Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

A stochastic optimization approach to the aggregation of electric vehicles for the provision of ancillary services

M. Rossini * **C. Sandroni** ** **R. Vignali** **

* *Università degli Studi di Milano, Milano, Italy (e-mail: matteo.rossini@unimi.it).*
** *Ricerca sul Sistema Energetico – RSE S.p.A., Milano, Italy (e-mail: {carlo.sandroni, riccardo.vignali}@rse-web.it).*

Abstract: We address the problem of the optimal management of an aggregate of electric vehicles (EVs) for the provision of ancillary services to the grid, by means of a bidirectional vehicle-to-grid (V2G) infrastructure. We consider the case of a charging point operator that acts as an aggregator and has to optimally choose the charge/discharge power profile of each vehicle so as to maximize its profits, while satisfying technical constraints and final user constraints (the latter expressed as a minimum desired charge for motion). In this setting the aggregator can operate on both an *energy market* and an *ancillary services market*: in the latter, the deployed power depends on a signal received by the aggregator after the market closing time; this signal can be discrete or continuous. We formulate the problem via stochastic programming, under the assumptions of optimal bidding strategy and known vehicle arrivals and departures. We obtain, via mixed-integer linear programming, an exact robust counterpart of the constraints and an expected value cost function, which is exact if the signal is discrete. If the signal is continuous, the cost function varies depending on the probability distribution of the signal and could require an approximation to obtain a computationally tractable formulation. We then show that, in the case of uniform probability, an efficient formulation can be obtained by introducing a negligible approximation of the cost function; a numerical example shows the validity of the approach.

Keywords: Stochastic optimal control problems, Robust control applications, Power Systems.

1. INTRODUCTION

The increase in electric vehicle (EV) penetration is rapidly opening the opportunity for the introduction of vehicle-to-grid (V2G) power. This is the possibility for an EV to provide ancillary services to the main grid by appropriately charging and discharging. Because of the limited power capacity of each vehicle, the participation to the market of a single EV must be mediated by an aggregator, that is the owner or the manager of a parking lot equipped with EV charging stations. This actor operates on the market by joining the capacity of the vehicles in its fleet and earns profits by offering ancillary services: these profits can be then shared with the EV users.

The topic of V2G is gaining more and more interest in the literature of both control and power systems communities. In Kempton and Tomić (2005a) and Kempton and Tomić (2005b) the positive impacts and the actual applicability of V2G are discussed on both the technical and economical side. The problem discussed in this work, i.e., how to optimally charge and discharge a fleet of EVs, is discussed in Sortomme and El-Sharkawi (2010), Sortomme and El-Sharkawi (2011), He et al. (2012), Vagropoulos and Bakirtzis (2013), Sarker et al. (2015), to name a few. However, these works either consider a deterministic setting or consider an expected value reformulation of the constraints. Since the deployment level of the ancillary services to be provided is both unknown in advance and binding on the aggregator, a *robust counterpart* (Ben-Tal et al. (2009)) of the problem should be sought, so as to ensure the satisfaction of the constraints for any possible value of the received signal. The problem of computing a robust counterpart of constraints involving the state of a dynamical system is strictly related to the reachability analysis problem (Bemporad and Morari (1999b)). In the case of piecewise affine systems (as the ones considered in this paper), this is in general a hard problem (Bemporad and Morari (1999b)); however, in our setting, thanks to the particular structure of the considered systems, we are able to provide an exact tightening of the constraints via mixed-integer linear programming. As for the cost function, we choose to maximize the expected value of the profits. Finally, we validate the overall stochastic approach on a numerical case study.

The rest of the paper unfolds as follows. In Section 2 we introduce our assumptions and nomenclature of variables. In Section 3 we assume the services signal known, so that a deterministic formulation of the problem is obtained. In Section 4 this assumption is removed and we derive the stochastic reformulation of constraints and cost function. In Section 5 we validate the approach on a numerical example, whereas in Section 6 we draw some conclusion remarks.

2. ASSUMPTIONS AND NOMENCLATURE

A schematic of the parking lot with the names of the variables is depicted in Fig. 1; the complete lists of model variables and parameters are shown in Tables 1 and 2.

2.1 Sets, indices and units of measurement

The parking lot is composed by n slots, indexed as $\mathcal{I} = \{1, \dots, n\}$. We assume that each slot $i \in \mathcal{I}$ is equipped with one charging station that is able to charge one EV at a time. The considered time horizon is denoted by $\mathcal{T} = \{1, \dots, T\}$; each optimization period $t \in \mathcal{T}$ has a duration of τ hours. Time is expressed in hours, power in kW, energy in kWh. The variables and parameters involving power follow the load convention: positive when power flows from the grid to the EVs.

2.2 EV charge/discharge

EV users pay the aggregator a tariff c^{v+}, proportional to the charged energy; whenever the aggregator decides to temporarily discharge a vehicle, it returns the EV user a tariff $c^{v-} > c^{v+}$, in order to compensate for the greater battery degradation.

In every period t in which an EV is parked at slot i, the charge in its battery is allowed to range between parameters $E_{t,i}^{\min}$ and $E_{t,i}^{\max}$, which can be respectively set e.g. to 0 and the battery capacity, or other less extreme values; at departure time, $E_{t,i}^{\min}$ represents the minimum charge for motion required by the user.

We consider that the availability of each vehicle at a given time is known in advance: this assumption is satisfied in, e.g., public transport parking lots, where arrivals and departures are scheduled. Extending the model to deal with stochastic availability of vehicles will be object of future research work.

We consider both the charging station efficiency (η^{g+}, η^{g-}) and the vehicle efficiency (η^{v+}, η^{v-}); the $^+$ and $^-$ superscripts stand for charge and discharge case, respectively. Distinguishing between charging station and vehicle efficiencies allows to accommodate both the AC and the DC charge/discharge scenarios. Furthermore, it enables to compute the power flow at grid interface (needed for evaluating market bids), at the interface between charging stations and vehicles (needed for computing the charging/discharging profit), and at vehicles batteries (needed for the state of charge estimation).

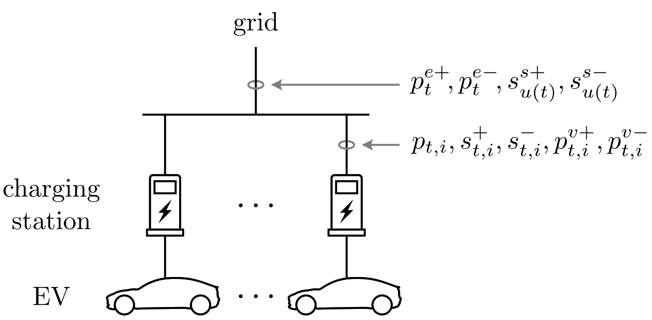

Fig. 1. Parking lot schematic with variable names.

Table 1. List of model variables.

