matrix with $k_{0,i} > 0, i = 1, 2, 3$
- Ω_{ref} and V_{ref} are the desired orientation and speed respectively,

the aircraft overall closed-loop system

$$\begin{cases} \|\dot{X}\|^2 = V \\ \dot{\Omega} = \dot{\Omega}_{ref} - \eta\left(\Omega - \Omega_{ref}\right) \\ \dot{\Phi} = \dot{\Phi}_{ref} - \mathbf{K}_0\left(\Phi - \Phi_{ref}\right) + \left(\Omega - \Omega_{ref}\right) \\ \dot{V} = \dot{V}_{ref} - l_1\left(V - V_{ref}\right) - V_{ref} + 2W^T\left(\Psi + \mathbf{B}_2 U\right) \end{cases}$$

converges towards the desired trajectories Ω_{ref} and V_{ref} while avoiding stalling condition.

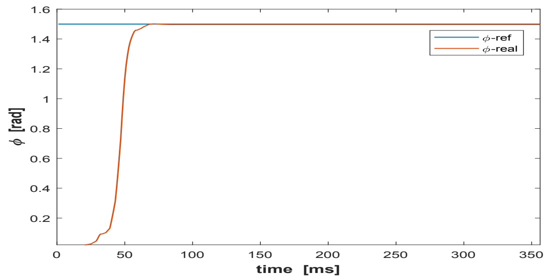

Fig. 3. Tracking of roll angle Φ.

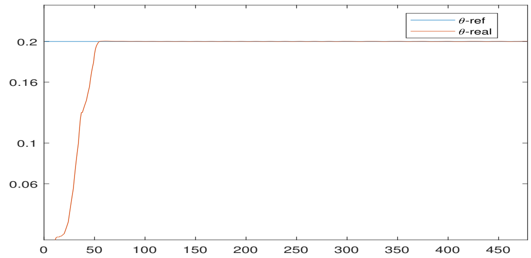

Fig. 4. Tracking of pitch angle .

4. SIMULATION RESULTS

A MATLAB/Simulink model is developed for Jetstream-3102 aircraft using the equations described in Section 2.The aerodynamic coefficients are taken from [13]. Simulation results are carried out corresponding to roll, pitch and yaw angles respectively as:

$\Phi_{ref} = \left(\Phi_{ref}, \theta_{ref}, \Psi_{ref}\right)^T = (1.5, 0.2, 1)^T$ and the reference speed $V_{ref} = 80\text{ms}^{-1}$. Figures 4, 5, 6, 7 and 8 shows the tracking results for respectively roll, pitch, yaw angles and speed, with $\eta = 1500$ and $\mathbf{K}_0 = diag(150, 20, 200)$.

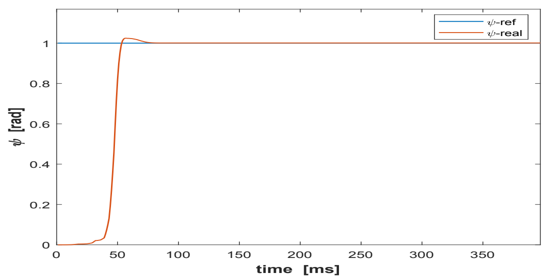

Fig. 5. Tracking of yaw angleΨ.

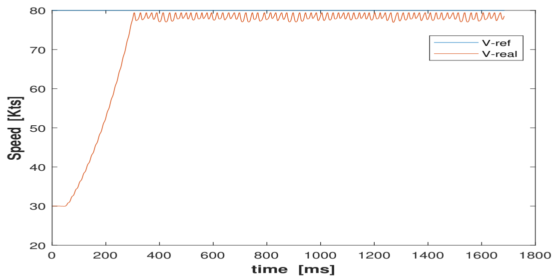

Fig. 6. Speed tracking.

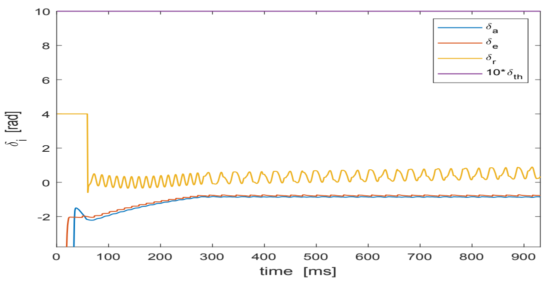

Fig. 7. Control surfaces.

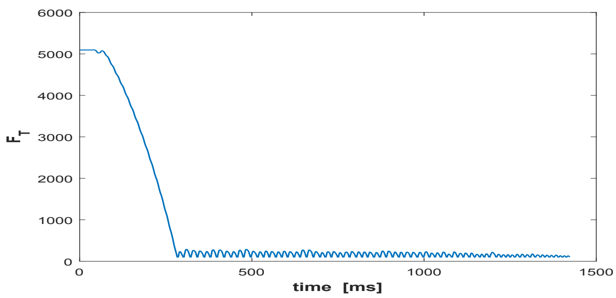

Fig. 8. Thrust force.

According to the obtained results we make the following observation:

- The parameter η affects the rise time of the system and at the same time the amplitude of the overflow.
- The parameter $\mathbf{K}_0$ affect the precision. Therefore, one have to make a compromise between precision, rising time and overflow.
- This strategy of control give a good tracking of attitude references but to the detriment of the speed control as shown on Figure 9. This is normal since the convergence is asymptotic rather than exponential. The asymptotic convergence of the speed to its reference value is not critical in practice. In fact, it is more important to have a precise and exponential convergence of the orientation rather than the speed.

5. CONCLUSION

In this paper we have proposed a new nonlinear feedback control design methodology for velocity and attitude control. For this we first reduce the aircraft model so that its is suitable for the specific control design objective. The proposed strategy consists of three control loops each realising a specific task. The key feature of the control strategy is the introduction of a virtual control input in order to cater for the underactuation property of such vehicles. Simulation results on a Jetstream-3102 aircraft have showed very good performance in terms of convergence towards its desired reference trajectories and in terms of robustness with respect to modeling uncertainties. The methodology developed here can be easily extended to other underactuated mechanical systems.

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