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Virtual Holonomic Constraints Control for port-Hamiltonian Systems: A Case Study of Fully Actuated Mechanical Systems

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Abstract: In this paper, virtual holonomic constraints control of port-Hamiltonian systems is proposed. In this research we especially focus on controller design for fully actuated mechanical systems as a case study. A virtual holonomic constraint force is calculated as a nonlinear feedback input by introducing the coordinate transformation. When some assumptions hold, this feedback successfully converts the original mechanical system into the reduced order port-Hamiltonian system with desired holonomic constraints. A numerical example shows the effectiveness and the property of the proposed virtual holonominc control.

Keywords: nonlinear control, port-Hamiltonian systems, tracking control, mechanical systems

1. INTRODUCTION

The progress of the declining birthrate and aging society is one of the big social problems, and it is possibly necessary to cope with the problem of a serious shortage of manpower. In this paper, as one of the solutions to the problem, controller design for good human-machine cooperation is considered.

Let us imagine a human-machine cooperative manipulation. For example, imagine that an operator and a machine grasp opposite ends of a long object and cooperate in moving the object based on force applied by the operator. In order to move the object to a target position, one of the popular approaches is impedance control. On fully actuated mechanical system, there are several researches about impedance control (see Salisbury (1980); Hogan (1984)). For example, the impedance control with time-varying impedance center is proposed and its passivity is studied. In Kishi et al. (2003), by adjusting the impedance center to hold the passivity condition, the passive impedance control is achieved. Arai et al. (2000) reports a robot assistant system to enable a human operator to manipulate large three-dimensional objects by anisotropic impedance control. This anisotropic impedance realizes a pseudo artificial nonholonomic constraint as if the system were equipped with fixed wheels.

As another approach for good human-machine cooperative manipulation, path following control is also one of the approaches. There are several researches about path following control, for example, see Salisbury (1980); Hogan (1984); Li and Horowitz (2001a,b); Duindam and Stramigioli (2004); Nielsen and Maggiore (2006); Hladio et al. (2013); Akhtar et al. (2015); Aguiar and Hespanha (2007); Nakamura (2016). In these control methods, a virtual potential function which take its minimum value on the desired path is designed to achieve the objective, and by changing the gradient of this potential function, the magnitude of the control input of the mechanical system can be adjusted.

In this paper, as another approach for human-machine cooperation in moving the object based on the operator's force, we propose a virtual holonomic constraint control of port-Hamiltonian system. By designing virtual holonomic constraints, the controlled system assist the human operation. As a case study, fully actuated mechanical systems formulated as port-Hamiltonian systems are considered in this paper. By introducing the coordinate transformation written in Matsumoto and Fujimoto (2017), a virtual holonomic constraint force can be calculated as a nonlinear feedback input. When some assumptions hold, this feedback successfully converts the original mechanical system into the reduced order port-Hamiltonian system with desired holonomic constraints.

The organization of this paper is as follows. First of all, mathematical formulation of port-Hamiltonian systems with holonomic constraint is briefly outlined. In Section 3, the design procedure of the proposed virtual holonomic constraint control for the port-Hamiltonian system is discussed. Finally, the effectiveness and the property of the proposed virtual holnomic constraint control are illustrated by using a numerical example in Section 4.

2. PRELIMINARIES

This section briefly refers to the existing results on formulation of port-Hamiltonian systems with holonomic constraints and useful coordinate transformation written in Matsumoto and Fujimoto (2017).

2.1 Port-Hamiltonian systems

Port-Hamiltonian systems are described as

$$\begin{cases} \dot{x} = (J(x,t) - R(x,t)) \dfrac{\partial H(x,t)}{\partial x}^{\mathrm{T}} + g(x,t)u \\ y = g(x,t)^{\mathrm{T}} \dfrac{\partial H(x,t)}{\partial x}^{\mathrm{T}} \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $y \in \mathbb{R}^m$ is the output, $H(x,t) \in \mathbb{R}$ is the Hamiltonian function, $J \in \mathbb{R}^{n\times n}$ is a skew symmetric matrix, and $R \in \mathbb{R}^{n\times n}$ is a symmetric semi-positive definite matrix. They are known as expansion of Hamilton's canonical equations. Fully actuated mechanical systems, systems with a class of nonholonomic constraints and dynamics of LC-circuits can be expressed in this form (see van der Schaft (1996); van der Schaft and Maschke (1994); Maschke et al. (1995); Fujimoto et al. (2012)).

