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Extremum Seeking Under Distributed Input Delay ⋆

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Abstract: In this paper, we propose an extremum seeking scheme for single-parameter static maps in the presence of distributed input delay. A probing signal is newly developed so that the delayed signal has a conventional form in standard extremum seeking problems. An update law for an estimate of the unknown argument of the extremum is designed based on the idea of the predictor feedback law. We prove the convergence of the estimation error to a neighborhood of the origin by means of the method of averaging. The effectiveness of the proposed scheme is confirmed by a numerical simulation.

Keywords: Extremum seeking and model free adaptive control, delay compensation for linear and nonlinear systems, averaging for infinite dimensional systems, Lyapunov methods

1. INTRODUCTION

Extremum seeking is known as a real-time and model-free optimization tool for tuning parameters in dynamic nonlinear systems as well as nonlinear static maps. In a usual setting, a model of the system or map is unavailable, but we can measure the value of the performance output to be optimized. To obtain information used to search the optimal parameter, we change the value of the parameter in accordance with a sinusoidal perturbation signal. Based on the corresponding output to this oscillatory parameter, an estimate of the optimal parameter is updated. In this way, we seek the optimal parameter.

Development of extremum seeking has a long history (see, for example, (Tan et al., 2010)). Let us focus on the theoretical side. A rigorous stability analysis is conduced by Krstić and Wang (2000); Ariyur and Krstić (2003). The semi-global stability is investigated in (Tan et al., 2006, 2009). Extremum seeking schemes corresponding to Newton's method are proposed in (Moase et al., 2010; Nešić et al., 2010; Ghaffari et al., 2012). In addition, extremum seeking schemes using a stochastic signal instead of a sinusoidal signal is developed by Manzie and Krstic (2009); Liu and Krstic (2010, 2014). Approaches exploiting the Lie bracket motion induced by sinusoidal signals are also reported in (Dürr et al., 2013; Labar et al., 2019).

In those studies, the perturbation signal is assumed to be transmitted to the system immediately. However, sensor and actuator delays or transmission delays are unavoidable in practical situations. Then, the delay might destabilize the extremum seeking loop. Recently, the authors proposed an extremum seeking scheme for static maps that can compensate a class of delays (Oliveira et al., 2017). The proposed scheme is constructed based on the idea of predictor feedback control laws (Manitius and Olbrot, 1979; Artstein, 1982; Krstic, 2009). Since the map to be optimized is unknown, a complete predictor can not be implemented. Hence, we introduce a signal corresponding to a predictor in the average sense.

The class of delays considered in our previous work is the point delay. In this paper, we deal with distributed delays to handle more general situations. As the first step of this study, only a single parameter case is considered. For the extension of the results in (Oliveira et al., 2017) to the distributed delay case, the idea of exploiting predictor feedback also works since the original predictor feedback laws proposed in (Manitius and Olbrot, 1979; Artstein, 1982) can handle the distributed delays. We have to pay attention to the design of the perturbation and probing signals. Modification of their definition is necessary so that the delayed signals at the input of the nonlinear static map to be optimized have conventional forms.

We analyze the stability of closed-loop system with the aid of the method of averaging. In this approach, the stability of the associated average system must be shown. This will be proved by constructing a Lyapunov functional. The Lyapunov stability of predictor feedback laws for linear systems with distributed input delays was studied in (Bekiaris-Liberis and Krstic, 2011). A Lyapunov functional is constructed by using the backstepping transformation. Although it is possible to follow this approach, we do not employ it. Since our system has a simple structure, we can construct a Lyapunov functional without back-

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stepping transformation. The effectiveness of the proposed scheme is also examined by a numerical simulation.

Notation: For an open interval $I \subset \mathbb{R}$, the space of (equivalent classes of) Lebesgue square-integrable functions on I is denoted by $L^2(I)$. The first order Sobolev space as a subset of $L^2(I)$ is denoted by $H^1(I)$. Let $f : [0,\infty) \times [0,\infty) \to [0,\infty)$ be a nonnegative-valued function. Then, we write $f(t,\epsilon) = O(\epsilon)$ when there exists $k > 0$ and $\bar{\epsilon} > 0$ such that $f(t,\epsilon) \leq k\epsilon$ for any $t \in [0,\infty)$, $\epsilon \in [0,\bar{\epsilon}]$. Given an interval $I \subset \mathbb{R}$, a normed vector space $\mathcal{X}$, and a natural number $k \in \mathbb{N}$, $C(I,\mathcal{X})$ and $C^k(I,\mathcal{X})$ denote the spaces of continuous functions from I to $\mathcal{X}$ and k-times continuously differentiable functions from I to $\mathcal{X}$, respectively.

