Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Redundant disturbance rejection controller applied to quadrotors for 3D trajectory tracking

Y. BOUZID * **H. SIGUERDIDJANE** ** **M. Guiatni** * **M. Zareb** ***
L. H. Choukri *

* *CSCS Laboratory, Ecole Militaire Polytechnique, Algiers, Algeria (e-mail: yasseremp@gmail.com).*
** *L2S, Centralesupelec, University Paris-Saclay, France (e-mail: Houria.Siguerdidjane@centralesupelec.fr).*
*** *USTO, Oran University, Algeria.*

Abstract: The objective is to develop a control algorithm for quadrotors that guarantees a good compromise robustness/performance in presence of external disturbances. Thus, we investigate and apply a nominal model-based control strategy doted by a robustness boosting mechanism. This latter, uses an Extended State-based Observer (ESO) to estimate the uncertainties and the various disturbances. The obtained controller is augmented by an additional input, which is derived via a sliding modes framework to handle the estimation errors and ensure asymptotic stability. The primary results are shown through numerical simulations.

Keywords: Quadrotor, Trajectory tracking, Robust control, ADRC, Extended State-based Observer.

1. INTRODUCTION

During the last two decades, hundreds of control strategies have been studied in the literature and applied successfully to quadrotors. For the sake of increasing the performance of the flight controllers, several combinations, between different strategies, have been proposed. A backstepping control scheme combined with Neural Networks (NN) is proposed by Wang et al. (2017). One year later, terminal sliding modes controller with a gains tuning stage is proposed by Miranda-Colorado et al. (2018) to reduce the power consumption. A fractional backstepping combined with sliding modes based controller is proposed by Shi et al. (2019). The interested readers may refer to the reviews Mo and Farid (2019) for more information.

Recently, some advanced and robust control strategies have been proposed to deal with the various disturbances and uncertainties. Most of them estimate the overall dynamics of the system with the disturbances affecting its behavior during the flight. Then, they proceed by compensation of the estimated terms. Such king of controllers is usually employed besides a conventional PID controller. This principle is known as Active Disturbance Rejection Control (ADRC) Chenlu et al. (2016). For instance, reference Cai et al. (2019) considers sliding-modes based observer, which is employed for the suppression of disturbances. This latter is applied only for the translational sub-system of a quadrotor where a PID controller is used for the rotational sub-system. Also, reference Miranda-Colorado (2019) combines a conventional PID controller with a sliding mode disturbance observer for trajectory tracking tasks. In reference Zhao et al. (2019), attitude control is investigated for a quadrotor. ADRC is adopted in the inner loop to estimate the internal uncertain dynamics and external wind disturbances. Very recently, many research papers have been carried out investigating the control of quadrotors using ADRC such as Yuan et al. (2018) Lotufo et al. (2019) Castillo et al. (2019).

The popular ADRC assumes no available model, which is not, according to our point of view, a nice assumption. Therefore, involving the available information about the controlled system (dynamics model) will considerably improve the effectiveness of control and bring an additional benefit.

In this paper, we envisage the use a reference-model based control strategy as a main controller. Via such kind of controllers, we can ensure a good tracking of the 3D trajectory according to the desired requirements. The desired behavior can be easily fixed through the asymptotic behavior of the tracking error dynamics. Due to the limitation of this controller (model-based controller) via-a-vis the disturbances and the uncertainties, we propose to involve an auxiliary input that boosts the robustness level of the main controller (i.e. Robustness Booster (RB)). The booster is built upon an estimation principle considering an Extended State-based Observer (ESO). This latter is involved to cope with the unknown part of the system only (e.g. disturbances, uncertainties, unmodeled dynamics, etc.). Moreover, a sliding modes-like term is also added to enhance the ESO performance and to deal with the estimation errors. The elaborated simulations have shown satisfactory results.

The document is organized as follows: in Section 2, a simple control-oriented model is presented. Section 3 introduces the reference model-based control strategy. Section 4 shows the design of the proposed nonlinear booster and observer. The sliding modes auxiliary input is exposed in Section 5. Numerical simulations are shown in Section 6. Conclusions are taken in Section 7.

2. CONTROL-ORIENTED MODEL

Simplified models are usually introduced in the literature that are considered as control-oriented models to obtain simple control laws. Moreover, through these simplified models, we investigate the effectiveness of the designed controllers if they

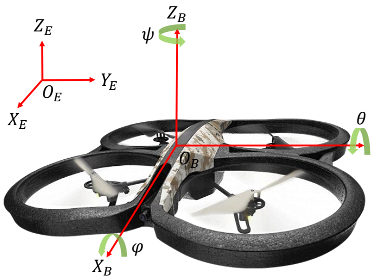

Fig. 1. Frames representation.

are able to handle the neglected dynamics, the parameters uncertainties and the disturbances.

In low-speed flight conditions, the used control-oriented model neglects some effects such as: blade-flapping moments, hub forces, gyroscopic moments, etc. These effects have a minor impact on the vehicle and will be gathered in one term Δ that includes all the external disturbances, neglected and unmodeled dynamics, uncertainties, etc.

The vehicle operates in two coordinate frames: the Earth-fixed frame $R_E(O_E, X_E, Y_E, Z_E)$ and the Body-fixed frame $R_B(O_B, X_B, Y_B, Z_B)$ (as shown in Figure 1). It is supposed with rigid structure.