NAME	DOMAIN	DESCRIPTION
p_t^{e+}	$\mathbb{R}^+$	Positive part of power (buy bid) traded on energy market in period t
p_t^{e-}	$\mathbb{R}^+$	Negative part of power (sell bid) traded on energy market in period t
$s_{u(t)}^{s+}$	$\mathbb{R}^+$	Positive part of power (buy bid) traded on ancillary services market in period $u(t)$
$s_{u(t)}^{s-}$	$\mathbb{R}^+$	Negative part of power (sell bid) traded on ancillary services market in period $u(t)$
$p_{t,i}$	$\mathbb{R}$	Portion of $p_t^{e+} - p_t^{e-}$ relating to slot i
$s_{t,i}^{+}$	$\mathbb{R}^+$	Portion of $s_{u(t)}^{s+}$ relating to slot i
$s_{t,i}^{-}$	$\mathbb{R}^+$	Portion of $s_{u(t)}^{s-}$ relating to slot i
$p_{t,i}^{v+}$	$\mathbb{R}^+$	Positive part of power exchanged with slot i in period t [deterministic formulation only]
$p_{t,i}^{v-}$	$\mathbb{R}^+$	Negative part of power exchanged with slot i in period t [deterministic formulation only]

Table 2. List of model parameters.

NAME	DOMAIN	DESCRIPTION
ω_t^+	[0,1]	Positive part of ancillary services signal in period t [deterministic formulation only]
ω_t^-	[0,1]	Negative part of ancillary services signal in period t [deterministic formulation only]
$d_{t,i}$	$\mathbb{N}$	0 if slot i is empty in period t; else, number of periods since EV arrival, counting from 0
$E_{t,i}^{\max}$	$\mathbb{R}^+$	Maximum allowed energy for slot i at end of period t
$E_{t,i}^{\min}$	$\mathbb{R}^+$	Minimum allowed energy for slot i at end of period t
$E_{t,i}^{0}$	$\mathbb{R}^+$	Energy injection in slot i at begin of period t resulting from the arrival of an EV in the preceding period
$p_{t,i}^{v,\max}$	$\mathbb{R}^+$	Max charge power of the EV in slot i in period t
$p_{t,i}^{v,\min}$	$\mathbb{R}^-$	Max discharge power of the EV in slot i in period t
$p^{g,\max}$	$\mathbb{R}^+$	Max imported power from the grid
$p^{g,\min}$	$\mathbb{R}^-$	Max exported power to the grid
c_t^{e+}	$\mathbb{R}^+$	Price of buy bids on the energy market in period t
c_t^{e-}	$\mathbb{R}^+$	Price of sell bids on the energy market in period t
$c_{u(t)}^{s+}$	$\mathbb{R}^+$	Price of buy bids on the ancillary services market in period t
$c_{u(t)}^{s-}$	$\mathbb{R}^+$	Price of sell bids on the ancillary services market in period t
c^{v+}	$\mathbb{R}^+$	EVs charging tariff
c^{v-}	$\mathbb{R}^+$	EVs discharging tariff
η^{g+}	$(0,1)$	Average charge efficiency of charging stations
η^{g-}	$(0,1)$	Average discharge efficiency of charging stations
η^{v+}	$(0,1)$	Average charge efficiency of EVs
η^{v-}	$(0,1)$	Average discharge efficiency of EVs
λ	$\mathbb{R}^+\backslash\{0\}$	Average self-discharge rate of EVs
τ	$\mathbb{R}^+$	Duration of each period
m	$\mathbb{N}\backslash\{0\}$	Ratio between ancillary services market and energy market period duration

2.3 Markets

In our framework, the aggregator can trade energy by operating in an *energy market* and in an *ancillary services market*. Both the terminology related to ancillary services and their implementation can vary significantly from country to country, due to development in different technical and normative contexts: interested readers are referred to overviews such Banshwar et al. (2018) and Kaushal and Van Hertem (2019).

The supply of the energy market is sure, i.e. the deployed power is determined exclusively by the accepted bids. As for the ancillary services market, we assume that the deployed power depends also on a *signal* $\{\omega_t\}_{t\in\mathcal{T}}$ distributed after the market closing time by the Transmission System Operator (TSO) or by the Distribution System Operator (DSO). This signal assumes values in a set Ω that can be discrete or continuous. If discrete, the possible values for ω_t are: -1 "deploy the sell bid", 0 "do nothing", and $+1$ "deploy the buy bid"; else, in the continuous case, the signal represents the deployment level in the $[-1,+1]$ interval, where the values -1, 0 and $+1$ have the same meaning of the discrete case.

For every period t, the signal ω_t is an uncertain coefficient for the optimization problem: we assume that its probability distribution is known (in practice, it can be estimated from historical data). If Ω is discrete, for every $t\in\mathcal{T}$ the probability distribution of ω_t is completely characterized by its values: $\pi_t^+ = P(\omega_t = +1)$, $\pi_t^- = P(\omega_t = -1)$, and clearly $P(\omega_t = 0) = 1 - \pi_t^+ - \pi_t^-$. In the case where Ω is continuous, the cost function of our stochastic model depends on the probability distribution of each random variable ω_t with $t\in\mathcal{T}$; we expose the model with two further assumptions: that $\{\omega_t\}_{t\in\mathcal{T}}$ is a collection of independent random variables, and that all of them are uniformly distributed over $[-1,+1]$.

We consider the duration τ of the optimization periods equal to that of the energy market periods. The ancillary services market is allowed to have a periodicity which is a multiple m of τ; a function $u\colon t\mapsto \lceil \frac{t}{m}\rceil$ is used to obtain the period indices of the ancillary services market.

We also assume that the aggregator has an optimal bidding strategy, such that its bids in both markets are always accepted. The prices at which its bids are placed in the energy and ancillary services markets are denoted by c_t^{e+}, c_t^{e-} and $c_{u(t)}^{s+}$, $c_{u(t)}^{s-}$ respectively, where the $^+$ and $^-$ superscripts stand for buy bid and sell bid respectively. We further assume that

$$c_t^{e+} > c_t^{e-} \qquad \forall t\in\mathcal{T}. \tag{1}$$

3. DETERMINISTIC PROBLEM FORMULATION

In this section we assume that the ancillary services signal $\{\omega_t\}_{t\in\mathcal{T}}$ is known.

3.1 Constraints

Power grid connection Power exchanged with the grid is the sum of the power exchanged on the energy market and on the ancillary services market; it is limited by:

$$p^{g,\min} \le \sum_{i\in\mathcal{I}} p_{t,i} + \omega_t^+ s_{u(t)}^{s+} - \omega_t^- s_{u(t)}^{s-} \le p^{g,\max} \quad \forall t\in\mathcal{T}. \tag{2}$$

The actual power bought (sold) on ancillary services market is the product of $s_{u(t)}^{s+}$ ($s_{u(t)}^{s-}$) and the positive (negative) part ω_t^+ (ω_t^-) of the signal.