2. PROBLEM SETTING

2.1 Review of a conventional single parameter extremum seeking problem

Consider a static map $f : \mathbb{R} \to \mathbb{R}$. Suppose that f is twice continuously differentiable and that the first and second derivatives of f satisfy $f'(\theta^*) = 0$ and $f''(\theta^*) = H \neq 0$ at some point $\theta^* \in \mathbb{R}$. Then, the map f takes an extremum $f^* := f(\theta^*)$ at θ^*. Without loss of generality, we can assume that the Hessian H satisfies $H < 0$. Then, f^* is a local maximum. If f is three-times continuously differentiable, the quadratic map

$$Q(\theta) = f^* + \frac{1}{2}H\left(\theta - \theta^*\right)^2. \tag{1}$$

is a local approximation of f around $\theta = \theta^*$.

We assume that f^*, H, and θ^* are unknown. The objective of the extremum seeking is to find f^* without knowledge of H and θ^* by measuring the signal $y(t) = f(\theta(t))$ for an appropriately designed probing signal $\theta(t)$.

A conventional gradient-based extremum seeking scheme is given by

$$\dot{\hat{\theta}}(t) = k\frac{2}{a}\sin(\omega t)y(t) = k\frac{2}{a}\sin(\omega t)f(\theta(t)), \tag{2}$$

$$\theta(t) = \hat{\theta}(t) + a\sin(\omega t), \tag{3}$$

where $k > 0$, $a > 0$, and $\omega > 0$ are design parameters. The variable $\hat{\theta}$ is an estimate of θ^*. Set the error variable $\tilde{\theta}$ as $\tilde{\theta}(t) := \hat{\theta}(t) - \theta^*$. The closed-loop system can be expressed in terms of the error variable $\tilde{\theta}$ as

$$\dot{\tilde{\theta}} = k\frac{2}{a}\sin(\omega t)\, f\left(\tilde{\theta} + a\sin(\omega t) + \theta^*\right). \tag{4}$$

The stability analysis of this closed-loop system is normally conducted with the help of the averaging analysis. Assume that f is approximated by (1). Namely, f in (4) is replaced with Q. Then, the average system of (4) with $f = Q$ is calculated as

$$\dot{\tilde{\theta}}^{\mathrm{a}} = kH\tilde{\theta}^{\mathrm{a}}. \tag{5}$$

Since $H < 0$, this system clearly has a unique exponentially stable equilibrium $\tilde{\theta}^{\mathrm{a}} = 0$. The averaging theorem (Khalil, 2002) guarantees some sort of convergence property of the original error variable $\tilde{\theta}$.

2.2 Extremum seeking under a distributed input delay

In this paper, we assume that there is a certain kind of delay in the transmission process from the input signal $\theta(t)$ to the measurement output $y(t)$. More precisely, the measurement output y is given by

$$y(t) = f\left(\int_0^D \theta(t-\sigma)d\beta(\sigma)\right) \tag{6}$$

for some constant $D > 0$. The integral on the right-hand side of (6) is the Riemann-Stieltjes integral with respect to a function $\beta : [0,D] \to \mathbb{R}$ of bounded variation. Recall that a function of bounded variation can be discontinuous. If β is such that

$$\beta(x) = \begin{cases} 0, & 0 \leq x < D_0, \\ 1, & D_0 \leq x \leq D, \end{cases} \tag{7}$$

for some $D_0 \in (0,D)$, then, we have $y(t) = f(\theta(t - D_0))$. The extremum seeking scheme developed in our previous paper (Oliveira et al., 2017) can handle this case. However, for the case with general β, a new scheme is necessary.

We make an assumption on the class of the function β. For each $\omega > 0$, define $\gamma(\omega)$ by

$$\gamma(\omega) = \left(\int_0^D \cos(\omega\sigma)d\beta(\sigma)\right)^2 + \left(\int_0^D \sin(\omega\sigma)d\beta(\sigma)\right)^2. \tag{8}$$

Clearly, $\gamma(\omega) \geq 0$ for all $\omega > 0$.