Based on the symmetry property and with the appropriate choice of $R_B(O_B, X_B, Y_B, Z_B)$ (see Figure 1), the inertia matrix is diagonal $I = diag(I_x, I_y, I_z)$. Explicitly, the quadrotor model can be written as

$$\begin{cases} \ddot{x} &= u_1 \dfrac{c_\psi s_\theta c_\varphi + s_\psi s_\varphi}{m} + \Delta_x \\ \ddot{y} &= u_1 \dfrac{s_\psi s_\theta c_\varphi - c_\psi s_\varphi}{m} + \Delta_y \\ \ddot{z} &= -g + u_1 \dfrac{c_\theta c_\varphi}{m} + \Delta_z \\ \ddot{\varphi} &= \dfrac{(I_y - I_z)}{I_x} \dot{\theta}\dot{\psi} + \dfrac{u_2}{I_x} + \Delta_\varphi \\ \ddot{\theta} &= \dfrac{(I_z - I_x)}{I_y} \dot{\varphi}\dot{\psi} + \dfrac{u_3}{I_y} + \Delta_\theta \\ \ddot{\psi} &= \dfrac{(I_x - I_y)}{I_z} \dot{\varphi}\dot{\theta} + \dfrac{u_4}{I_z} + \Delta_\psi \end{cases} \tag{1}$$

where m is the mass and g denotes the gravitation coefficient. Let $\eta = (\varphi, \theta, \psi)^T \in \mathbb{R}^3$ be the orientation (Roll, Pitch, Yaw) of the quadrotor and $\chi = (x, y, z)^T \in \mathbb{R}^3$ be the absolute position in R_E with $\varphi \neq \frac{\pi}{2} + k\pi, \theta \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$. Let $u = (u_1, u_2, u_3, u_4)^T$ be the control input vector. It contains the thrust, and the attitude moments. $s_{(.)}$ and $c_{(.)}$ are abbreviations for $sin(.)$ and $cos(.)$ respectively. $\Delta = (\Delta_x, \Delta_y, \Delta_x, \Delta_\varphi, \Delta_\theta, \Delta_\psi)^T \in \mathbb{R}^6$ is a disturbance vector.

Model (1) can arranged to appear in a compact form as

$$\ddot{\mathbf{q}} = F(\mathbf{q}, \dot{\mathbf{q}}) + G(\mathbf{q})u + \Delta \tag{2}$$

where $F(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^6$ are nonlinear functions and $G(\mathbf{q}) \in \mathbb{R}^{6\times 4}$ is the input matrix. $\mathbf{q} = (\chi, \eta)^T$ is the state vector. $\Delta \in \mathbb{R}^6$ is a bounded and unknown term. All the terms can be identified readily from system (1).

3. REFERENCE MODEL-BASED CONTROL STRATEGY

Assuming the nominal (i.e. without disturbances) and the general case, we consider the class of nonlinear MIMO systems that are affine in the control for $t \in [0, \infty)$

$$\ddot{\mathbf{q}}(t) = F(\mathbf{q}, \dot{\mathbf{q}}) + G(\mathbf{q})\, u(t) \tag{3}$$

where $\mathbf{q} \in \mathscr{Q} \subset \mathbb{R}^n$ is an n-dimensional vector and $u \in \mathscr{U} \subset \mathbb{R}^m$ is m-dimensional input vector. $\mathbf{q}$ contains the states of the system. $F(.) : \mathscr{Q} \to \mathbb{R}^n$ is a vector of multi-variable functions smooth on $\mathscr{Q}$ and $G(.) : \mathscr{Q} \to \mathbb{R}^{n\times m}$ denotes the input matrix, which is non-singular in $\mathscr{Q}$.

The control input $u(t)$ should be designed in order to ensure that the tracking error of system (3) goes toward the origin according to some requirements. Thus, reference-model based control strategies are more adequate.

Our proposed procedure, in order to design the controller, can be achieved in two main steps. In the **first step**, a dynamic inversion of the nonlinear model is required. Thus, we investigate a relationship between the input vector $u(t) \in \mathscr{U}$ and the output vector $\mathbf{q} \in \mathscr{Q}$. Various techniques already published resolving this classic problem for certain class of systems. Recently, some heuristic and learning algorithms are exploited such as the neuronal network Nahas et al. (1992); Liu et al. (2016). However, the inaccuracies of the experimental tests and the computational complexity render these strategies not recommended for real time applications. Herein, for this particular case, a forward inversion leads to

$$u = G(\mathbf{q})^{\perp}(\ddot{\mathbf{q}}(t) - F(\mathbf{q}, \dot{\mathbf{q}})) \tag{4}$$

The matrix $G(\mathbf{q})$ can be not invertible. Thus, the pseudo inverse matrix is used by multiplying both sides of equation (3) by $G^T(\mathbf{q})$. Notice that $G^T(\mathbf{q})G(\mathbf{q})$ is square and it is invertible. Finally, $G(\mathbf{q})^{\perp} = \left(G(\mathbf{q})^T G(\mathbf{q})\right)^{-1} G(\mathbf{q})^T$ is the pseudo-inverse of $G(\mathbf{q})$.