In this research we especially focus on fully actuated mechanical systems written as following formulations.

$$\begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 & I \\ -I & -R \end{pmatrix} \begin{pmatrix} \dfrac{\partial H}{\partial q}^{\mathrm{T}} \\ \dfrac{\partial H}{\partial p}^{\mathrm{T}} \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} u \tag{2}$$

$$y = \frac{\partial H}{\partial p}^{\mathrm{T}} (= \dot{q})$$

$$H(q,p) = \frac{1}{2} p^{\mathrm{T}} M(q)^{-1} p$$

where $q \in \mathbb{R}^m$ is the position, $p \in \mathbb{R}^m$ is the momentum, $x = (q^{\mathrm{T}}, p^{\mathrm{T}})^{\mathrm{T}}$ is the state, $R \in \mathbb{R}^{m\times m}$ is a symmetric positive semi-definite matrix, $M(q) \in \mathbb{R}^{m\times m}$ is a symmetric matrix satisfying $p = M(q)\dot{q}$.

2.2 Port-Hamiltonian systems with holonomic constraints

Here the formulation of the port-Hamiltonian system with holonomic constraints is briefly explained.

Given the Hamiltonian $H(q,p)$ of a dynamical system, subjected to l holonomic constraints

$$c_i(q) = 0, \quad i = 1, ..., l, \tag{3}$$

the port-Hamiltonian system is written as

$$\begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 & I \\ -I & -R \end{pmatrix} \begin{pmatrix} \dfrac{\partial H}{\partial q}^{\mathrm{T}} \\ \dfrac{\partial H}{\partial p}^{\mathrm{T}} \end{pmatrix} + \begin{pmatrix} 0 \\ \dfrac{\partial C}{\partial q}^{\mathrm{T}} \end{pmatrix} \lambda + \begin{pmatrix} 0 \\ I \end{pmatrix} u \tag{4}$$

$$C(q) = 0 \tag{5}$$

$$y = \frac{\partial H}{\partial p}^{\mathrm{T}} (= \dot{q})$$

$$H(q,p) = \frac{1}{2} p^{\mathrm{T}} M(q)^{-1} p,$$

where $C(q) = (c_1(q), ..., c_l(q))^{\mathrm{T}}$ and $\lambda \in \mathbb{R}^l$ is the unknown Lagrangian multipliers.

It was shown in Matsumoto and Fujimoto (2017) that the Hamiltonian system defined above can be described by a port-Hamiltonian system by the following procedure.

First a matrix $J_{12}(q) \in \mathbb{R}^{m\times l}$ is chosen in such a way that $(\partial C/\partial q)J_{12}(q) = 0$ holds and that $(\partial C/\partial q^{\mathrm{T}}, J_{12}(q))$ is nonsingular. Using the matrix $J_{12}(q)$, we can define the following coordinate transformation.

$$\bar{p} := \begin{pmatrix} \hat{p} \\ \check{p} \end{pmatrix} = \begin{pmatrix} \hat{\phi}(q,p) \\ \check{\phi}(q,p) \end{pmatrix} = \begin{pmatrix} J_{12}(q)^{\mathrm{T}} p \\ \dfrac{\partial C}{\partial q} p \end{pmatrix} \tag{6}$$

Then $\partial H/\partial \check{p} = 0$ holds where $\check{p}$ coordinate does not affect the input-output behavior. Consequently the constrained system can be described by the following reduced order port-Hamiltonian system.

$$\begin{pmatrix} \dot{q} \\ \dot{\hat{p}} \end{pmatrix} = \begin{pmatrix} 0 & J_{12} \\ -J_{12}^{\mathrm{T}} & J_{22} - \hat{R}_{22} \end{pmatrix} \begin{pmatrix} \dfrac{\partial H}{\partial q}^{\mathrm{T}} \\ \dfrac{\partial H}{\partial \hat{p}}^{\mathrm{T}} \end{pmatrix} + \begin{pmatrix} 0 \\ G \end{pmatrix} u \tag{7}$$

$$y = G(q)^{\mathrm{T}} \frac{\partial H}{\partial \hat{p}}^{\mathrm{T}}$$

$$H(q,p) = \frac{1}{2} \hat{p}^{\mathrm{T}} \hat{M}(q)^{-1} \hat{p}$$

3. MAIN RESULTS

This section discusses how to design a nonlinear feedback input for virtual holonomic constraints control of port-Hamiltonian systems.