Bids We introduce a limit on each term appearing in (2). The maximum range for power traded on the energy market is given by:

$$p^{g,\min} \le \sum\nolimits_{i\in\mathcal{I}} p_{t,i} \le p^{g,\max} \qquad \forall t\in\mathcal{T}. \tag{3}$$

The maximum range for $s_{u(t)}^{s+}$ and $s_{u(t)}^{s-}$ is modeled so as to allow the possibility of providing a power that span the entire capacity of the parking lot:

$$s_{u(t)}^{s+} \le p^{g,\max} - p^{g,\min} \qquad \forall t\in\mathcal{T} \tag{4}$$

$$s_{u(t)}^{s-} \le p^{g,\max} - p^{g,\min} \qquad \forall t\in\mathcal{T}. \tag{5}$$

Nonnegativity of the variables $s_{u(t)}^{s+}$ and $s_{u(t)}^{s-}$ will be enforced by constraints (8), (9), and (10).

Balance constraints Bids on both markets are split among the slots via constraints:

$$p_t^{e+} - p_t^{e-} = \sum\nolimits_{i\in\mathcal{I}} p_{t,i} \qquad \forall t\in\mathcal{T} \tag{6}$$

$$p_t^{e+} \ge 0, \quad p_t^{e-} \ge 0 \qquad \forall t\in\mathcal{T} \tag{7}$$

$$s_{u(t)}^{s+} = \sum\nolimits_{i\in\mathcal{I}} s_{t,i}^+ \qquad \forall t\in\mathcal{T} \tag{8}$$

$$s_{u(t)}^{s-} = \sum\nolimits_{i\in\mathcal{I}} s_{t,i}^- \qquad \forall t\in\mathcal{T} \tag{9}$$

$$s_{t,i}^+ \ge 0, \quad s_{t,i}^- \ge 0 \qquad \forall t\in\mathcal{T},\ \forall i\in\mathcal{I}. \tag{10}$$

Thanks to function u, the values of $s_{u(t)}^{s+}$ and $s_{u(t)}^{s-}$ are constant for m consecutive periods, fulfilling the periodicity requirement of the ancillary services market; nevertheless, the values of $s_{t,i}^+$ and $s_{t,i}^-$ can change in consecutive periods. The reason to split the bids among all slots is to facilitate the extension of the model to the stochastic formulation and will be more clear in Section 4.

EVs charging/discharging power Power exchanged with EVs is limited by:

$$\eta^{g-} p_{t,i}^{v,\min} \le p_{t,i} + \omega_t^+ s_{t,i}^+ - \omega_t^- s_{t,i}^- \le \frac{1}{\eta^{g+}} p_{t,i}^{v,\max} \qquad \forall t\in\mathcal{T},\ \forall i\in\mathcal{I} \tag{11}$$

where $p_{t,i}^{v,\min}\in\mathbb{R}^-$ and $p_{t,i}^{v,\max}\in\mathbb{R}^+$ are quantified at the interface between the charging station and the EV. The energetic modeling of the EVs requires to separate the above power in its positive and negative parts:

$$p_{t,i}^{v+} - p_{t,i}^{v-} = p_{t,i} + \omega_t^+ s_{t,i}^+ - \omega_t^- s_{t,i}^- \quad \forall t\in\mathcal{T},\ \forall i\in\mathcal{I} \tag{12}$$

$$p_{t,i}^{v+} \ge 0 \qquad \forall t\in\mathcal{T},\ \forall i\in\mathcal{I} \tag{13}$$

$$p_{t,i}^{v-} \ge 0 \qquad \forall t\in\mathcal{T},\ \forall i\in\mathcal{I}. \tag{14}$$

These constraints do not prevent $p_{t,i}^{v+}$ and $p_{t,i}^{v-}$ from being both positive for the same t and i, but this never happens in optimal solutions, as will be explained in Section 3.3.

Energy stored in the EVs The energy stored in the EVs is modeled considering the self-discharge as proportional to the energy, i.e. according to the differential equation $\frac{\mathrm{d}}{\mathrm{d}t}\overline{E}(t) = \overline{p}(t) - \lambda\overline{E}(t)$, where $\overline{E}$ is the energy in the

EV battery, $\overline{p}$ is its charging/discharging power and the constant λ is the self-discharge rate expressed in h^{-1}. By solving the above equation with constant power and discretizing the time over one period t we get at its end

$$\overline{E}_{t,i} = e^{-\lambda\tau}\overline{E}_{t-1,i} + \frac{1-e^{-\lambda\tau}}{\lambda}\overline{p}_{t,i}. \tag{15}$$

Unrolling this expression over time since the EV arrival we obtain

$$\overline{E}_{t,i} = e^{-(d_{t,i}+1)\lambda\tau}E^0_{t-d_{t,i},i} + \frac{1-e^{-\lambda\tau}}{\lambda}\sum_{k=0}^{d_{t,i}} e^{-k\lambda\tau}\overline{p}_{t-k,i}. \tag{16}$$

Here, the power at the EV battery $\overline{p}_{t-k,i}$ can be readily expressed in terms of the power upstream of the charging station, by substituting, with the appropriate indices,

$$\overline{p}_{t,i} = \eta^{g+}\eta^{v+}p^{v+}_{t,i} - \frac{1}{\eta^{g-}\eta^{v-}}p^{v-}_{t,i}. \tag{17}$$

We can now impose a lower and an upper bound on the energy stored in each EV, via constraints:

$$E^{\min}_{t,i} \le \overline{E}_{t,i} \le E^{\max}_{t,i} \quad \forall t\in\mathcal{T},\ \forall i\in\mathcal{I}. \tag{18}$$

Observe that $\overline{E}_{t,i}$ is not a model variable: its presence in (18) is just as a placeholder for (16); the same holds for $\overline{p}_{t-k,i}$ in (16), which is a placeholder for (17), with the appropriate indices.

3.2 Cost function

The aim of the aggregator is to maximize its profit. We equivalently consider the minimization of the negative profit, which is given by $z = z^e + z^s + z^v$, where

$$z^e = \tau\sum\nolimits_{t\in\mathcal{T}}\left(c^{e+}_t p^{e+}_t - c^{e-}_t p^{e-}_t\right) \tag{19}$$

is the net cost of the operations on the energy market,

$$z^s = \tau\sum\nolimits_{t\in\mathcal{T}}\left(c^{s+}_{u(t)}\omega^+_t s^{s+}_{u(t)} - c^{s-}_{u(t)}\omega^-_t s^{s-}_{u(t)}\right) \tag{20}$$

is the net cost of the operations on the ancillary services market, and

$$z^v = \tau\sum_{t\in\mathcal{T}}\left(-c^{v+}\eta^{g+}\sum_{i\in\mathcal{I}}p^{v+}_{t,i} + c^{v-}\frac{1}{\eta^{g-}}\sum_{i\in\mathcal{I}}p^{v-}_{t,i}\right) \tag{21}$$

is the net cost derived from charging and discharging the EVs. Significantly, possible time-dependent costs might also be present for the aggregator, but they would not be relevant to the optimization problem, since they would add a constant term to the cost function.