Obviously, to raise the performance of the control, the inversion should occur in a closed-loop architecture. For this purpose, in this **second step**, we seek to formulate a forward mapping between the tracking errors and the derivatives of the output. In other words,

$$\mathbf{q}^{(i)}(t) = \Theta_i\left(e_{\mathbf{q}}, \dot{e}_{\mathbf{q}} \ldots, e_{\mathbf{q}}^{(i-1)}\right) \quad i = 1, \ldots, \nu \tag{5}$$

where Θ_i are scalar functions and $e_{\mathbf{q}}(t) \in \mathbb{R}^n$ is the tracking error between the reference trajectory $\mathbf{q}_r(t) \in \mathscr{Q} \subset \mathbb{R}^n$ and the output $\mathbf{q}(t)$.

Consequently, we define a surface $S(t) \in \mathbb{R}^n$ in state-space as

$$S(t) = \left(\frac{d}{dt} + \mu^{-1}\right)^{\nu - 1} e_{\mathbf{q}}(t) \tag{6}$$

where $\frac{d}{dt}$ is a time derivative operator. $\mu \in \mathbb{R}^{n\times n}$ is a diagonal positive definite scale matrix. It is selected by the user in order to adjust the behavior of convergence with a good compromise robustness/performance. ν denotes the order of the system. In our case, it equals to $\nu = 2$.

The main goal is to push $e_{\mathbf{q}}(t)$ and its dynamics toward the origin. Therefore, we set $S(t) = 0$. For $\nu = 2$, we obtain

$$\dot{e}_{\mathbf{q}}(t) + \mu^{-1} e_{\mathbf{q}}(t) = 0 \tag{7}$$

Assumption 1. Taking into consideration further implementation purpose, $\mathbf{q}_r(t)$ may be considered as a set of piecewise constant reference trajectories. In other words,

$$\mathbf{q}_r(t) = \mathbf{q}_r(t_i), \quad t \in [t_i, \quad t_{i+1}] \tag{8}$$

where $t_{i+1} = t_i + \delta t$, $\quad i = 0, \ldots N$ and δt denotes the sampling time.

If Assumption 1 holds, from equation (7), $\mathbf{q}^{(i)}(t)$ for $i = 0, \ldots, 2$ can be written as

$$\mathbf{q}(t) = \mu^{-1} \int_{t_0}^{t} e_{\mathbf{q}}(\tau)\, d\tau + \mathbf{q}_0 \tag{9}$$

$$\dot{\mathbf{q}}(t) = \mu^{-1} e_{\mathbf{q}}(t) \tag{10}$$

$$\ddot{\mathbf{q}}(t) = \mu^{-1} \dot{e}_{\mathbf{q}}(t) \tag{11}$$

where $\mathbf{q}_0$ denotes the initial value of $\mathbf{q}$.

By introducing two weighting diagonal matrices $\alpha \in \mathbb{R}^{n\times n}$ and $\beta \in \mathbb{R}^{n\times n}$ satisfying the condition $\alpha + \beta = I_{n\times n}$, equation (11) becomes

$$\ddot{\mathbf{q}}(t) = \mu^{-1} \alpha\, \dot{e}_{\mathbf{q}}(t) + \mu^{-1} \beta\, \dot{e}_{\mathbf{q}}(t) \tag{12}$$

If Assumption 1 holds, (12) may be written as

$$\ddot{\mathbf{q}}(t) = \mu^{-1} \alpha\, \dot{e}_{\mathbf{q}}(t) - \mu^{-1} \beta\, \dot{\mathbf{q}}(t) \tag{13}$$

Notice that $\dot{\mathbf{q}}(t)$ in equation (13) can be also expressed using equation (10). Substituting $\dot{\mathbf{q}}(t)$, we get,

$$\ddot{\mathbf{q}}(t) = \mu^{-1}(\alpha \dot{e}_{\mathbf{q}}(t) - \mu^{-1} \beta e_{\mathbf{q}}(t)) \tag{14}$$

By using relationships (9)-(10) and (14), controller (4) can be written as

$$\begin{aligned} u &= \mathscr{U}(e_{\mathbf{q}}, \dot{e}_{\mathbf{q}}) \\ &= G(e_{\mathbf{q}})^{\perp}(\mu^{-1}(\alpha \dot{e}_{\mathbf{q}}(t) - \mu^{-1} \beta e_{\mathbf{q}}(t)) - F(e_{\mathbf{q}}, \dot{e}_{\mathbf{q}})) \end{aligned} \tag{15}$$

4. ACTIVE DISTURBANCE REJECTION BASED BOOSTER

4.1 Nominal controller robustness analysis

Usually, model (3) is a simplified version of the real system where some dynamics are unmodeled or neglected. Furthermore, external disturbances as the wind are constantly affecting the system leading to a remarkable mismatch between the nominal model and the real plant.

For a complete representation of the system, an additive disturbance term $\Delta \in \mathbb{R}^n$ is introduced. Consequently, model (3) becomes

$$\ddot{\mathbf{q}}(t) = F(\mathbf{q}, \dot{\mathbf{q}}) + G(\mathbf{q})\, u(t) + \Delta \tag{16}$$

where Δ may result from external disturbances (constant or time-varying), parametric uncertainties, neglected dynamics, etc. $\|\Delta\| \leq \Delta_{max}$ with $\Delta_{max} \geq 0$ considers the system physical limits.