Assumption 1. The function $\beta : [0,D] \to \mathbb{R}$ of bounded variation satisfies $\beta(0) = 0$ and $\beta(1) = 1$, and there exists a non-decreasing sequence $\{\omega_i\}_{i=1}^{\infty} \subset (0,\infty) \subset \mathbb{R}$ of positive real numbers such that $\omega_i \to \infty$ as $i \to \infty$ and that γ defined by (8) satisfies $\gamma(\omega_i) \neq 0$ for any $i \in \mathbb{N}$.

The condition on the value of β at $\sigma = 0$ does not cause any loss of generality since we only consider the Stieltjes integrals with respect to β. The condition at $x = 1$ ensures that the transmission is lossless when a constant signal is applied to the map. The last condition is necessary to let the frequency ω be arbitrarily large. Note that, for β given in (7), we have $\beta(0) = 0$, $\beta(1) = 1$, and $\gamma(\omega) = 1$ for any $\omega \in (0,\infty)$. Hence, Assumption 1 is fulfilled in this case.

The objective of this paper is to develop an extremum seeking scheme for a static map f that is locally approximated by the quadratic map (1) in the presence of distributed input delay represented by (6). While the map f is unknown, our approach requires complete information about the delay. Namely, D and β are assumed to be known. This is a rather strong assumption. Nevertheless, construction of the proposed extremum seeking scheme will follow a model-free approach.

3. EXTREMUM SEEKING SCHEME

In this section, we propose an extremum seeking scheme as a solution to our problem.

3.1 Proposed scheme

Let $\hat{\theta}(t) \in \mathbb{R}$ be an estimate of θ^* and $\bar{\theta}(t) \in \mathbb{R}$ be an auxiliary variable. We temporary write an update law for the auxiliary variable $\bar{\theta}$ as

$$\dot{\bar{\theta}} = U. \tag{9}$$

The signal $U(t) \in \mathbb{R}$ is to be determined. We define $\hat{\theta}$ by

$$\hat{\theta}(t) = \int_0^D \bar{\theta}(t-\sigma)d\beta(\sigma). \tag{10}$$

The use of these two variables $\hat{\theta}$ and $\bar{\theta}$ is a feature of the proposed extremum seeking scheme.

In the proposed scheme, we will use three perturbation signals $S(t), M(t), N(t) \in \mathbb{R}$ defined by

$$S(t) = \frac{a}{\gamma(\omega)} \int_0^D \sin\big(\omega(t+\xi)\big) d\beta(\xi), \tag{11}$$

$$M(t) = (2/a)\sin(\omega t), \tag{12}$$

$$N(t) = (8/a^2)\left(2\sin^2(\omega t) - 1\right). \tag{13}$$

The additive perturbation signal $S(t)$ has a complicated form, whereas $M(t)$ and $N(t)$ are relatively simple. The signal $M(t)$ is used in a conventional extremum seeking. The third signal $N(t)$ is introduced in (Ghaffari et al., 2012) to estimate the unknown Hessian H. The signal $N(t)$ will be used for the same purpose. It should be noted that, as reviewed in the previous section, the Hessian H is not necessary in the standard gradient-based extremum seeking in the delay-free case. As we will see later, our approach to delay compensation involves prediction of future values of the estimation error between $\hat{\theta}$ and θ^*. For the prediction, an estimate of H will be necessary.

We set the probing signal $\theta(t)$ as

$$\theta(t) = \bar{\theta}(t) + S(t). \tag{14}$$

The signal θ includes $\bar{\theta}$, but does not include $\hat{\theta}$ unlike conventional extremum seeking schemes. We claim that substituting (14) into (6) leads to

$$y(t) = f\Big(\hat{\theta}(t) + a\sin(\omega t)\Big). \tag{15}$$

Indeed, integrating (14) with respect to β yields

$$\begin{aligned}
&\int_0^D \theta(t-\sigma) d\beta(\sigma) \\
&= \hat{\theta}(t) + \frac{a}{\gamma(\omega)} \int_0^D \int_0^D \sin\Big(\omega(t-\sigma+\xi)\Big) d\beta(\sigma) d\beta(\xi) \\
&= \hat{\theta}(t) + \frac{a\sin(\omega t)}{\gamma(\omega)} \int_0^D \int_0^D \cos\Big(\omega(\xi-\sigma)\Big) d\beta(\sigma) d\beta(\xi) \\
&\quad + \frac{a\cos(\omega t)}{\gamma(\omega)} \int_0^D \int_0^D \sin\Big(\omega(\xi-\sigma)\Big) d\beta(\sigma) d\beta(\xi),
\end{aligned} \tag{16}$$

