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Model-Predictive-Type Signal Control for a Burgers' Cellular Automaton Traffic Flow Model Based on Particle Swarm Optimization

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Abstract: This study is devoted to development of a new systematic signal control method for a traffic flow model represented by the Burgers' cellular automaton. First, it is shown that an optimal signal control problem with an objective function on the total number of traffic jam is formulated as a nonlinear integer programming problem. Then, a new algorithm to solve the optimal signal control problem based on particle swarm optimization (PSO), and the method is extended to a model-predictive-type control method in order to treat inflow and outflow of cars. Some numerical simulations indicate that the new signal control method can reduce the total number of traffic jam the most in the four methods.

Keywords: Signal Control, Model Predictive Control, Particle Swarm Optimization, Traffic Flow, Burgers' Cellular Automaton

1. INTRODUCTION

In recent years, traffic jam of cars on roads is one of the most serious problems in the world. Especially, in Japan, traffic jam frequently occurs mainly in large metropolitan areas. Economic loss on traffic jam of Japan for one year is estimated at 12 trillion yen, and total time loss for all Japanese is also estimated at 3.8 billion hours. It is well known that traffic jam causes various problems; environmental matters by exhaust gas, traffic accidents by inflow of cars into community roads, negative effects to drivers' bodies (Kerner (2009)). The causes of traffic jam are considered as natural jam by unconscious speed degradation of drivers, closures and detours by road accidents, decrease of roadway width by roadworks and on-street parking, bottlenecks of cars at tollgates and junctions. Moreover, signals installed at roads and intersections are also a contributory factor of traffic jam. Inappropriate switching of signals causes bottlenecks of cars, and then they are evolved to traffic jam. Currently, there are mainly two ways to switch signals; one is called "constant period control" that switches signals periodically, and the other is called "traffic sensing control" that switches signals bases on sensing information about cars and pedestrians. The latter control is more intelligent, and we can expect that it will be developed through the fusion of not only control theory and optimization methods but also newest technologies such as IoT (Internet of Things), big data, ITS (Intelligent Transportation Systems) (Gordon (2015); Moridpour (2019)).

The purpose of this study is to develop a new signal control method to minimize the total of traffic jam for a Burgers' cellular automaton traffic flow model. The contents of this paper are as follows. First, Section 2 introduces a Burgers' cellular automaton traffic flow model. Next, Section 3 formulates an optimization problem for the Burgers' cellular automaton traffic flow model, and develop a solving method based on particle swarm optimization (PSO). In addition, the method is extended to a model-predictive-type control method Finally, a numerical simulation is preformed to confirm the effectiveness of the proposed control method.

2. BURGERS' CELLULAR AUTOMATON TRAFFIC FLOW MODEL

In this section, a Burgers' cellular automaton traffic flow model will be formulated. First, a Burgers' cellular automaton with signals is explained. Consider a single-line traffic illustrated in Fig. 1. It is assumed that cars move from left to right, and each cell can contain up to C cars. We use indices on the time step and the cell number by $k = 1, \cdots, K$ and $l = 1, \cdots, L$, respectively. Let us denote the number of cars at the time k and in the l-th cell by $U_{k,l} \in \{0, 1, \cdots, C\}$. We also refer $S_{k,l} \in \{0, C\}$ as a signal variable, which is installed between the $(l-1)$-th and l-th cells, and Δ as the indices set of signals. If the sign of a signal installed at the cell l is blue, then $S_{k,l} = C$ holds. Conversely, if the sign of a signal installed at the cell l is red, then $S_{k,l} = 0$ holds. If a signal is not installed at the cell l, cars always can move through the cell, and $S_{k,l} = C$ $(l \notin \Delta)$ always holds. Hence, the signal variables $S_{k,l}$ can be regard as control inputs. Under the setting explained above, *the Burgers' cellular automaton with signals*, which represents time evolution of the cars in the cells, is given by

$$\begin{aligned} U_{k+1,l} = U_{k,l} &+ \min(S_{k,l}, U_{k,l-1}, C - U_{k,l}) \\ &- \min(S_{k,l+1}, U_{k,l}, C - U_{k,l+1}), \qquad (1) \\ &(k = 1, \cdots, K-1;\ l = 2, \cdots, L-1). \end{aligned}$$

See Hirota and Takahashi (2003) for derivation and details of it. In (1), the second term of the right-hand side shows the inflow number of the cars from the cell $l-1$ to l, and the third term of the right-hand side shows the outflow number of the cars from the cell l to $l+1$.