The nominal reference model-based controller (15) is designed in the ideal case (i.e. without considering the disturbances effect Δ). This is because that this strategy is a model-based control technique. Therefore, the performance of control will degrade leading to the instability in the extreme conditions of disturbances. For this reason, we seek to see the effect of the disturbances when using just the nominal controller.

Applying controllers (15) to system (16), we get the following

$$\ddot{e_{\mathbf{q}}}(t) = \mu^{-1}(-\alpha \dot{e}_{\mathbf{q}}(t) + \mu^{-1} \beta e_{\mathbf{q}}(t)) - \Delta \tag{17}$$

We check, in the following, the stability conditions of the disturbed system.

Theorem: *System* (17) *is stable if*

$$\Delta_{max} \leq eig_{min}(K_D) \|\dot{e}_{\mathbf{q}}\| \tag{18}$$

Proof: *To prove the stability of system* (17)*, $\mathscr{V} \in \mathbb{R}$ is chosen as a Lyapunov candidate function. It is given by*

$$\mathscr{V} = \frac{1}{2} e_{\mathbf{q}}^T K_1 e_{\mathbf{q}} + \frac{1}{2} \dot{e}_{\mathbf{q}}^T K_2 \dot{e}_{\mathbf{q}} \tag{19}$$

where $K_1 \in \mathbb{R}^{n\times n}$ and $K_2 \in \mathbb{R}^{n\times n}$ are positive definite matrices. Thus, we compute the first time derivative of $\mathscr{V}$ along system dynamics (17)*.*

$$\begin{aligned} \dot{\mathscr{V}} &= \dot{e}_{\mathbf{q}}^T K_1 e_{\mathbf{q}} + \ddot{e}_{\mathbf{q}}^T K_2 \dot{e}_{\mathbf{q}} \\ &= \dot{e}_{\mathbf{q}}^T K_1 e_{\mathbf{q}} + (\mu^{-1}(-\alpha \dot{e}_{\mathbf{q}} + \mu^{-1} \beta e_{\mathbf{q}}) - \Delta)^T K_2 \dot{e}_{\mathbf{q}} \end{aligned} \tag{20}$$

Select the adequate matrices K_1, K_2, α and β, system can be simplified as

$$\dot{\mathscr{V}} = (-\mu^{-1} \alpha \dot{e}_{\mathbf{q}} - \Delta)^T K_2 \dot{e}_{\mathbf{q}} \tag{21}$$

Then

$$\begin{aligned} \dot{\mathscr{V}} &\leq -\dot{e}_{\mathbf{q}}^T \overbrace{\alpha^T (\mu^{-1})^T}^{K_D} K_2 \dot{e}_{\mathbf{q}} + \Delta_{max} \|K_2 \dot{e}_{\mathbf{q}}\| \\ &= -\dot{e}_{\mathbf{q}}^T K_D K_2 \dot{e}_{\mathbf{q}} + \Delta_{max} \|K_2 \dot{e}_{\mathbf{q}}\| \end{aligned} \tag{22}$$

Taking

$$\Delta_{max} \leq eig_{min}(K_D) \|\dot{e}_{\mathbf{q}}\| \tag{23}$$

Obviously, $\dot{\mathscr{V}}$ is negative semidefinite. As result, the closed-loop of disturbed system (16)*, applying the nominal controller* (15)*, is at least stable.*

According to this result, more the disturbance effect is big more the stability of the system is questioned due to the fact that it depends on the control parameters.

4.2 Booster design

Most of model-based control strategies require a deep analysis of the system non-linearities. However, extracting the complete model of a given system is almost a challenging task. For this reason, the exploited models are almost simplified. Notice that ADRC employs usually observers to estimate the overall dynamics and disturbances of the system assuming no available model.

Therefore, in our approach, we will involve the active disturbance rejection principle to handle the unknown parts of the system (i.e. disturbances, unmodeled dynamics, etc.) only and thus boost the abilities of the nominal controller (reference model-based controller). This latter deals with the modeled part only considering a target behavior for the desired performance.

Let us expose, in more details, our proposed approach through class of systems (16). Therefore, we suggest a control law given by

$$u = \mathscr{U}(e_{\mathbf{q}}, \dot{e}_{\mathbf{q}}) + \delta_u \tag{24}$$

We seek now to design the auxiliary input δ_u, which can certainly increase the performance of nominal input $\mathscr{U}(e_{\mathbf{q}}, \dot{e}_{\mathbf{q}})$. Notice that $\mathscr{U}(e_{\mathbf{q}}, \dot{e}_{\mathbf{q}})$ is derived, in the previous section.

Using the experimentally available data, we can estimate the unknown quantity Δ. This estimation denoted by $\widehat{\Delta}$ is valid for a short time δ_t only and should be continuously updated at each instant t. Many approaches are used for the estimation as the model-free principle. However, here an Extended State-based Observer (ESO) is considered where a detailed explanation is provided in Sub-section 4.3.