where we have used the definition (10) of $\hat{\theta}$ and the addition formula of trigonometric functions. The addition formula also gives

$$\int_0^D \int_0^D \cos\Big(\omega(\xi-\sigma)\Big) d\beta(\sigma) d\beta(\xi) = \gamma(\omega), \tag{17}$$

$$\int_0^D \int_0^D \sin\Big(\omega(\xi-\sigma)\Big) d\beta(\sigma) d\beta(\xi) = 0. \tag{18}$$

Substituting (17) and (18) into (16), we arrive at

$$\int_0^D \theta(t-\sigma) d\beta(\sigma) = \hat{\theta}(t) + a\sin(\omega t). \tag{19}$$

This immediately implies (15). Although the estimate $\hat{\theta}$ of θ^* and the signal θ are defined in a non-conventional manner, the output y is expressed as if the map f received the signal $\hat{\theta} + a\sin(\omega t)$ as in the traditional extremum seeking scheme.

We next derive the equation that the error variable $\tilde{\theta}(t) := \hat{\theta}(t) - \theta^*$ satisfies. Notice that $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$. Then, differentiating (10) with respect to t leads to

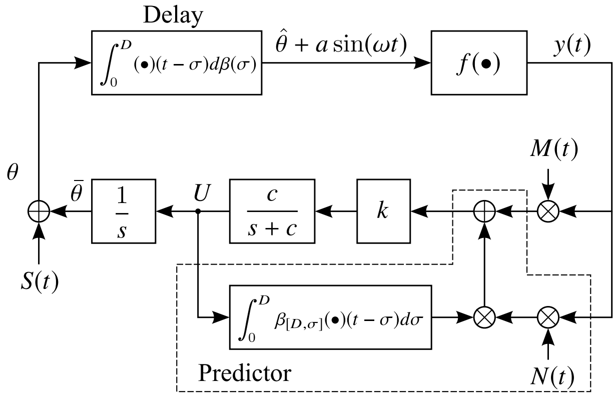


Fig. 1. Block diagram of the proposed extremum seeking scheme for systems with distributed input delays.

$$\dot{\tilde{\theta}}(t) = \int_0^D U(t-\sigma) d\beta(\sigma). \tag{20}$$

The error variable $\tilde{\theta}$ evolves in accordance with the equation (20). If U is regarded as the control input, the equation (20) is in a typical form of a driftless linear system with distributed input delay. It is well-known that such a system can be stabilized by a predictor feedback control law. In (Manitius and Olbrot, 1979; Artstein, 1982), the following control law is proposed:

$$U(t) = -k\left(\tilde{\theta}(t) + \int_t^{t+D} \int_{\tau-t}^D U(\tau-\sigma) d\beta(\sigma) d\tau\right), \tag{21}$$

where $k > 0$. The terms in the parentheses correspond to a predicted future value of $\tilde{\theta}$. Unfortunately, this control law is not implementable because $\tilde{\theta}$ is unavailable. Our approach aims at realizing (21) in the average sense.

To this end, we propose to close the loop of our extremum seeking scheme by setting the signal U in the update law (9) for $\bar{\theta}$ as the solution to the following time-varying ordinary differential equation:

$$\dot{U}(t) = -cU(t) + ckP(t), \tag{22}$$

where $\beta_{[D,\sigma]} := \beta(D) - \beta(\sigma)$, $c > 0$, and $P(t)$ is defined by

$$P(t) = M(t)y(t) + N(t)y(t) \int_0^D \beta_{[D,\sigma]} U(t-\sigma) d\sigma. \tag{23}$$

The corresponding block diagram is given in Fig. 1. The equation (22) means that $U(t)$ is a filtered version of the signal $kP(t)$. The filter is represented by the transfer function $c/(s+c)$. As we will see later, kP mimics (21) in the average sense. Introducing the filter turns U into a part of the state variable. Then, we can apply the averaging theorem to the closed-loop system.