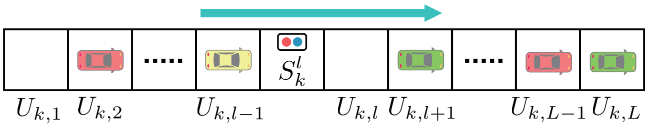

Fig. 1. A Burgers' cellular automaton with signals. ($C=3$, Red: 3, Yellow: 2, Green: 1)

Next, we shall derive a Burgers' cellular automaton traffic flow model based on the Burgers' cellular automaton with signals. In this study, we consider a traffic flow model as shown in Fig. 2. The model consists of 6 Burgers' cellular automaton traffic flow models, and 3 models are in a transverse direction (U^1, U^2, U^3) and 3 models are in a longitudinal direction (V^1, V^2, V^3). The number of cars on the i-th (j-th) transverse (longitudinal) road at the time k and the cell l is denoted by $U^i_{k,l} \in \{0,1,\cdots,C\}$ ($i=1,2,3$; $k=1,\cdots,K$; $l=1,\cdots,L$) ($V^j_{k,l} \in \{0,1,\cdots,C\}$ ($j=1,2,3$; $k=1,\cdots,K$; $l=1,\cdots,L$)). It is assumed that the locations of signals in the transverse and longitudinal roads are the same, and their indices sets are represented as Δ_U, Δ_V. We also denote the signal of the i-th transverse (longitudinal) road at the (i,j) intersection of the i-th transverse road and the j-th longitudinal road by $S^i_{k,l} \in \{0,C\}$ ($i=1,2,3$; $k=1,\cdots,K$; $l \in \Delta_U$) and $T^j_{k,m} \in \{0,C\}$ ($j=1,2,3$; $k=1,\cdots,K$; $m \in \Delta_V$), respectively. Since one signal is red (blue) and the other signal is blue (red) at the same intersection, $S^i_{k,l} + T^j_{k,m} = C$; $l \in \Delta_U$; $m \in \Delta_V$ holds. Hence, it must be noted that $T^j_{k,m} = C - S^i_{k,l}$ holds, and $S^i_{k,l}$ can indicate two signals at one time. Using the above variables, *the Burgers' cellular automaton traffic flow model* can be represented by (2) and (3).

3. MODEL-PREDICTIVE-TYPE SIGNAL CONTROL METHOD VIA PARTICLE SWARM OPTIMIZATION

This section will formulate an optimal signal control problem to minimize the total number of traffic jam for the Burgers' cellular automaton traffic flow model introduced in Section 2, and consider a solving method of the problem based on particle swarm optimization.

First, we define the total number of traffic jam. Let us consider the i-th transverse road (2) and the number of traffic jam can be calculated by subtracting the outflow of the cars: $\min(S^i_{k,l+1}, U^i_{k,l}, C-U^i_{k,l+1})$ from the current number of cars: $U_{k,l}$. Therefore, by calculating the sum total of the number of traffic jam, we can have the total number of traffic jam as (4). Now, an optimal signal control problem for the Burgers' cellular automaton traffic flow is stated as follows.