The term $\widehat{\Delta}$ captures all the unknown dynamics of the system (e.g. the external disturbances, neglected dynamics, etc.) during each iteration of the control algorithm and then brings the necessary changes in the main control loop by compensation. Thus, from (24), the additional term is selected to be

$$\delta_u = G(\mathbf{q})^{\perp}(-\widehat{\Delta} + \ddot{\mathbf{q}}_r) \tag{25}$$

$\delta_u(t)$ enhances the nominal controller. The complete controller is then

$$u = G(e_{\mathbf{q}})^{\perp}(\mu^{-1}(\alpha \dot{e}_{\mathbf{q}}(t) - \mu^{-1}\beta e_{\mathbf{q}}(t)) - F(e_{\mathbf{q}}, \dot{e}_{\mathbf{q}})) + G(\mathbf{q})^{\perp}(-\widehat{\Delta} + \ddot{\mathbf{q}}_r) \tag{26}$$

From (26), we recognize that the strategy is not pure data-driven. In other words, the differential equations of the system are used to design the nominal control laws where the input-output data are involved for the compensation of the unknown parts (see equation (25)).

4.3 Observer design

The main idea is to consider an Extended State Observer (ESO) that provides an estimate $\widehat{\Delta}$. This estimated term is updated at each iteration in order to compensate the unknown term Δ. This means that a rejection is ensured on-line continuously, which bring a robustness property to the reference model-based control strategy.

In order to built the estimator, a state space description is necessary. Let the vector.

$$X = [\mathbf{q}, \mathbf{p}, \mathbf{r}]^T = [\mathbf{q}, \dot{\mathbf{q}}, \Delta]^T \tag{27}$$

be the extended state vector where $\mathbf{r}$ denotes the disturbance term.

Disturbed model (16) can be rewritten as

$$\begin{bmatrix}\dot{\mathbf{q}}\\ \dot{\mathbf{p}}\\ \dot{\mathbf{r}}\end{bmatrix} = \begin{bmatrix} O_{n\times n} & I_{n\times n} & O_{n\times n}\\ O_{n\times n} & F(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}^{\perp} & I_{n\times n}\\ O_{n\times n} & O_{n\times n} & O_{n\times n}\end{bmatrix}\begin{bmatrix}\mathbf{q}\\ \mathbf{p}\\ \mathbf{r}\end{bmatrix} + \begin{bmatrix}O_{n\times m}\\ G(\mathbf{q})\\ O_{n\times m}\end{bmatrix} u + \begin{bmatrix}O_{n\times n}\\ O_{n\times n}\\ I_{n\times n}\end{bmatrix}\dot{\Delta} \tag{28}$$

with output vector $\mathbf{Y}$ given by

$$\mathbf{Y} = [I_{n\times n}\ O_{n\times n}\ O_{n\times n}]\begin{bmatrix}\mathbf{q}\\ \mathbf{p}\\ \mathbf{r}\end{bmatrix} \tag{29}$$

The residual virtual input $\dot{\Delta}$ cannot be measured. Thus the observer for system (28) can only be built using the systems data that are the $u(t)$, and the output $\mathbf{Y}(t)$. The estimated state $\widehat{\mathbf{r}}$ will provide an approximate value of Δ.

The equations for the extended state observer are given in equation (30). Herein, we adapt the well-known Luenberger based observer to form (28).

$$\begin{bmatrix}\dot{\widehat{\mathbf{q}}}\\ \dot{\widehat{\mathbf{p}}}\\ \dot{\widehat{\mathbf{r}}}\end{bmatrix} = \begin{bmatrix} O_{n\times n} & I_{n\times n} & O_{n\times n}\\ O_{n\times n} & F(\widehat{\mathbf{q}},\widehat{\dot{\mathbf{q}}})\widehat{\dot{\mathbf{q}}}^{\perp} & I_{n\times n}\\ O_{n\times n} & O_{n\times n} & O_{n\times n}\end{bmatrix}\begin{bmatrix}\widehat{\mathbf{q}}\\ \widehat{\mathbf{p}}\\ \widehat{\mathbf{r}}\end{bmatrix} + \begin{bmatrix}O_{n\times m}\\ G(\mathbf{q})\\ O_{n\times m}\end{bmatrix} u + \begin{bmatrix}L_1\\ L_2\\ L_3\end{bmatrix}(\mathbf{Y} - \widehat{\mathbf{q}}) \tag{30}$$

where $L_i \in R^{n\times n}, i = 1,2,3$ are diagonal positive definite matrices.

The above observer can be arranged as

$$\begin{bmatrix}\dot{\widehat{\mathbf{q}}}\\ \dot{\widehat{\mathbf{p}}}\\ \dot{\widehat{\mathbf{r}}}\end{bmatrix} = \begin{bmatrix} -L_1 & I_{n\times n} & O_{n\times n}\\ -L_2 & F(\widehat{\mathbf{q}},\widehat{\dot{\mathbf{q}}})\widehat{\dot{\mathbf{q}}}^{\perp} & I_{n\times n}\\ -L_3 & O_{n\times n} & O_{n\times n}\end{bmatrix}\begin{bmatrix}\widehat{\mathbf{q}}\\ \widehat{\mathbf{p}}\\ \widehat{\mathbf{r}}\end{bmatrix} + \begin{bmatrix}O_{n\times m}\\ G(\mathbf{q})\\ O_{n\times m}\end{bmatrix} u + \begin{bmatrix}L_1\\ L_2\\ L_3\end{bmatrix}\mathbf{Y} \tag{31}$$

One can now use the estimated variables, $\widehat{\mathbf{q}}$, $\widehat{\mathbf{p}}$, $\widehat{\mathbf{r}}$, to implement the disturbance rejection where the controller is shown in equation (26).