3.2 Abstract formulation of the closed-loop system

Recall that the measurement signal y is given by (15). This can be rewritten in terms of the error variable $\tilde{\theta}$ as

$$y(t) = f\left(\tilde{\theta}(t) + a\sin(\omega t) + \theta^*\right). \tag{24}$$

Hence, the closed-loop system consists of (20) and (22) with the initial condition $\tilde{\theta}(0) = \tilde{\theta}_0 \in \mathbb{R}$ and $U(\tau - D) = \phi(\tau)$ for each $\tau \in [0, D]$. The initial function ϕ is an element of some function space. To conduct stability analysis, we introduce a partial differential equation (PDE) representation.

Set $u(x,t) = U(x+t-D)$ for each $x \in (0,D]$ and $t \geq 0$ with $x+t \geq 0$. Then, the closed-loop system (20) and (22) can be expressed as

$$\dot{\tilde{\theta}}(t) = \int_0^D u(D-\sigma,t)d\beta(\sigma), \tag{25}$$
$$u_t(x,t) = u_x(x,t),\ x \in (0,D), \tag{26}$$
$$u(D,t) = U(t), \tag{27}$$
$$\dot{U}(t) = -cU(t) + ckP(t) \tag{28}$$

with the initial condition $\tilde{\theta}(0) = \tilde{\theta}_0$, $u(x,0) = u_0(x) = \phi(x)$, $x \in (0,D)$ and $U(0) = U_0 = \phi(D)$. The signal P is given in (23). In this representation, the state variable is composed of the finite dimensional components $\tilde{\theta}$, U and the infinite dimensional one u.

Let the state space be $\mathcal{H} := \mathbb{R} \times L^2(0,D) \times \mathbb{R}$ equipped with the inner product

$$\Big([X_1,X_2,X_3]^\top, [Y_1,Y_2,Y_3]^\top\Big)_{\mathcal{H}} := X_1Y_1 + \int_0^D X_2(\xi)Y_2(\xi)d\xi + X_3Y_3 \tag{29}$$

for each $[X_1,X_2,X_3]^\top, [Y_1,Y_2,Y_3]^\top \in \mathcal{H}$. The space $\mathcal{H}$ is a Hilbert space. The norm induced by the inner product (29) is denoted by $\|\cdot\|_{\mathcal{H}}$. Define a linear operator $A : D(A) \subset \mathcal{H} \to \mathcal{H}$ by

$$A\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} \int_0^D X_2(D-\sigma)d\beta(\sigma) & \dfrac{dX_2}{dx} & -cX_3 \end{bmatrix}^\top \tag{30}$$

with the domain

$$D(A) = \big\{[X_1,X_2,X_3] \in \mathcal{H} \ \big|\ X_2 \in H^1(0,D),\ X_2(D) = X_3\big\}. \tag{31}$$

The time-varying nonlinear perturbation term is given by

$$F(\omega t, X) = [\,0\ 0\ F_3(\omega t, X)\,]^\top, \tag{32}$$

where $F_3 : \mathbb{R} \times \mathcal{H} \to \mathbb{R}$ is defined by

$$F_3(\omega t, X) = ck\Bigg(\frac{2}{a}\sin(\omega t)\, f\big(X_1 + a\sin(\omega t) + \theta^*\big) + \frac{16}{a^2}\Big(\sin^2(\omega t) - \frac{1}{2}\Big) f\big(X_1 + a\sin(\omega t) + \theta^*\big) \times \int_0^D \beta_{[D,\sigma]} X_2(D-\sigma)d\sigma\Bigg). \tag{33}$$

Thus, we arrive at the following abstract evolution equation corresponding to the closed-loop system (25)–(28):

$$\frac{dX}{dt} = AX + F(\omega t, X), \tag{34}$$

where $X(t) = [\tilde{\theta}(t), u(\cdot,t), U(t)]^\top$.

Clearly, $F(\omega t + 2\pi, X) = F(\omega t, X)$ for any $X \in \mathcal{H}$. If $1/\omega$ is considered as a small parameter ϵ, we can apply the method of averaging for infinite dimensional systems developed in (Hale and Verduyn Lunel, 1990). Then, the stability of the closed-loop system can be investigated through the corresponding average system.

4. STABILITY ANALYSIS

The main goal of this section is to prove our main theorem.