3.1 Problem formulation

Consider a port-Hamiltonian system in (2) with external input u_{ext} and desired l virtual holonomic constraints $C_{\mathrm{d}}(q) \in \mathbb{R}^l$

$$C_{\mathrm{d}}(q) = \begin{pmatrix} c_1(q) \\ c_2(q) \\ \vdots \\ c_l(q) \end{pmatrix}. \tag{8}$$

The objective of the virtual holonomic constraints control is to transform the system (2) into the port-Hamiltonian system (7) by the nonlinear feedback $u = f(q, p, u_{\mathrm{ext}})$

subject to the initial value $(q(0)^{\mathrm{T}}, p(0)^{\mathrm{T}})^{\mathrm{T}}$ which holds the following assumptions.

Assumption 1. The initial value of the state $q(0) = q_{\mathrm{ini}}$ satisfies $C_{\mathrm{d}}(q_{\mathrm{ini}}) = 0 (i = 1, .., l)$

Assumption 2. The initial value of the state $p(0) = p_{\mathrm{ini}}$ satisfies $\check{p}(q_{\mathrm{ini}}, p_{\mathrm{ini}}) = (\partial C(q_{\mathrm{ini}})/\partial q) p_{\mathrm{ini}} = 0$

3.2 Virtual holonomic constraints controller design

A design procedure of the proposed virtual holonomic constraints control is explained here.

When the external input u_{ext} is added, the fully actuated mechanical system is written as following formulation.

$$\begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 & I \\ -I & -R \end{pmatrix} \begin{pmatrix} \dfrac{\partial H}{\partial q}^{\mathrm{T}} \\ \dfrac{\partial H}{\partial p}^{\mathrm{T}} \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} u + \begin{pmatrix} 0 \\ I \end{pmatrix} u_{\mathrm{ext}} \quad (9)$$

On the other hand, by using the Lagrangian multipliers λ, the constrained system by the desired virtual holonomic constraints is described as follows.

$$\begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 & I \\ -I & -R \end{pmatrix} \begin{pmatrix} \dfrac{\partial H}{\partial q}^{\mathrm{T}} \\ \dfrac{\partial H}{\partial p}^{\mathrm{T}} \end{pmatrix} + \begin{pmatrix} 0 \\ \dfrac{\partial C_{\mathrm{d}}}{\partial q}^{\mathrm{T}} \end{pmatrix} \lambda + \begin{pmatrix} 0 \\ I \end{pmatrix} u_{\mathrm{ext}} \quad (10)$$

Comparing expression (9) and expression (10), it is realized that if the feedback controller u is calculated as

$$u = \frac{\partial C_{\mathrm{d}}}{\partial q}^{\mathrm{T}} \lambda$$

and λ is calculated satisfying the equation $\partial H/\partial \check{p} = 0$, then the virtual holonomic constraint control and the reduced order port-Hamiltonian system (7) are obtained. Therefore, first of all, we calculate the Lagrangian multiplier λ which holds the condition $\partial H/\partial \check{p} = 0$.