This controller presents several advantages. It has an acceptable robustness level. Its structure is quite simple and combines the ADRC principle with a reference-model based control strategy. This combination leads to a nice compromise performance/robustness. This is due to the reference model-based strategy ability in meeting the desired control performance and the ability of ADRC in estimating disturbances and the modeling errors.

4.4 Estimation error and analysis

Applying controller (26) to system (16), leads to the closed-loop system

$$\ddot{e_{\mathbf{q}}}(t) = \mu^{-1}(-\alpha \dot{e}_{\mathbf{q}}(t) + \mu^{-1}\beta e_{\mathbf{q}}(t)) + \tilde{\Delta} \tag{32}$$

where $e_{\mathbf{q}} = \mathbf{q}_r - \mathbf{q}$ is the tracking error and $\tilde{\Delta} = \widehat{\Delta} - \Delta$ is the estimation error. This latter is assumed bounded.

Without loss of generality, equation (32) can be normalized and rewritten as

$$\ddot{e}_{\mathbf{q}} + K_2\dot{e}_{\mathbf{q}} + K_1 e_{\mathbf{q}} = \tilde{\Delta} \tag{33}$$

with $K_1 \in \mathbb{R}^{n\times n}$ and $K_2 \in \mathbb{R}^{n\times n}$ are positive definite matrices. They are chosen to ensure the stability of the system [1]

Based on equation (33), we can claim that the performance of the control is highly related to the accuracy of the estimation of Δ. If $\tilde{\Delta}$ is small, the estimation is good. Because of the estimation error $\tilde{\Delta}$, the tracking errors $e_{\mathbf{q}}$ will never go to the origin but it will nevertheless remain in its neighborhood, which is related to the boundedness of $\tilde{\Delta}$. Thus, in order to bring additional improvements, we propose to introduce an additional term involving a sliding modes framework.

5. SLIDING MODES BASED AUXILIARY INPUT

5.1 Controller design

As stated above, controller (26) is highly related to the accuracy of the estimation $\widehat{\Delta}$. So, to deal with the steady-state error, an additional effort $v(t) \in \mathbb{R}^n$ is needed where (24) becomes

$$u = \mathscr{U}(e_{\mathbf{q}}, \dot{e}_{\mathbf{q}}) + \delta_u + G(\mathbf{q})^{\perp}(\beta_\delta v(t)) \tag{34}$$

where $\beta_\delta \in \mathbb{R}^{n\times n}$ is a scale matrix fixed by the user.

Substituting (34) in disturbed model (16), we get

$$\ddot{e}_{\mathbf{q}} + K_2\dot{e}_{\mathbf{q}} + K_1 e_{\mathbf{q}} = \widetilde{\Delta}(t) - \beta_\delta v(t) \tag{35}$$

where $||\widetilde{\Delta}(t)|| \leq \delta_{max}$.

[1] They should satisfy the Hurwitz criterion.

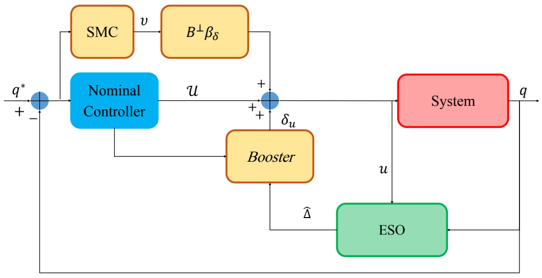

Fig. 2. Overall proposed control architecture.

The additional input $v(t)$ is designed to compensate the estimation error. Therefore, we investigate the sliding modes framework by selecting the following sliding surface $\mathbf{S}(t) \in \mathbb{R}^n$:

$$\mathbf{S}(t) = \dot{e}_{\mathbf{q}}(t) + \lambda e_{\mathbf{q}} \tag{36}$$

where $\lambda \in \mathbb{R}^{n\times n}$ is a positive definite diagonal matrix to refine the rate of convergence.

The first time derivative of $\mathbf{S}(t)$ is

$$\dot{\mathbf{S}}(t) = \ddot{e}_{\mathbf{q}}(t) + \lambda \dot{e}_{\mathbf{q}}(t) \tag{37}$$

Substituting $\ddot{e}_{\mathbf{q}}$ from (35) in equation (37), we obtain

$$\dot{\mathbf{S}}(t) = \widetilde{\Delta}(t) - \beta_\delta v(t) + \lambda \dot{e}_{\mathbf{q}}(t) - K_2 \dot{e}_{\mathbf{q}} - K_1 e_{\mathbf{q}} \tag{38}$$

Usually, the sliding modes controller contains two terms, which are the equivalent term v_{eq} and the switching term v_{sw} (also known as discontinuous term). Notice that v_{eq} occurs when $\dot{\mathbf{S}} = 0$ and v_{sw} drives the system states toward the sliding mode. Thus, the overall sliding-mode controller can be written as following

$$\begin{aligned} v(t) &= v_{eq}(t) + v_{sw}(t) \\ &= v_{eq}(t) + \gamma_1 \mathbf{S}(t) + \gamma_2 tanh(\mathbf{S}(t)) \end{aligned} \tag{39}$$

where $\gamma_1 \in \mathbb{R}^{n\times n}$ and $\gamma_2 \in \mathbb{R}^{n\times n}$ are positive definite diagonal matrices. $tanh(.)$ is the hyperbolic tangent function that acts on all the components of $\mathbf{S}(t)$.