Theorem 2. Let $H < 0$, $D > 0$ and $a \in \mathbb{R} \setminus \{0\}$. Consider the system (34) for f being the quadratic map (1). Suppose that the function $\beta : [0,D] \to \mathbb{R}$ of bounded variation satisfies Assumption 1 and the constants $k > 0$ and $c > 0$ are chosen so that $c > -kH$. Then, for each $\rho > 0$, there exist constants $\omega^* > 0$ and $\rho_0 \in (0,\rho)$ such that, for any $\omega > \omega^*$ with $\gamma(\omega) \neq 0$, any solution to (34) for an initial value $X_0 = [\tilde{\theta}_0, u_0(\cdot), U_0]^\top \in D(A)$ with $\|X_0\|_{\mathcal{H}} \leq \rho_0$ converges to an $O(1/\omega)$-neighborhood of the origin. In addition, the following estimate holds:

$$\limsup_{t\to\infty} |y(t) - f^*| = O\big(1/\omega^2 + |a|^2\big). \tag{35}$$

To prove the theorem, we consider the averaged version of the system (34).

4.1 Average system

Let us obtain the average system associated with the closed-loop system (25)–(28). The expression (24) of the output y has the same form as the one in common delay-free extremum seeking problems. Hence, average computation done in the literature, especially in (Ghaffari et al., 2012), also works for our problem.

If f is the quadratic map (1), the average of (33) can be explicitly computed as

$$\frac{\omega}{2\pi}\int_0^{2\pi/\omega} F_3(\omega\tau, X)d\tau = HX_1 + H\int_0^D \beta_{[D,\sigma]} X_2(D-\sigma)d\sigma \tag{36}$$

for each $X = [X_1,X_2,X_3]^\top \in \mathcal{H}$. From the argument above, the average system associated with the closed-loop system (25)–(28) is given by

$$\dot{\tilde{\theta}}^{\mathrm{a}}(t) = \int_0^D u^{\mathrm{a}}(D-\sigma,t)d\beta(\sigma), \tag{37}$$
$$u^{\mathrm{a}}_t(x,t) = u^{\mathrm{a}}_x(x,t), \quad x \in (0,D), \tag{38}$$
$$u^{\mathrm{a}}(D,t) = U^{\mathrm{a}}(t), \tag{39}$$
$$\dot{U}^{\mathrm{a}}(t) = -cU^{\mathrm{a}}(t) + ckH\Bigg(\tilde{\theta}^{\mathrm{a}}(t) + \int_0^D \beta_{[D,\sigma]} u^{\mathrm{a}}(D-\sigma,t)d\sigma\Bigg). \tag{40}$$

It can be inferred from the relation $u^{\mathrm{a}}(x,t) = U^{\mathrm{a}}(x+t-D)$ that U^{a} is a filtered value of the signal

$$kH\left(\tilde{\theta}^{\mathrm{a}}(t) + \int_0^D \beta_{[D,\sigma]} U^{\mathrm{a}}(t-\sigma)d\sigma\right), \tag{41}$$

This signal is strongly related to the predictor feedback control law (21).

Indeed, applying the integration by parts for the Riemann-Stieltjes integral, changing the variable of integration, reversing the order of integrations leads to

$$kH\left(\tilde{\theta}^{\mathrm{a}}(t) + \int_0^D \beta_{[D,\sigma]} U^{\mathrm{a}}(t-\sigma)d\sigma\right) = kH\left(\tilde{\theta}^{\mathrm{a}}(t) + \int_t^{t+D}\int_{\tau-t}^D U^{\mathrm{a}}(\tau-\sigma)d\beta(\sigma)d\tau\right). \tag{42}$$

The right-hand side completely coincides with (21) if the gain $-k$ is replaced with kH. Hence, the proposed U given by (22) with (23) realizes a filtered predictor feedback control law with the gain kH in the average sense.

4.2 Stability of the average system

The stability of the average system (37)–(40) is analyzed in this subsection.

Lemma 3. Consider the system (37)–(40) for some $D > 0$ and $H < 0$. Let $\beta : [0, D] \to \mathbb{R}$ be a function of bounded variation satisfying Assumption 1. Then, for each $k > 0$, there exists $c^* > 0$ such that, for any $c > c^*$, the following assertions hold:

(i) The system (37)–(40) admits a unique solution in $C^1([0, +\infty); \mathcal{H}) \cap C([0, +\infty); D(A))$ for any given initial data $[\tilde{\theta}^{\rm a}(0), u^{\rm a}(\cdot, 0), U^{\rm a}(0)]^\top \in D(A)$, where the subspace $D(A) \subset \mathcal{H}$ is defined by (31).