By utilizing the coordinate transformation in (6), the system is transformed into the following equations.

$$\begin{aligned} \begin{pmatrix} \dot{q} \\ \dot{\hat{p}} \\ \dot{\check{p}} \end{pmatrix} &= \begin{pmatrix} I & 0 \\ \dfrac{\partial \phi}{\partial q} & \begin{pmatrix} J_{12}^{\mathrm{T}} \\ \dfrac{\partial C_{\mathrm{d}}}{\partial q} \end{pmatrix} \end{pmatrix} \left(\begin{pmatrix} 0 & I \\ -I & -R \end{pmatrix} \begin{pmatrix} \dfrac{\partial H}{\partial q}^{\mathrm{T}} \\ \dfrac{\partial H}{\partial p}^{\mathrm{T}} \end{pmatrix} \right. \\ &\quad \left. + \begin{pmatrix} 0 \\ \dfrac{\partial C_{\mathrm{d}}}{\partial q}^{\mathrm{T}} \end{pmatrix} \lambda + \begin{pmatrix} 0 \\ I \end{pmatrix} u_{\mathrm{ext}} \right) \\ &= \begin{pmatrix} 0 & I \\ -J_{12}^{\mathrm{T}} \dfrac{\partial \hat{\phi}}{\partial q} & -J_{12}^{\mathrm{T}} R \\ -\dfrac{\partial C_{\mathrm{d}}}{\partial q} \dfrac{\partial \check{\phi}}{\partial q} & -\dfrac{\partial C_{\mathrm{d}}}{\partial q} R \end{pmatrix} \begin{pmatrix} \dfrac{\partial H}{\partial q}^{\mathrm{T}} \\ \dfrac{\partial H}{\partial p}^{\mathrm{T}} \end{pmatrix} \\ &\quad + \begin{pmatrix} 0 \\ 0 \\ \dfrac{\partial C_{\mathrm{d}}}{\partial q} \dfrac{\partial C_{\mathrm{d}}}{\partial q}^{\mathrm{T}} \lambda \end{pmatrix} + \begin{pmatrix} 0 \\ J_{12}^{\mathrm{T}} \\ \dfrac{\partial C_{\mathrm{d}}}{\partial q} \end{pmatrix} u_{\mathrm{ext}} \end{aligned} \quad (11)$$

Here J_{12} satisfies $(\partial C_{\mathrm{d}}/\partial q) J_{12}(q) = 0$ and the matrix $(\partial C_{\mathrm{d}}/\partial q^{\mathrm{T}}, J_{12}(q))$ is nonsingular

In order to achieve the reduced order port-Hamiltonian system in (7), the condition $\partial H/\partial \check{p} = 0$ is needed, and this equation holds when $\check{p}(t) = 0, \ \forall t \geq 0$. Thanks to the Assumption. 2, $\check{p}(0) = 0$. Thus, in order to hold the equation $\check{p}(t) = 0$, $\dot{\check{p}} = 0$ is required. That is, $\dot{\check{p}}$ in (11) is equal to 0.

$$\begin{aligned} \dot{\check{p}} =& -\frac{\partial C_{\mathrm{d}}}{\partial q} \frac{\partial H}{\partial q}^{\mathrm{T}} + \left(\frac{\partial \check{\phi}}{\partial q} - \frac{\partial C_{\mathrm{d}}}{\partial q} R \right) \frac{\partial H}{\partial p}^{\mathrm{T}} \\ &+ \frac{\partial C_{\mathrm{d}}}{\partial q} \frac{\partial C_{\mathrm{d}}}{\partial q}^{\mathrm{T}} \lambda + \frac{\partial C_{\mathrm{d}}}{\partial q} u_{\mathrm{ext}} \\ =& 0 \end{aligned}$$

Therefore, where $(\partial C_{\mathrm{d}}/\partial q) \cdot (\partial C_{\mathrm{d}}/\partial q^{\mathrm{T}})$ is nonsingular, the virtual constraint force λ is obtained as

$$\begin{aligned} \lambda = & -\left(\frac{\partial C_{\mathrm{d}}}{\partial q} \frac{\partial C_{\mathrm{d}}}{\partial q}^{\mathrm{T}} \right)^{-1} \left(-\frac{\partial C_{\mathrm{d}}}{\partial q} \frac{\partial H}{\partial q}^{\mathrm{T}} \right. \\ & \left. + \left(\frac{\partial \check{\phi}}{\partial q} - \frac{\partial C_{\mathrm{d}}}{\partial q} R \right) \frac{\partial H}{\partial p}^{\mathrm{T}} + \frac{\partial C_{\mathrm{d}}}{\partial q} u_{\mathrm{ext}} \right). \end{aligned} \quad (12)$$