Problem 1 : For the Burgers' cellular automaton traffic flow model, find optimal signal inputs $S^i_{k,l}$ ($i=1,2,3$; $k=1,\cdots,K-1$; $l \in \Delta_U$) such that the total number of traffic jam (4) is minimized. □

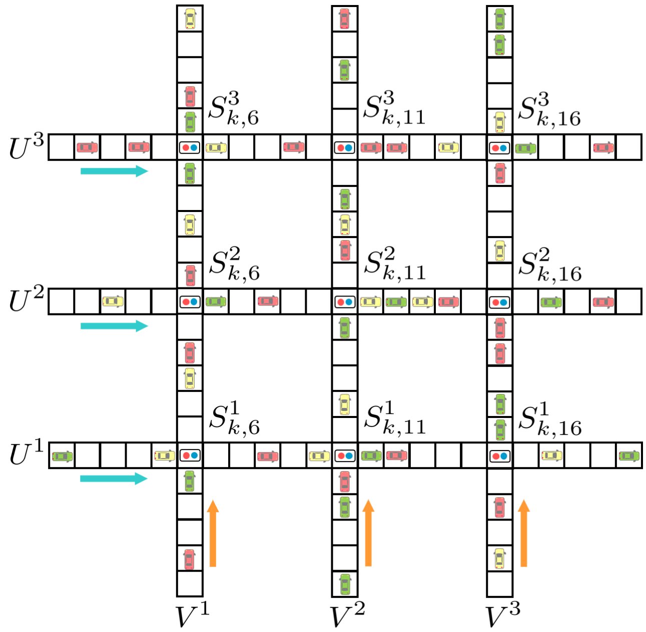

Fig. 2. A Burgers' cellular automaton traffic flow model. ($C=3$, Red: 3, Yellow: 2, Green: 1)

Problem 1 can be formulated as the next optimization problem:

$$\begin{aligned} &\mathsf{min}\ (4), \\ &\mathsf{subject\ to}\ (2),\ (3), \\ &\mathsf{given}\ \text{initial and boundary conditions.} \end{aligned} \tag{5}$$

All the variables in (5) are integer with upper and lower limits, and the objective function (4) and the Burgers' cellular automaton traffic flow model (2), (3) are nonlinear, hence (5) is represented as a nonlinear integer programming problem. In general, it is quite hard to solve this class of optimization problem and calculate an optimal solution. In this study, we utilize "particle swarm optimization (PSO)," which is one of the heuristic optimization methods (Clerc (2006); Parsopoulos (2010)). We also refer the modified version of PSO to solve a nonlinear integer programming problem (Matsui et al. (2008)). We set a particle for PSO as $X = [\ S^1_{1,l_1} \cdots S^3_{K-1,l_3}\ ]^\mathsf{T} \in \{0,1\}^{9(K-1)}$ ($l_1, l_2, l_3 \in \Delta_U$), which are constructed by list all the signal variables. The update rule in PSO is given by

$$\begin{aligned} V_n^{t+1} &= w^t V_n^t + c_1 r_1^t (P_n^t - X_n^t) + c_2 r_2^t (P_g^t - X_n^t), \\ X_n^{t+1} &= X_n^t + V_n^{t+1}, \end{aligned} \tag{6}$$

where X_n^t is the n-th particle ($n=1,\cdots,N$) at the search step t, V_n^{t+1} is the value of update, P_n^t is the best searching point for the n-th particle, P_g^t is the best searching point for all the particles, w^t, c_1, c_2, r_1^t, r_2^t are parameters. We first generate an initial set of particles X_n^0 ($n=1,\cdots,N$) at random, then update particles by using (6). If the serach step comes up the maximum value or the value of the best particle does not change, then search by PSO is terminated and the best particle is referred as the signal control inputs.

Moreover, we extend the above method based on PSO to a model-predictive-type control method in order to consider inflow and outflow of cars. The algorithm is shown as follows.

$$U_{k+1,l}^i = U_{k,l}^i + \min(S_{k,l}^i, U_{k,l-1}^i, C - U_{k,l}^i) - \min(S_{k,l+1}^i, U_{k,l}^i, C - U_{k,l+1}^i) \tag{2}$$

$$V_{k+1,m}^j = V_{k,m}^j + \min(T_{k,m}^j, V_{k,m-1}^j, C - V_{k,m}^j) - \min(T_{k,m+1}^j, V_{k,m}^j, C - V_{k,m+1}^j) \tag{3}$$

$$(i, j = 1, 2, 3;\ k = 1, \cdots, K;\ l = 1, \cdots, L;\ m \in 1, \cdots, M)$$

$$J = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{K}\sum_{l=1}^{L-1}\sum_{m=1}^{M-1}\Big[\{U_{k,l}^i - \min(S_{k,l+1}^i, U_{k,l}^i, C - U_{k,l+1}^i)\} + \{V_{k,m}^j - \min(T_{k,m+1}^j, V_{k,m}^j, C - V_{k,m+1}^j)\}\Big] \tag{4}$$