3.3 Deterministic optimization problem

Combining constraints and cost function defined in this section we obtain the following optimization problem:

$$\begin{cases} \quad\ \min & (19)+(20)+(21) \\ \text{subject to} & (2),(3),(4),(5),(6),(7),(8),(9), \\ & (10),(11),(12),(13),(14),(18). \end{cases} \tag{22}$$

Problem (22) is a linear programming problem that can be solved even for large instances by means of standard solvers like CPLEX (2019).

Note that this formulation does not prevent p^{e+}_t and p^{e-}_t from being both positive for the same t. Nevertheless, because: (a) these variables appear only in the defining constraints (6), (7), and in the cost function; and (b) thanks to assumption (1), the cost function would be convex in the actual power $p^{e+}_t - p^{e-}_t$ traded on the energy market; the formulation with positive and negative parts is equivalent to an epigraph reformulation (see Boyd and Vandenberghe (2004)) and solutions having both p^{e+}_t and p^{e-}_t greater than zero are never optimal. The same argument cannot be used with $p^{v+}_{t,i}$ and $p^{v-}_{t,i}$, because, in addition to appearing in the defining constraints (12), (13), and (14) and in the cost function, these variables appear also in energy constraint (18); however, the reader could easily verify that setting both $p^{v+}_{t,i}$ and $p^{v-}_{t,i}$ to positive values for the same t and i would result in an energy leakage and, therefore, a worse cost (see Baker et al. (2012)).

4. EXTENSION TO STOCHASTIC PROGRAMMING

In this section we extend the model to the more realistic case where the signal $\{\omega_t\}_{t\in\mathcal{T}}$ is uncertain.

4.1 Robust constraints

In our application, it is natural to consider a robustified version of the constraints, so that technical limits on power flowing on the lines and in the EV batteries can be satisfied for each possible signal realization. Therefore, our aim is to construct a *robust counterpart* (Ben-Tal et al. (2009)) of the constraints of problem (22). Constraint $g(x)\le 0$ is a robust counterpart of $h(x,\omega)\le 0$, where ω is an uncertain parameter, if

$$g(x)\le 0 \quad\Rightarrow\quad h(x,\omega)\le 0 \qquad \forall\omega. \tag{23}$$

If (23) holds with $\Leftrightarrow$, constraint $g(x)\le 0$ is exact and no conservatism is added.

The constraints obtained in this section are independent of the choice of Ω (discrete or continuous) because its convex hull is the same, i.e. $[-1,+1]$; see Ben-Tal et al. (2009).

Power grid connection Exploiting the fact that $\omega^+_t\le 1$, $\omega^-_t\le 1$, and either $\omega^+_t = 0$ or $\omega^-_t = 0$, the exact robust counterpart of constraint (2) is:

$$\sum\nolimits_{i\in\mathcal{I}}p_{t,i} + s^{s+}_{u(t)} \le p^{g,\max} \qquad \forall t\in\mathcal{T} \tag{24}$$

$$\sum\nolimits_{i\in\mathcal{I}}p_{t,i} - s^{s-}_{u(t)} \ge p^{g,\min} \qquad \forall t\in\mathcal{T}. \tag{25}$$

EVs charging/discharging power Similarly to grid connection limits, the robust counterpart of constraint (11) can be obtained:

$$p_{t,i} + s^+_{t,i} \le \frac{1}{\eta^{g+}}p^{v,\max}_{t,i} \qquad \forall t\in\mathcal{T},\ \forall i\in\mathcal{I} \tag{26}$$

$$p_{t,i} - s^-_{t,i} \ge \eta^{g-}p^{v,\min}_{t,i} \qquad \forall t\in\mathcal{T},\ \forall i\in\mathcal{I}. \tag{27}$$

Energy stored in the EVs The robust counterpart of (18) can not be immediately obtained, because in (17) we are not allowed to use $p^{v+}_{t,i}$ and $p^{v-}_{t,i}$ variables defined by equality constraint (12). Indeed, it does not make sense to impose constraint (12) $\forall\omega_t$, since this would have as a unique solution $s^+_{t,i} = s^-_{t,i} = 0$. So, we can only express the power upstream of the i-th charging station as an uncertain function of the signal

$$\widetilde{p}_{t,i} = p_{t,i} + \omega^+_t s^+_{t,i} - \omega^-_t s^-_{t,i} \tag{28}$$

and redefine (17) as

$$\overline{p}_{t,i} = \begin{cases} \eta^{g+}\eta^{v+}\widetilde{p}_{t,i} & \text{if } \widetilde{p}_{t,i} \geq 0 \\ \frac{1}{\eta^{g-}\eta^{v-}}\widetilde{p}_{t,i} & \text{if } \widetilde{p}_{t,i} < 0. \end{cases} \tag{29}$$

As a consequence, the energy $\overline{E}_{t,i}$ defined in (16) now depends on the signal; thus, bounds (18) must be robustified by imposing:

$$\min_{\omega_{t-d_{t,i}},\ldots,\omega_t \in \Omega} \overline{E}_{t,i} \geq E^{\min}_{t,i} \tag{30}$$

$$\max_{\omega_{t-d_{t,i}},\ldots,\omega_t \in \Omega} \overline{E}_{t,i} \leq E^{\max}_{t,i}. \tag{31}$$

Since $\overline{E}_{t,i}$ is a monotone function with respect to $\omega_{t-d_{t,i}},\ldots,\omega_t$ variables, its minimum and maximum are attained at the extrema of Ω and can be easily calculated:

$$\min_{\omega_{t-d_{t,i}},\ldots,\omega_t \in \Omega} \overline{E}_{t,i} = e^{-(d_{t,i}+1)\lambda\tau} E^0_{t-d_{t,i},i} + \frac{1-e^{-\lambda\tau}}{\lambda} \cdot \\ \cdot \sum_{k=0}^{d_{t,i}} e^{-k\lambda\tau} \begin{cases} \eta^{g+}\eta^{v+}(p_{t-k,i} - s^-_{t-k,i}) & \text{if } p_{t-k,i} - s^-_{t-k,i} \geq 0 \\ \frac{1}{\eta^{g-}\eta^{v-}}(p_{t-k,i} - s^-_{t-k,i}) & \text{if } p_{t-k,i} - s^-_{t-k,i} < 0 \end{cases} \tag{32}$$

$$\max_{\omega_{t-d_{t,i}},\ldots,\omega_t \in \Omega} \overline{E}_{t,i} = e^{-(d_{t,i}+1)\lambda\tau} E^0_{t-d_{t,i},i} + \frac{1-e^{-\lambda\tau}}{\lambda} \cdot \\ \cdot \sum_{k=0}^{d_{t,i}} e^{-k\lambda\tau} \begin{cases} \eta^{g+}\eta^{v+}(p_{t-k,i} + s^+_{t-k,i}) & \text{if } p_{t-k,i} + s^+_{t-k,i} \geq 0 \\ \frac{1}{\eta^{g-}\eta^{v-}}(p_{t-k,i} + s^+_{t-k,i}) & \text{if } p_{t-k,i} + s^+_{t-k,i} < 0. \end{cases} \tag{33}$$