By making $\mathbf{S}(t) = 0$, we extract the equivalent term. Doing some computations, the auxiliary sliding mode controller is obtained and is given by

$$v(t) = \beta_\delta^{-1}\left(\bar{\Delta} + \lambda \dot{e}_{\mathbf{q}}(t) - K_2 \dot{e}_{\mathbf{q}} - K_1 e_{\mathbf{q}}\right) + \gamma_1 \mathbf{S} + \gamma_2 tanh(\mathbf{S}) \tag{40}$$

where $\bar{\Delta} \in \mathbb{R}^n$ contains the positive upper limit of each component of $\widetilde{\Delta}(t)$ with $||\bar{\Delta}|| \leq \Delta_{max}$.

The overall control architecture, after involving this sliding mode - like term, is summarized by Figure 2.

6. RESULTS AND DISCUSSION

The proposed controller is applied to the quadrotor [2], considering an hierarchical control architecture, to show briefly its effectiveness. For more information about the application details, the reader may solicit one of our previous papers as for instance Bouzid et al. (2017b).

Herein, the quadrotor is requested to follow a smooth time varying trajectory where the 3D path represents a square [3]. The reference trajectories along the three axes as well as the 3D path are displayed in Figure 3.

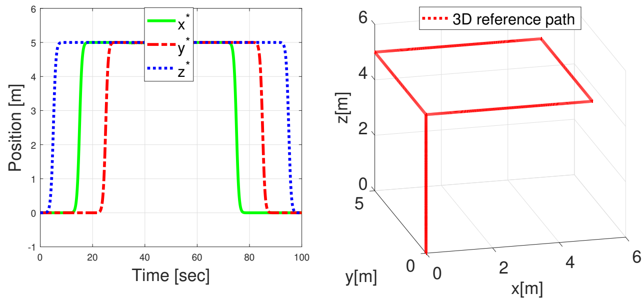

Fig. 3. Reference trajectory.

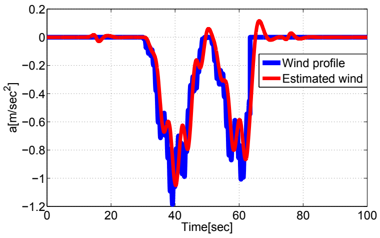

Fig. 4. Wind profile.

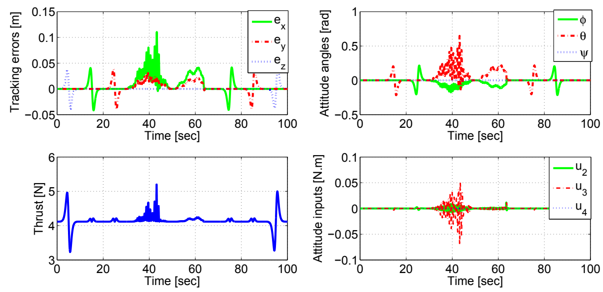

Fig. 5. Nominal controller (under wind).

We present a comparison between the nominal controller and the boosted one. For more realistic results, the quadrotor is considered flying under the effect of wind that is assumed acting in the longitudinal and the lateral directions. It has a momentary effect between the instants 30 and 65 seconds. This scenario is considered to investigate the stability of controlled system while encountering the wind.

The applied wind profile as well as the estimated one (using the proposed strategy) are displayed in Figure 4.

The results for the nominal controller are shown in Figure 5 and those of the ADRC are displayed in Figure 6. We plot separately, the attitude control inputs and the thrust, the tracking errors of the absolute position and the attitude angles.

From Figure 5, we observe that the vehicle is shifted from the reference trajectory during the gust of wind where an error of $0.15m$ is noticed along the X-axis. Moreover, the quadrotor is highly tilted from the origin especially the pitch angle. The control input are with very high magnitudes, which is logical for such technique to face the effect of disturbances. Obviously, these performances are not satisfactory and should be improved. Thus, the ADRC booster is applied. From Figure

[2] we used an AR.drone where the different parameters of the quadrotor are given in Bouzid et al. (2017a)

[3] the mathematical formulation is given in Bouzid et al. (2017b)

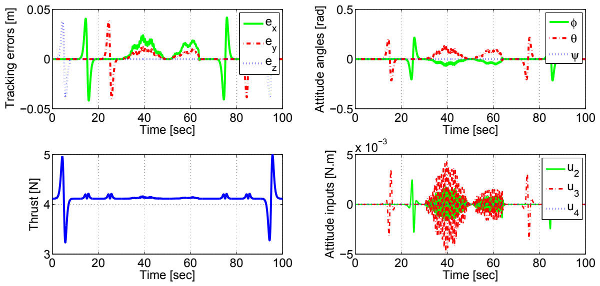

Fig. 6. Boosted controller (under wind).

6, we see that the results are clearly improved where the errors are reduced to a maximum of 0.05m with less tilted angles. Even these good results, the control inputs are with moderate values.

The obtained results confirm our claims where clear improvements are obtained through the application of the booster especially in the presence of the wind.

7. CONCLUSION

In this paper, we have presented a booster as an active disturbance rejection using an extended state based observer to deal with the various uncertainties and disturbances. The booster is applied to a reference model based control strategy and improved by involving a sliding modes-like term. This combination leads to a good compromise performance/robustness. The obtained results were very promising. The main disadvantaged of the strategy is the great number of gains, which require an optimal tuning. This last issue will be treated in our future works.