Algorithm 1 :

Step 1 At the time step k =1, solve the optimal signal control problem for the predictive horizon κ by the new method by PSO, and obtain an optimal signal switching law $S_{k,l}^i$ $(i = 1, 2, 3;\ k = 1, \cdots, \kappa - 1;\ l \in \Delta_U)$.

Step 2 Apply a part of the optimal signal switching law $S_{k,l}^i$ $(i = 1, 2, 3;\ k = 1, \cdots, h;\ l \in \Delta_U)$ to the Burgers' cellular automaton traffic flow model (2), (3) for the time interval h.

Step 3 Repeat Steps 1 and 2 until the time step reaches the maximum value K. □

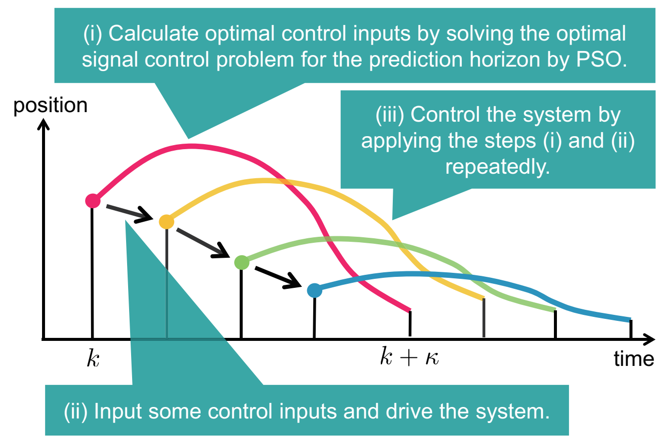

Fig. 3. An illustration on model-predictive-type signal control.

4. SIMULATIONS

In this section, some numerical simulations are performed to confirm effectiveness of the proposed method. We here consider the Burgers' cellular automaton traffic flow model illustrated in Fig. 2 with $K = 20$, $L = 20$, $M = 20$, and the locations of signals are set as $\Delta_U = \{6, 11, 16\}$, $\Delta_V = \{6, 11, 16\}$. We here consider inflow and outflow of cars as disturbances as shown in Fig. 4. The numbers of cars and locations on inflow and outflow of cars are determined in a random manner.

In this study, we consider the following three control method in order for comparison.

(a) Time Control: Switch signals by 3 time steps:

$$S_{k,l}^i = \begin{cases} 0 & (k = 1, 2, 3, 7, 8, 9, \cdots), \\ C & (k = 4, 5, 6, 10, 11, 12, \cdots). \end{cases} \tag{7}$$

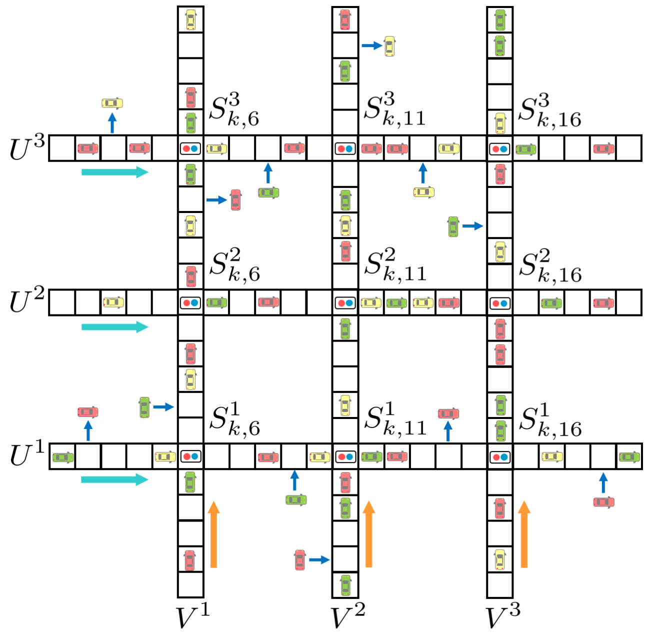

Fig. 4. The simulation setting of the Burgers' cellular automaton traffic flow model with inflow and outflow of cars.