Consequentially, the feedback control input u is calculated as

$$\begin{aligned} u =& \frac{\partial C_{\mathrm{d}}}{\partial q}^{\mathrm{T}} \lambda \\ =& -\frac{\partial C_{\mathrm{d}}}{\partial q}^{\mathrm{T}} \left(\frac{\partial C_{\mathrm{d}}}{\partial q} \frac{\partial C_{\mathrm{d}}}{\partial q}^{\mathrm{T}} \right)^{-1} \left(-\frac{\partial C_{\mathrm{d}}}{\partial q} \frac{\partial H}{\partial q}^{\mathrm{T}} \right. \\ & \left. + \left(\frac{\partial \check{\phi}}{\partial q} - \frac{\partial C_{\mathrm{d}}}{\partial q} R \right) \frac{\partial H}{\partial p}^{\mathrm{T}} + \frac{\partial C_{\mathrm{d}}}{\partial q} u_{\mathrm{ext}} \right), \end{aligned} \quad (13)$$

and reduced order port-Hamiltonian system can be obtained as follows.

$$\begin{pmatrix} \dot{q} \\ \dot{\hat{p}} \end{pmatrix} = \begin{pmatrix} 0 & J_{12} \\ -J_{12}^{\mathrm{T}} & J_{22} - \hat{R}_{22} \end{pmatrix} \begin{pmatrix} \dfrac{\partial H}{\partial q}^{\mathrm{T}} \\ \dfrac{\partial H}{\partial \hat{p}}^{\mathrm{T}} \end{pmatrix} + \begin{pmatrix} 0 \\ G \end{pmatrix} u_{\mathrm{ext}} \quad (14)$$

$$\begin{aligned} J_{22} &= -J_{12}^{\mathrm{T}} \frac{\partial \hat{\phi}}{\partial q}^{\mathrm{T}} + \frac{\partial \hat{\phi}}{\partial q} J_{12}, \\ \hat{R}_{22} &= J_{12}^{\mathrm{T}} R J_{12}, \\ G &= J_{12} \end{aligned}$$

3.3 Design procedure

Here we summarize the procedure to design the virtual holonomic constraint control of port-Hamiltonian systems as the following algorithm.

Design procedure for virtual holonomic constraints control

(i) Define the desired virtual holonomic constraints C_d in (8) and set the initial value of the state $(q(0), p(0))^\mathrm{T}$ which holds the Assumption 1 and 2.

(ii) Find the matrix $J_{12}(q) \in \mathbb{R}^{m\times l}$ in such a way that $(\partial C_\mathrm{d}/\partial q)J_{12}(q) = 0$ holds and that $(\partial C_\mathrm{d}/\partial q^\mathrm{T}, J_{12}(q))$ is nonsingular

(iii) By using J_{12} and C_d, calculate the coordinate transformation according to (6). Then, the transformed port-Hamiltonian system (11) is obtained by this coordinate transformation.

(iv) Calculate the virtual holonomic constraints controller u according to (13). Finally, the reduced order port-Hamiltonian system (14) is obtained.

Remark 1. In calculating the virtual holonomic constraint controller u in (13), only C_d and $\check{\phi} = (\partial C_\mathrm{d}/\partial q)p$ are utilized. Thus, when we need only the calculation of the controller, we don't have to find the matrix J_{12}.

4. NUMERICAL EXAMPLE

This section shows how to construct a virtual holonomic constraint control for a port-Hamiltonian system by using a numerical example.

A fully actuated mechanical system written as a following port-Hamiltonian system formulation is considered here.

$$\begin{cases} \begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \begin{pmatrix} \dfrac{\partial H}{\partial q}^\mathrm{T} \\ \dfrac{\partial H}{\partial p}^\mathrm{T} \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} u \\ y = \dfrac{\partial H}{\partial p}^\mathrm{T} (= \dot{q}) \\ H = \dfrac{1}{2} p^\mathrm{T} M^{-1} p \end{cases} \tag{15}$$

Here $q = (q_1,\ q_2)^\mathrm{T}$, $p = (p_1,\ p_2)^\mathrm{T}$, and $M = I \in \mathbb{R}^{2\times 2}$.