Because these are piecewise linear functions, the actual implementation of (30) and (31) in a mathematical programming model require further transformations. By exploiting the relation $\eta^{g+}\eta^{v+} < \frac{1}{\eta^{g-}\eta^{v-}}$, constraint (30) can be expressed via hypograph formulation introducing only continuous auxiliary variables $l_{t,i}$:

$$\sum_{k=0}^{d_{t,i}} e^{-k\lambda\tau} l_{t-k,i} \geq \frac{\lambda\left(E^{\min}_{t,i} - e^{-(d_{t,i}+1)\lambda\tau}E^0_{t-d_{t,i},i}\right)}{1-e^{-\lambda\tau}} \quad \forall t \in \mathcal{T},\ \forall i \in \mathcal{I} \tag{34}$$

$$l_{t,i} \leq \eta^{g+}\eta^{v+}\left(p_{t,i} - s^-_{t,i}\right) \qquad \forall t \in \mathcal{T},\ \forall i \in \mathcal{I} \tag{35}$$

$$l_{t,i} \leq \frac{1}{\eta^{g-}\eta^{v-}}\left(p_{t,i} - s^-_{t,i}\right) \qquad \forall t \in \mathcal{T},\ \forall i \in \mathcal{I} \tag{36}$$

Crucially, this formulation does not require binary variables because the hypograph defined by inequalities (35) and (36) is convex, thus allowing to enforce both inequalities simultaneously.

A similar epigraph formulation of constraint (31) can not be obtained by introducing only continuous auxiliary variables $u_{t,i}$, because their piecewise linear epigraphs are not convex; therefore we need to introduce binary variables $y_{t,i}$ to enforce only one inequality at a time. These considerations lead to the following formulation (where $\forall t \in \mathcal{T},\ \forall i \in \mathcal{I}$ is omitted for brevity):

$$\sum_{k=0}^{d_{t,i}} e^{-k\lambda\tau} u_{t-k,i} \leq \frac{\lambda\left(E^{\max}_{t,i} - e^{-(d_{t,i}+1)\lambda\tau}E^0_{t-d_{t,i},i}\right)}{1-e^{-\lambda\tau}} \tag{37}$$

$$u_{t,i} \geq \eta^{g+}\eta^{v+}\left(p_{t,i} + s^+_{t,i}\right) - M^1_{t,i}(1-y_{t,i}) \tag{38}$$

$$u_{t,i} \geq \frac{1}{\eta^{g-}\eta^{v-}}\left(p_{t,i} + s^+_{t,i}\right) - M^2_{t,i}y_{t,i}, \tag{39}$$

where we set the constants:

$$M^1_{t,i} = \eta^{g-}p^{v,\min}_{t,i}\left(\eta^{g+}\eta^{v+} - \frac{1}{\eta^{g-}\eta^{v-}}\right) \tag{40}$$

$$M^2_{t,i} = \frac{1}{\eta^{g+}}p^{v,\max}_{t,i}\left(\frac{1}{\eta^{g-}\eta^{v-}} - \eta^{g+}\eta^{v+}\right) \tag{41}$$

so as to achieve the tightest possible bounds; i.e., $M^1_{t,i}$ and $M^2_{t,i}$ are the smallest shifts that make superfluous the respective constraints when $p_{t,i} + s^+_{t,i} \in \left[\eta^{g-}p^{v,\min}, (1/\eta^{g+})\,p^{v,\max}\right]$. Finally, it can be noted that $y_{t,i}$ is not enforced to change according to the sign of $p_{t,i} + s^+_{t,i}$ in (38) and (39); however, if critical to the satisfiability of (37), $y_{t,i}$ will be set by the solver to the value that allows $u_{t,i}$ to assume its minimum value, i.e. $y_{t,i} = 1$ if and only if $p_{t,i} + s^+_{t,i} \geq 0$.

An alternative approach for formulating the bounds on the energy stored in the EVs would consist in rewriting the piecewise affine system resulting from (16) and (29) in its Mixed Logical Dynamical (MLD) formulation (see Bemporad and Morari (1999a)) and then tightening the obtained constraints (see Vignali et al. (2014)). Nonetheless, this would introduce in the problem a robust version of (29) that fixes the sign of the power independently on the realization of the signal, thus excluding the solutions where the battery can be charged or discharged depending on the realization of the signal.

Summary of constraints To summarize, the constraints of the stochastic formulation are the following:

$$\begin{cases} (24), (25) & \text{[grid connection]} \\ (6), (7), (8), (9), (10) & \text{[balance constr.]} \\ (26), (27) & \text{[EVs power]} \\ (34), (35), (36), (37), (38), (39). & \text{[EVs energy]} \end{cases} \tag{42}$$

4.2 Expected value cost function

When uncertainty is involved in the cost function, different criteria may be chosen. The worst-case scenario approach is standard in robust optimization (see Ben-Tal et al. (2009)) and, applied to this problem, consists in minimizing $\max_{\omega_1,\ldots,\omega_T \in \Omega}((19) + (20) + (21))$. This would lead to never exploiting the ancillary services market. In fact, for each $t \in \mathcal{T}$: if there exists an $\omega^\dagger_t \in \Omega$ such that exploiting the ancillary services market entails a greater cost, then any optimal solution would have $s^{s+}_t = s^{s-}_t = 0$; else, the maximum when ω_t varies would be attained at $\omega_t = 0$, making any choice of s^{s+} and s^{s-}_t equivalent to $s^{s+}_t = s^{s-}_t = 0$. Recalling that this problem has to be solved periodically, we deem the expected value to be a more appropriate criterion for the cost function, because by definition it represents the best estimate of the long-term cost.

Exploiting linearity, the expected value of the cost function is given by:

$$\mathbb{E}(z) = z^e + \mathbb{E}(z^s) + \mathbb{E}(z^v), \tag{43}$$

where:

$$\mathbb{E}(z^s) = \tau \sum_{t\in\mathcal{T}} \left(c^{s+}_{u(t)} s^{s+}_{u(t)} \mathbb{E}(\omega^+_t) - c^{s-}_{u(t)} s^{s-}_{u(t)} \mathbb{E}(\omega^-_t) \right) \quad (44)$$

$$\mathbb{E}(z^v) = \tau \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{I}} \mathbb{E}\left(z^v_{t,i}\right) \quad (45)$$

$$\begin{aligned}\mathbb{E}(z^v_{t,i}) = \mathbb{E}\Big(&- c^{v+}\eta^{g+} \left[p_{t,i} + s^+_{t,i}\omega^+_t - s^-_{t,i}\omega^-_t\right]^+ + \\ &+ c^{v-}\frac{1}{\eta^{g-}} \left[p_{t,i} + s^+_{t,i}\omega^+_t - s^-_{t,i}\omega^-_t\right]^- \Big).\end{aligned} \quad (46)$$

The expressions of these expected values depend on whether Ω is discrete or continuous. In the following, for notational convenience, we set

$$a = c^{v+}\eta^{g+} \qquad \text{and} \qquad b = \frac{c^{v-}}{\eta^{g-}}. \quad (47)$$