REFERENCES

Bouzid, Y., Siguerdidjane, H., and Bestaoui, Y. (2017a). 3d trajectory tracking control of quadrotor UAV with on-line disturbance compensation. In *2017 IEEE Conference on Control Technology and Applications (CCTA)*, 2082–2087. doi:10.1109/CCTA.2017.8062760.

Bouzid, Y., Siguerdidjane, H., and Bestaoui, Y. (2017b). Nonlinear internal model control applied to VTOL multi-rotors UAV. *Mechatronics*, 47, 49–66. doi:10.1016/j.mechatronics.2017.08.002. URL `http://www.sciencedirect.com/science/article/pii/S0957415817301046`.

Cai, W., She, J., Wu, M., and Ohyama, Y. (2019). Disturbance suppression for quadrotor uav using sliding-mode-observer-based equivalent-input-disturbance approach. *ISA Transactions*, 92, 286 – 297. doi:https://doi.org/10.1016/j.isatra.2019.02.028. URL `http://www.sciencedirect.com/science/article/pii/S0019057819301028`.

Castillo, A., Sanz, R., Garcia, P., Qiu, W., Wang, H., and Xu, C. (2019). Disturbance observer-based quadrotor attitude tracking control for aggressive maneuvers. *Control Engineering Practice*, 82, 14 – 23. doi:https://doi.org/10.1016/j.conengprac.2018.09.016. URL `http://www.sciencedirect.com/science/article/pii/S096706611830563X`.

Chenlu, W., Zengqiang, C., Qinglin, S., and Qing, Z. (2016). Design of PID and ADRC based quadrotor helicopter control system. In *2016 Chinese Control and Decision Conference (CCDC)*, 5860–5865. doi:10.1109/CCDC.2016.7532046.

Liu, W., Cheng, L., Hou, Z.G., Yu, J., and Tan, M. (2016). An Inversion-Free Predictive Controller for Piezoelectric Actuators Based on a Dynamic Linearized Neural Network Model. *IEEE/ASME Transactions on Mechatronics*, 21(1), 214–226. doi:10.1109/TMECH.2015.2431819.

Lotufo, M.A., Colangelo, L., Perez-Montenegro, C., Canuto, E., and Novara, C. (2019). Uav quadrotor attitude control: An adrc-emc combined approach. *Control Engineering Practice*, 84, 13 – 22. doi:https://doi.org/10.1016/j.conengprac.2018.11.002. URL `http://www.sciencedirect.com/science/article/pii/S0967066118305148`.

Miranda-Colorado, R. (2019). Finite-time sliding mode controller for perturbed second-order systems. *ISA Transactions*. doi:https://doi.org/10.1016/j.isatra.2019.05.026. URL `http://www.sciencedirect.com/science/article/pii/S0019057819302587`.

Miranda-Colorado, R., Aguilar, L.T., and Herrero-Brito, J.E. (2018). Reduction of power consumption on quadrotor vehicles via trajectory design and a controller-gains tuning stage. *Aerospace Science and Technology*, 78, 280 – 296. doi:https://doi.org/10.1016/j.ast.2018.04.027. URL `http://www.sciencedirect.com/science/article/pii/S1270963817319028`.

Mo, H. and Farid, G. (2019). Nonlinear and Adaptive Intelligent Control Techniques for Quadrotor UAV A Survey. *Asian Journal of Control*, 21(2), 989–1008. doi:10.1002/asjc.1758. URL `https://onlinelibrary.wiley.com/doi/abs/10.1002/asjc.1758`.

Nahas, E.P., Henson, M.A., and Seborg, D.E. (1992). Nonlinear internal model control strategy for neural network models. *Computers & Chemical Engineering*, 16(12), 1039–1057. doi:10.1016/0098-1354(92)80022-2. URL `http://www.sciencedirect.com/science/article/pii/0098135492800222`.

Shi, X., Cheng, Y., Yin, C., Dadras, S., and Huang, X. (2019). Design of Fractional-Order Backstepping Sliding Mode Control for Quadrotor UAV. *Asian Journal of Control*, 21(1), 156–171. doi:10.1002/asjc.1946. URL `https://onlinelibrary.wiley.com/doi/full/10.1002/asjc.1946`.

Wang, X., Lin, X., Yu, Y., Wang, Q., and Sun, C. (2017). Backstepping control for quadrotor with BP neural network based thrust model. In *2017 32nd Youth Academic Annual Conference of Chinese Association of Automation (YAC)*, 292–297. doi:10.1109/YAC.2017.7967422.

Yuan, Y., Cheng, L., Wang, Z., and Sun, C. (2018). Position tracking and attitude control for quadrotors via active disturbance rejection control method. *Science China Information Sciences*, 62(1), 10201. doi:10.1007/s11432-018-9548-5. URL `https://doi.org/10.1007/s11432-018-9548-5`.

Zhao, L., Dai, L., Xia, Y., and Li, P. (2019). Attitude control for quadrotors subjected to wind disturbances via active disturbance rejection control and integral sliding mode control. *Mechanical Systems and Signal Processing*, 129, 531 – 545. doi:https://doi.org/10.1016/j.ymssp.2019.04.040. URL `http://www.sciencedirect.com/science/article/pii/S0888327019302791`.

.