(ii) There exist constants $\lambda > 0$ and $M > 0$, which are independent from the initial data, such that

$$\left\| \left[\tilde{\theta}^{\rm a}(t), u^{\rm a}(\cdot, t), U^{\rm a}(t)\right] \right\|_{\mathcal{H}} \leq M e^{-\lambda t} \left\| \left[\tilde{\theta}^{\rm a}(0), u^{\rm a}(\cdot, 0), U^{\rm a}(0)\right] \right\|_{\mathcal{H}}. \quad (43)$$

The complete proof is omitted due the space limitation. The idea is the use of a Lyapunov functional of the form

$$V(t) = \frac{c^2}{2}\vartheta(t)^2 + \frac{b}{2}\int_0^D e^{\alpha(x-D)} u^{\rm a}(x, t)^2 dx + \frac{1}{2}U^{\rm a}(t)^2, \quad (44)$$

where $\vartheta(t) \in \mathbb{R}$ is defined by

$$\vartheta(t) = \tilde{\theta}^{\rm a}(t) + \int_0^D \beta_{[D,\sigma]} u^{\rm a}(D - \sigma, t) d\sigma + \frac{1}{c} U^{\rm a}(t). \quad (45)$$

Differentiating ϑ with respect to t gives $\dot{\vartheta} = kH(\vartheta - U^{\rm a}/c)$. This property allows us to show the existence of $\lambda > 0$ such that $\dot{V} \leq -2\lambda V$. In this way, we can prove the lemma.

It should be emphasized that the construction of the Lyapunov function (44) does not involve a backstepping transformation unlike our previous study (Oliveira et al., 2017). Instead, the variable ϑ defined by (45) is introduced. Lyapunov functionals similar to (44) can be found in (Jankovic, 2009; Mazenc et al., 2012).

4.3 Proof of Theorem 2

The theorem is a consequence of Lemma 3 and the averaging theorem for infinite dimensional systems (Hale and Verduyn Lunel, 1990). Hence, we omit details. However, there is a remark that we should make. To apply the averaging theorem, we need to check that the operator A is a generator of strongly continuous semigroup T_A on $\mathcal{H}$ and the generated semigroup T_A has a smoothing property. The required property is as follows: for any $h : [0, \infty) \to \mathcal{H}$ being norm continuous, the following relations hold:

$$\text{(i)} \quad \int_0^t T_A(t - \tau) h(\tau) d\tau \in D(A), \qquad t \geq 0, \quad (46)$$

$$\text{(ii)} \quad \left\| A \int_0^t T_A(t - \tau) h(\tau) d\tau \right\| \leq M e^{\mu t} \max_{0 \leq \tau \leq t} \|h(\tau)\|_{\mathcal{H}}, \qquad t \geq 0, \quad (47)$$

where $M > 0$ and $\mu \in \mathbb{R}$ are independent from h. This is called the property (H).

The operator A defined by (30) is surely a generator of strongly continuous semigroup T_A on $\mathcal{H}$. However, T_A does not fulfill the smoothing property (H) in general. Fortunately, the perturbation F defined in (32) merely has the finite dimensional component F_3. Hence, in our problem, T_A only has to satisfy (46) and (47) for $h : [0, \infty) \to \mathcal{H}$ of the form $h(t) = [0, 0, h_3(t)]^\top$ for any continuous $h_3 : [0, \infty) \to \mathbb{R}$. We can explicitly compute the integral of $T_A(t - \tau)h(\tau)$ with respect to τ from 0 to t for given $h(t) = [0, 0, h_3(t)]^\top$. It can be inferred from the resulting expression that T_A satisfies the property (H). Then, the theorem follows from the exponential stability of the average system and the averaging theorem.