Discrete Ω The expression of (44) is trivial:

$$\mathbb{E}(z^s) = \tau \sum_{t\in\mathcal{T}} \left(c^{s+}_{u(t)} \pi^+_t s^{s+}_{u(t)} - c^{s-}_{u(t)} \pi^-_t s^{s-}_{u(t)} \right); \quad (48)$$

that of (46), instead, requires a piecewise formulation: after some calculations we get

$$\mathbb{E}(z^v_{t,i}) = \\ = \begin{cases} -ap - a\pi^+ s^+ + a\pi^- s^- & \text{if } p \geq 0 \wedge p \geq s^- \\ -(a(1-\pi^-) + b\pi^-)\, p - a\pi^+ s^+ + b\pi^- s^- & \text{if } p \geq 0 \wedge p < s^- \\ -(a\pi^+ + b(1-\pi^+))\, p - a\pi^+ s^+ + b\pi^- s^- & \text{if } p < 0 \wedge p > -s^+ \\ -bp - b\pi^+ s^+ + b\pi^- s^- & \text{if } p < 0 \wedge p \leq -s^+, \end{cases} \quad (49)$$

where we omitted the period and slot indices for brevity. Observing that the inequality $\eta^{g+}\eta^{g-} < 1 < \frac{c^{v-}}{c^{v+}}$ holds by definition of its elements, so $a < b$, it is rather easy to verify that (49) can be recast to

$$\mathbb{E}(z^v_{t,i}) = \max \begin{cases} -ap - a\pi^+ s^+ + a\pi^- s^- \\ -(a(1-\pi^-) + b\pi^-)\, p - a\pi^+ s^+ + b\pi^- s^- \\ -(a\pi^+ + b(1-\pi^+))\, p - a\pi^+ s^+ + b\pi^- s^- \\ -bp - b\pi^+ s^+ + b\pi^- s^-, \end{cases} \quad (50)$$

thus proving that is a convex function; as such, (50) can be expressed via an epigraph formulation. In conclusion, in this case, minimizing (43) translates to minimizing

$$\begin{aligned}\tau \sum_{t\in\mathcal{T}} \Big(& c^{e+}_t p^{e+}_t - c^{e-}_t p^{e-}_t + \\ & + c^{s+}_{u(t)} \pi^+_t s^{s+}_{u(t)} - c^{s-}_{u(t)} \pi^-_t s^{s-}_{u(t)} + \sum_{i\in\mathcal{I}} h_{t,i} \Big),\end{aligned} \quad (51)$$

where the auxiliary variables $h_{t,i}$ are, $\forall t \in \mathcal{T}$, $\forall i \in \mathcal{I}$, subject to:

$$h_{t,i} \geq -ap_{t,i} - a\pi^+_t s^+_{t,i} + a\pi^-_t s^-_{t,i} \quad (52)$$

$$h_{t,i} \geq -(a(1-\pi^-_t) + b\pi^-_t)\, p_{t,i} - a\pi^+_t s^+_{t,i} + b\pi^-_t s^-_{t,i} \quad (53)$$

$$h_{t,i} \geq -(a\pi^+_t + b(1-\pi^+_t))\, p_{t,i} - a\pi^+_t s^+_{t,i} + b\pi^-_t s^-_{t,i} \quad (54)$$

$$h_{t,i} \geq -bp_{t,i} - b\pi^+_t s^+_{t,i} + b\pi^-_t s^-_{t,i}. \quad (55)$$

Continuous Ω In this case, the form assumed by the expected values in (44) and (46) depends on the probability distributions of $\{\omega_t\}_{t\in\mathcal{T}}$ variables. For example, with the further assumptions mentioned in Section 2 (independence and uniform distribution), it is easy to observe that $\mathbb{E}(\omega^+_t) = \mathbb{E}(\omega^-_t) = \frac{1}{4}$, so that (44) becomes

$$\mathbb{E}(z^s) = \frac{1}{4}\tau \sum_{t\in\mathcal{T}} \left(c^{s+}_{u(t)} s^{s+}_{u(t)} - c^{s-}_{u(t)} s^{s-}_{u(t)} \right), \quad (56)$$

while expected value (46) can be expressed as

$$\mathbb{E}(z^v_{t,i}) = \begin{cases} -ap - \frac{a}{4}s^+ + \frac{a}{4}s^- & \text{if } p \geq 0 \wedge p \geq s^- \\ -\frac{a+b}{2}p - \frac{a}{4}s^+ + \frac{b}{4}s^- + \frac{b-a}{4}\frac{p^2}{s^-} & \text{if } p \geq 0 \wedge p < s^- \\ -\frac{a+b}{2}p - \frac{a}{4}s^+ + \frac{b}{4}s^- + \frac{b-a}{4}\frac{p^2}{s^+} & \text{if } p < 0 \wedge p > -s^+ \\ -bp - \frac{b}{4}s^+ + \frac{b}{4}s^- & \text{if } p < 0 \wedge p \leq -s^+, \end{cases} \quad (57)$$

where we omitted the period and slot indices for brevity. Again depending on the probability distributions of $\{\omega_t\}_{t\in\mathcal{T}}$, expected values (44) and (46) can result in nonlinear functions, as is for (57) in this case, due to the presence of terms $p^2_{t,i}/s^-_{t,i}$ and $p^2_{t,i}/s^+_{t,i}$. To obtain a computationally tractable model it is necessary to construct a linear surrogate of the nonlinear expected values. In the case under consideration we linearize (57) by imposing continuity on the border of the piecewise regions:

$$\mathbb{E}(z^v_{t,i}) \approx \begin{cases} -ap - \frac{a}{4}s^+ + \frac{a}{4}s^- & \text{if } p \geq 0 \wedge p \geq s^- \\ -\frac{3a+b}{4}p - \frac{a}{4}s^+ + \frac{b}{4}s^- & \text{if } p \geq 0 \wedge p < s^- \\ -\frac{a+3b}{4}p - \frac{a}{4}s^+ + \frac{b}{4}s^- & \text{if } p < 0 \wedge p > -s^+ \\ -bp - \frac{b}{4}s^+ + \frac{b}{4}s^- & \text{if } p < 0 \wedge p \leq -s^+. \end{cases} \quad (58)$$

By the same argument used for (49) and (50) it can be proven that this approximation is a convex function. This enables us to proceed as in the discrete Ω case, obtaining the following approximated cost function:

$$\begin{aligned}\tau \sum_{t\in\mathcal{T}} \Big(& c^{e+}_t p^{e+}_t - c^{e-}_t p^{e-}_t + \\ & + \frac{c^{s+}_{u(t)}}{4} s^{s+}_{u(t)} - \frac{c^{s-}_{u(t)}}{4} s^{s-}_{u(t)} + \frac{1}{4}\sum_{i\in\mathcal{I}} h_{t,i} \Big),\end{aligned} \quad (59)$$

where the auxiliary variables $h_{t,i}$ are subject to:

$$h_{t,i} \geq -4ap_{t,i} - as^+_{t,i} + as^-_{t,i} \qquad \forall t \in \mathcal{T},\ \forall i \in \mathcal{I} \quad (60)$$

$$h_{t,i} \geq -(3a+b)\, p_{t,i} - as^+_{t,i} + bs^-_{t,i} \qquad \forall t \in \mathcal{T},\ \forall i \in \mathcal{I} \quad (61)$$

$$h_{t,i} \geq -(a+3b)\, p_{t,i} - as^+_{t,i} + bs^-_{t,i} \qquad \forall t \in \mathcal{T},\ \forall i \in \mathcal{I} \quad (62)$$

$$h_{t,i} \geq -4bp_{t,i} - bs^+_{t,i} + bs^-_{t,i} \qquad \forall t \in \mathcal{T},\ \forall i \in \mathcal{I}. \quad (63)$$