The objective of the virtual holonomic constraint control is to make the system track on the virtual holonomic constraint as illustrated in Figure 1. This desired holonomic constraint is written as

$$C_\mathrm{d}(q) = q_1(q_1 - 3) + q_2$$

In this simulation, we add the external input u_ext described as follows,

$$u_\mathrm{ext} = \begin{pmatrix} 0.2\sin(t) \\ -0.4\cos(t) \end{pmatrix}$$

In this example, J_{12} satisfies

$$\frac{\partial C_\mathrm{d}}{\partial q} J_{12}(q) = (q_1 - 3,\ 1)\, J_{12}(q) = 0$$

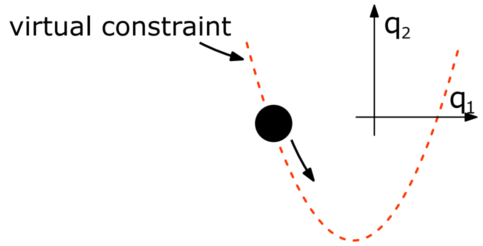

Fig. 1. The settings of the numerical simulation

For example, $J_{12}(q)$ is chosen as follows.

$$J_{12}(q) = \begin{pmatrix} 1 \\ -q_1 + 3 \end{pmatrix}$$

Thus, the coordinate transformation (6) is calculated as

$$\begin{pmatrix} \hat{p} \\ \check{p} \end{pmatrix} = \begin{pmatrix} p_1 - p_2 q_1 + 3p_2 \\ p_1 q_1 - 3p_1 + p_2 \end{pmatrix}. \tag{16}$$

Finally, virtual holonomic constraints controller u is calculated according to (13) and the reduced order port-Hamiltonian system (14) is obtained as follows.

$$\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{\hat{p}} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & -q_1 + 3 \\ -1 & q_1 - 3 & 0 \end{pmatrix} \begin{pmatrix} \dfrac{\partial H}{\partial q_1}^\mathrm{T} \\ \dfrac{\partial H}{\partial q_2}^\mathrm{T} \\ \dfrac{\partial H}{\partial \hat{p}}^\mathrm{T} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & -q_1 + 3 \end{pmatrix} u_\mathrm{ext} \tag{17}$$

For comparison, the system without the proposed controller is also numerically simulated.

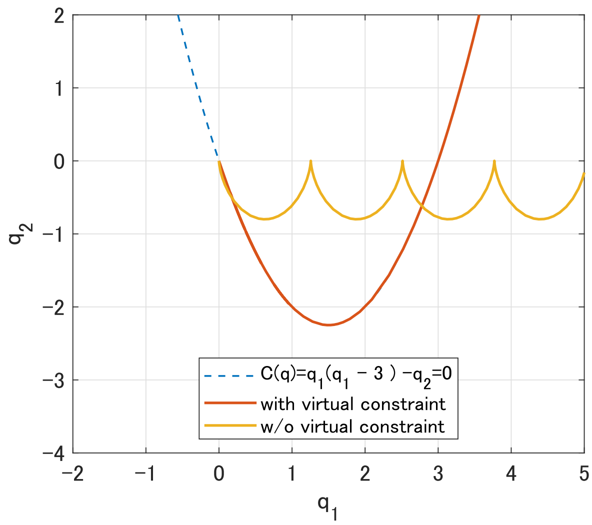

Fig. 2. Motion on q space when Assumption 1 and 2 hold

Figure 2 shows the simulation result of the virtual holonomic constraint control. The motion of the system with