(b) Comparing Control: Compare the two numbers of cars at the fronts of cells in a transverse and a longitudinal directions, then switch the signal at the cell whose numbers of cars is larger to blue. If the number of cars are same, switch the signal at the cell in a longitudinal direction to blue:

$$S_{k,l}^i = \begin{cases} 0 & (U_{k,l-1}^i \le V_{k,m-1}^j), \\ C & (U_{k,l-1}^i > V_{k,m-1}^j). \end{cases} \tag{8}$$

(c) PSO Control: Switch signals by using the proposed control method based on PSO.

(d) PSO-MPC Control: Switch signals by using Algorithm 1.

20 patterns of initial locations of cars and boundary conditions on inflow of cars to the model are generated at random, and numerical simulations are performed by using the control methods (a)–(d) for the 20 patterns. The parameters for PSO are set as follows; the number of particles: $N = 100$, the maximum serach step: $T = 100$, ω^t is the time-varying type proposed in Matsui et al. (2008):

Table 1. The total numbers of traffic jam for the four methods (a)–(d).

No.	(a)Time	(b) Compare	(c) PSO	(d) PSO-MPC
1	1309	1003	1305	758
2	1235	1165	1223	780
3	1874	1548	1603	1320
4	1275	1064	1257	776
5	1408	1087	1128	828
6	1321	1000	1239	796
7	1145	943	1272	817
8	1387	860	1308	755
9	1439	776	1082	709
10	1398	1003	1284	859
11	1330	933	1099	821
12	1085	946	1119	692
13	1124	766	1027	607
14	1300	954	1193	790
15	1285	793	1057	651
16	1626	1359	1557	1184
17	1392	1254	1323	882
18	1598	1100	1284	895
19	1665	1427	1619	1317
20	1455	1141	1435	952
Ave.	1382.55	1056.1	1270.7	859.45

$$\omega^t = \begin{cases} \omega^0 - \dfrac{t(\omega^0 - \omega^T)}{0.75T} & t \le 0.75T, \\ \omega^T & t > 0.75T, \end{cases} \tag{9}$$

where $c_1 = c_2 = 1000000000$, and r_1^t, r_2^t are random numbers in [0, 1].

Table 1 shows the total numbers of traffic jam obtained by the control methods (a)–(d) for the 20 patterns. From this result, it turns out that the proposed method (d) can reduce the total number of traffic jam the most in the three methods for al the 20 patterns. The averages of the total number of traffic jam for al the 20 patterns are also shown at the last column of Table 1. From these averages, we can see that the proposed method (d) realizes about 38% reduction in comparison with (a). We can also see that the proposed method (d) realizes about 32% reduction in comparison with (c) since (d) can take into account inflow and outflow of cars, hence the proposed method (d) has the robustness on inflow and outflow of cars. In addition, the simulations show that the proposed method (d) needs small amount of calculation since it is a model-predictive-type. Consequently, it can be confirmed that the proposed signal control method via PSO is effective in terms of optimization and computation efficiency from the simulation results.

5. CONCLUSIONS

In this study, an optimal signal control problem to minimize the total number of traffic jam for the Burgers' cellular automaton traffic flow model has been formulated, and a solving method based on particle swarm optimization has been developed. In addition, we extend the method to a model-predictive-type control method. Numerical simulations show that the proposed method can considerably reduce the total number of traffic jam in comparison with other methods.

Our future work are as follows: construction of new Burgers' cellular automaton traffic flow models in consideration of left and right turns, and the optimal velocity model, development of a distributed control method and numerical simulations for large-sized urban areas.

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