4.3 Stochastic optimization problems

Combining constraints and cost function described in this section we finally get the following stochastic optimization problem for the case in which Ω is discrete:

$$\begin{cases} \min & (51) \\ \text{subject to} & (52), (53), (54), (55), (42). \end{cases} \quad (64)$$

In case of continuous Ω and with the further assumptions mentioned in Section 2, we get an approximated stochastic optimization problem:

$$\begin{cases} \min & (59) \\ \text{subject to} & (60), (61), (62), (63), (42). \end{cases} \quad (65)$$

Both (64) and (65) are mixed-integer linear programming (MILP) problems and can be solved via standard solvers like CPLEX (2019). An optimal solution of (64) or (65) maximizes the expected value of the aggregator's profits and is such that the constraints on power flowing from/to the grid, power flowing from/to the EVs and energy in the EVs are satisfied for any possible realization of the ancillary services market signal $\{\omega_t\}_{t\in\mathcal{T}}$.

5. NUMERICAL CASE STUDY

As a case study let us consider the following instance. The parking lot is composed by 25 slots, in which a total of 100 vehicles stop during a time horizon of 24 hours, with an average stop duration of around 6 hours (the arrival time and departure time of each vehicle can be inferred from Fig. 7). The power limit at the grid interface is $p^{g,\max} = -p^{g,\min} = 500$ kW. The energy market has

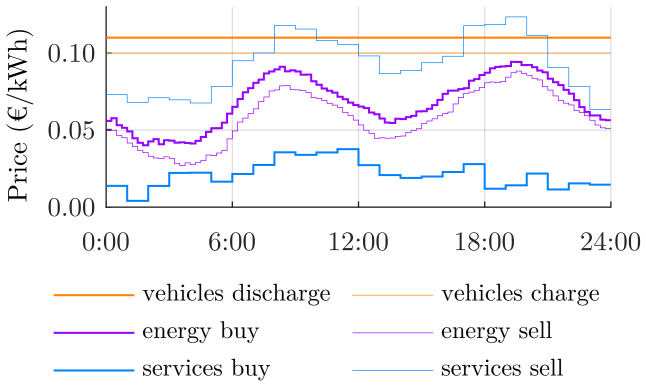

Fig. 2. Prices: the energy market has a time discretization of 15 minutes, the services market of 1 hour, and the user prices are constant.

15 minutes time periods (τ), whilst the ancillary services market has 1 hour periods ($m = 4$). The input prices are represented in Fig. 2 (only the energy component – without system charges and VAT – has been considered). The ancillary services market signal is continuous.

The approximated formulation (65) has been coded using MATLAB programming language with YALMIP modeling toolbox (see Löfberg (2004)) and solved with CPLEX (2019) optimizer. The optimal solution of an instance of this size can be computed in about 1 minute using a standard PC.

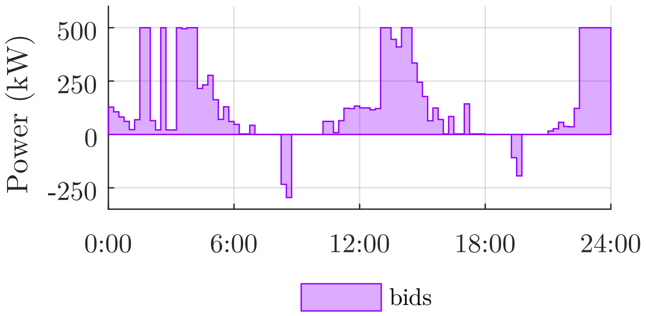

Fig. 3. Bids in energy market.

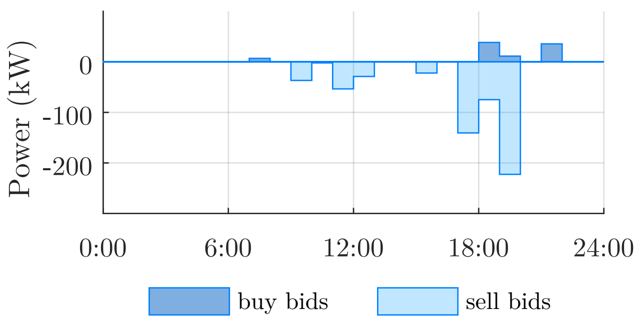

Fig. 4. Bids in ancillary services market. Note that is possible to simultaneously place a buy and a sell bid. Nonetheless, only one of them will be realized, depending on the value of ω_t.

The bids in energy and services markets can be compared in Figs. 3 and 4. Note that ancillary services market bids can be concurrent, i.e. the aggregator can place buy and sell bids for the same period: nonetheless, only one of the bids will be realized, to an amount defined by the value of the signal ω_t.

Fig. 5 represents the power exchanged at the grid interface. We can note that the minimization of costs produces a shift of power consumption towards the hours in which the energy is cheaper. Fig. 5 highlights the power variations as a function of the signal: as can be noted, the bounds on maximum and minimum absorption ($p^{g,\max}$, $p^{g,\min}$) are always satisfied. The total energy absorbed from the grid, computed as the integral of the exchanged power, ranges from 2.7 MWh to 3.3 MWh, depending on the signal realization.

Fig. 6 shows the parking lot equivalent energy and capacity, both obtained as the sum of the corresponding quantity for each EV. Placing a bid in the services market generates an effect on the equivalent energy of the parking lot to an extent that depends not only on the signal but also on vehicles departures: for example, the effect of the bids from 7:00 to 13:00 will vanish at 15:00, because the involved vehicles will have left the parking lot by that time.

Finally, in Fig. 7 is shown a representation of the charging state for each period and slot. Interestingly enough, we are able to include in the solution those cases when, depending on the realization of the signal, the batteries are either charged or discharged (yellow area); as outlined

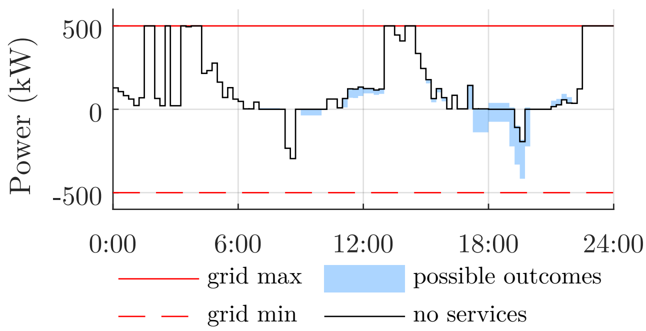

Fig. 5. Power at grid interface. For any possible outcome, bounds are satisfied.