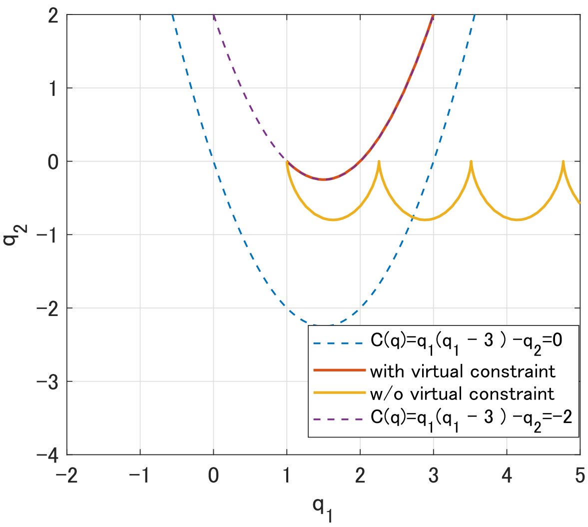

Fig. 3. Motion on q space when Assumption 1 dose not hold

virtual holonomic constraint control is depicted by the orange sold line, whereas The motion of the system without the control input is depicted by the yellow sold line. The desired holonomic constraint $C_\mathrm{d}(q)$ is depicted by the broken blue line. The figure shows that the controlled system starting from $q(0) = (0,0)^\mathrm{T}, p(0) = (0,0)^\mathrm{T}$ successfully tracks on the desired holonomic constraint.

4.1 Discussions

The simulation result when the proposed virtual holonomic constraints control successfully work has been explained above. Here, the results of the numerical simulation when the assumptions in Section 3.1 does not hold are shown here.

Figure 3, 4 shows the simulation result when the virtual holonomic constraint control does not work well. In Figure 3 and Figure 4, the initial value of the system is selected as $q(0) = (1,0)^\mathrm{T}, p(0) = (0,0)^\mathrm{T}$ and $q(0) = (0,0)^\mathrm{T}, p(0) = (1,0)^\mathrm{T}$ respectively. These initial values do not hold the assumptions in Section 3.1. The motion of the system with virtual holonomic constraint control is depicted by the orange sold line, whereas The motion of the system without the control input is depicted by the yellow sold line. The desired holonomic constraint $C_\mathrm{d}(q)$ is depicted by the broken blue line.

As illustrated in Figure 3, when the initial value of the system does not hold the Assumption 1, the controlled system starting from $q(0) = (1,0)^\mathrm{T}, p(0) = (0,0)^\mathrm{T}$ tracks on the level set $C_d(q) = C_d(q(0)) = -2$ depicted in broken purple line.

On the other hand, as illustrated in Figure 4, when the initial value of the system does not hold the Assumption 2, the controlled system starting from $q(0) = (0,0)^\mathrm{T}, p(0) = (1,0)^\mathrm{T}$ can not track on the desired holonomic constraint. The reason is because $\partial H/\partial \check{p}$ is not equal to 0 when the Assumption 2 fails.

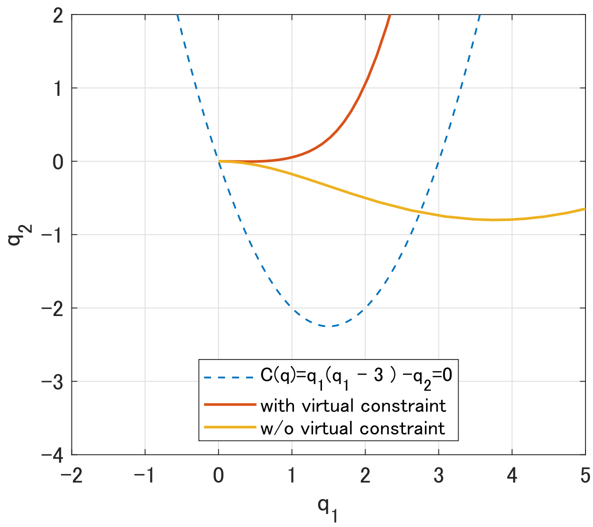

Fig. 4. Motion on q space when Assumption 2 dose not hold

5. CONCLUSION

In this paper, the virtual holonomic constraints control for the port-Hamiltonian systems has been proposed. In this research we especially focus on controller design for fully actuated mechanical systems as a case study. A virtual holonomic constraint force has been calculated as a nonlinear feedback input by introducing the coordinate transformation. Some assumptions are made for initial values, and when some assumptions hold, this feedback has successfully converted the original mechanical system into the reduced order port-Hamiltonian system with desired holonomic constraints. Moreover, the numerical example has shown the effectiveness and the property of the proposed virtual holonomic constraint control.

Future work will include the extension of the proposed framework to more generalized port-Hamiltonian systems, construction of virtual nonholonomic constraints control, and its application to the human-robot cooperative manipulation.

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