5. NUMERICAL EXAMPLE

We confirm the effectiveness of the proposed scheme. Let β be given by

$$\beta(x) = \begin{cases} 0, & 0 \leq x \leq \dfrac{D}{2}, \\ \dfrac{2x}{D} - 1, & \dfrac{D}{2} < x \leq D. \end{cases} \quad (48)$$

Since it is clear that this β satisfies the first condition in Assumption 1. We have

$$\int_0^D \theta(t - \sigma) d\beta(\sigma) = \frac{2}{D}\int_{D/2}^D \theta(t - \sigma) d\sigma. \quad (49)$$

Hence, the map f receives an average of the signal θ over the past interval $[t-D, t-D/2]$ at each t. We next calculate γ in (8). Direct computation shows that

$$\gamma(\omega) = \frac{16}{\omega^2 D^2} \sin\left(\frac{\omega D}{4}\right), \quad (50)$$

which implies that $\gamma(\omega) \neq 0$ as long as $\omega \neq 4m\pi/D$ for each $m \in \mathbb{Z}$. Thus, Assumption 1 holds. For β in (48), S can be explicitly calculated as

$$S(t) = \frac{a\omega D}{4\sin(\omega D/4)} \sin\left(\omega\left(t + \frac{3}{4}D\right)\right). \quad (51)$$

Consequently, S is a sinusoidal function, but its amplitude and phase have specific forms.

We conduct a numerical simulation. The unknown parameters of the map are set as $f^* = 15$, $\theta^* = 5$, and $H = -1$. The maximum delay is $D = 5$. The parameters in the proposed extremum seeking scheme are chosen as $a = 0.25$, $\omega = 6$, $k = 0.5$, and $c = 1$. To improve the stability of numerical computation, a high-pass filter $s/(s + \omega_h)$ is applied to y. The filtered signal is denoted by z. The signal My in (22) is placed with Mz. The signal Ny in (22) is also swapped with $\hat{H}$, which is defined as a filtered signal of Nz with the low-pass $\omega_l/(s + \omega_l)$. We set $\omega_h = 1$ and $\omega_l = 0.1$. Initial conditions are such that $\bar{\theta}(0) = 0$ and $U(\sigma) = 0$ for any $\sigma \in [0, D]$. The initial values of the filters' states are also set as 0.

We first show the output y in Fig. 2 when the predictor is not used. An instability is induced by the delay. We next show the simulation results for the proposed scheme in Fig. 3. The output y is plotted in (a) and it is observed that y approaches to a neighborhood of the extremum f^*. Similarly, the estimate $\hat{\theta}$ of θ^* converges to a neighborhood

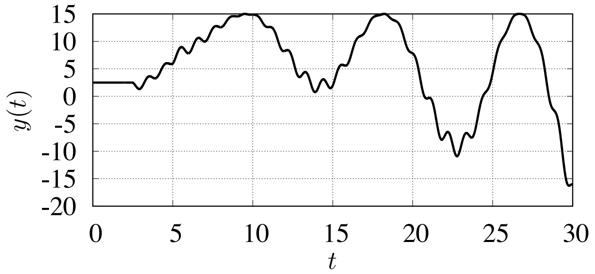
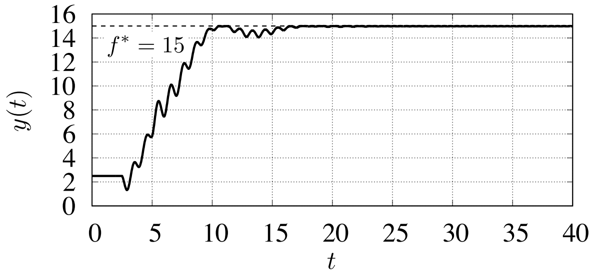
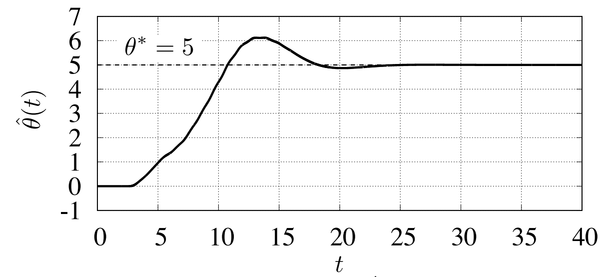


Fig. 2. The measured output of the closed-loop system without a prediction term in (22).



(a) The measurement output y.



(b) The estimate $\hat{\theta}$ of θ^*.

Fig. 3. Simulation Results for the proposed scheme.

of θ^*, as plotted in (b). Therefore, the proposed scheme successfully seeks the extremum while compensating the effect of the distributed delay.

6. CONCLUSION

In this paper, we have proposed a single-parameter extremum seeking scheme for a static map in the presence of distributed input delays. To compensate the distributed delay, we have introduced a new perturbation and probing signals. Then, the extremum seeking scheme is developed based on the predictor feedback control law. The effectiveness of the proposed scheme is demonstrated by a numerical simulation.

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