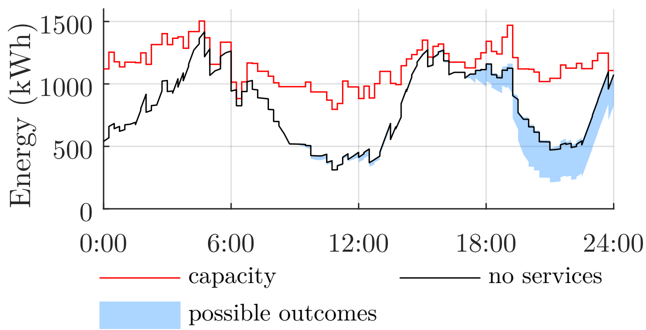

Fig. 6. Parking lot equivalent energy as a sum of the energy stored in each EV. For any possible outcome, bounds on maximum capacity are satisfied.

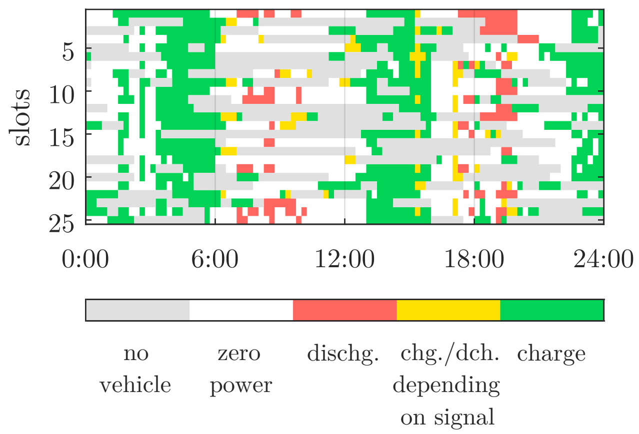

Fig. 7. Charging state of the parking slots during the considered time horizon.

in previous sections, this has some interesting links with reachability analysis of piecewise affine systems (see Bemporad and Morari (1999b), Schürmann et al. (2020)) and, in particular, with the case of reachable sets splitting among different modes. Yellow regions represent also those cases where an actual approximation of the cost function is performed. Since the number of these regions is quite limited (3 % of the total) and since the error introduced with the linearization is very small (less than 0.5 % for every reasonable value of p, s^+ and s^-), we can safely conclude that we incur in a negligible approximation error.

6. CONCLUSIONS

We have discussed the problem of optimal management of an aggregate of electric vehicles for the provision of ancillary services, under the assumptions of optimal bidding strategy and known vehicle arrivals and departures. We have considered that the deployed power on the ancillary services market depends on a signal that can be discrete or continuous; we have formulated one stochastic optimization problem for each of this two cases. Both formulations are MILP, have robust constraints and have an expected value cost function, which is exact if the ancillary services market signal is discrete. If the signal is continuous, the cost function varies depending on the probability distribution of the signal and could require an approximation to obtain a computationally tractable formulation. We have then shown that, in the case of uniform probability, an efficient formulation can be obtained by introducing a negligible approximation of the cost function; the validity of the approach has been shown on a numerical example. In future research we will study the extension to the case of stochastic arrivals and departures of the vehicles, and the online application of the proposed algorithm.

ACKNOWLEDGEMENTS

This work has been financed by the Research Fund for the Italian Electrical System in compliance with the Decree of Minister of Economic Development April 16, 2018.

REFERENCES

Baker, K., Hug, G., and Li, X. (2012). Optimal integration of intermittent energy sources using distributed multi-step optimization. In *2012 IEEE Power and Energy Society General Meeting*, 1–8. IEEE.

Banshwar, A., Sharma, N.K., Sood, Y.R., and Shrivastava, R. (2018). An international experience of technical and economic aspects of ancillary services in deregulated power industry: Lessons for emerging BRIC electricity markets. *Renewable and Sustainable Energy Reviews*, 90, 774 – 801.

Bemporad, A. and Morari, M. (1999a). Control of systems integrating logic, dynamics, and constraints. *Automatica*, 35(3), 407–427.

Bemporad, A. and Morari, M. (1999b). Verification of hybrid systems via mathematical programming. In *International Workshop on Hybrid Systems: Computation and Control*, 31–45. Springer.

Ben-Tal, A., El Ghaoui, L., and Nemirovski, A. (2009). *Robust Optimization*. Princeton Series in Applied Mathematics. Princeton University Press.

Boyd, S. and Vandenberghe, L. (2004). *Convex optimization*. Cambridge university press.

CPLEX (2019). IBM ILOG CPLEX 12.9 user's manual. *IBM ILOG CPLEX Division, Incline Village, NV*.

He, Y., Venkatesh, B., and Guan, L. (2012). Optimal scheduling for charging and discharging of electric vehicles. *IEEE transactions on smart grid*, 3(3), 1095–1105.

Kaushal, A. and Van Hertem, D. (2019). An overview of ancillary services and HVDC systems in European context. *Energies*, 12(18).

Kempton, W. and Tomić, J. (2005a). Vehicle-to-grid power fundamentals: Calculating capacity and net revenue. *Journal of power sources*, 144(1), 268–279.

Kempton, W. and Tomić, J. (2005b). Vehicle-to-grid power implementation: From stabilizing the grid to supporting large-scale renewable energy. *Journal of power sources*, 144(1), 280–294.

Löfberg, J. (2004). YALMIP: A toolbox for modeling and optimization in matlab. In *Proceedings of the CACSD Conference*. Taipei, Taiwan.

Sarker, M.R., Dvorkin, Y., and Ortega-Vazquez, M.A. (2015). Optimal participation of an electric vehicle aggregator in day-ahead energy and reserve markets. *IEEE Trans. on Power Systems*, 31(5), 3506–3515.

Schürmann, B., Vignali, R., Prandini, M., and Althoff, M. (2020). Set-based control for disturbed piecewise affine systems with state and actuation constraints. *Nonlinear Analysis: Hybrid Systems*, 36, 100826.

Sortomme, E. and El-Sharkawi, M.A. (2010). Optimal charging strategies for unidirectional vehicle-to-grid. *IEEE Transactions on Smart Grid*, 2(1), 131–138.

Sortomme, E. and El-Sharkawi, M.A. (2011). Optimal scheduling of vehicle-to-grid energy and ancillary services. *IEEE Transactions on Smart Grid*, 3(1), 351–359.

Vagropoulos, S.I. and Bakirtzis, A.G. (2013). Optimal bidding strategy for electric vehicle aggregators in electricity markets. *IEEE Transactions on power systems*, 28(4), 4031–4041.

Vignali, R., Deori, L., and Prandini, M. (2014). Control input design: detecting non influential inputs while satisfying a reachability specification. *IFAC Proceedings Volumes*, 47(3), 1416–1421.