Path Following with Stable and Unstable Modes Subject to Time-Varying Dwell-Time Conditions

Duc M. Le*, Hsi-Yuan Chen*, Andrew R. Teel, Warren E. Dixon***

* *Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, FL, USA (Email: {ledan50, hychen,wdixon}@ufl.edu)*
** *Department of Electrical and Computer Engineering, University of California Santa Barbara, Santa Barbara, CA, USA (Email: teel@ece.ucsb.edu)*

Abstract: Systems are often tasked with operating in modes where state feedback is available intermittently. The analysis of such systems involves analyzing the behavior of individual subsystems and the time that each subsystem is active, i.e., dwell-time conditions. Often, these dwell-time conditions are conservative, potentially limiting the performance of the overall system. In an effort to reduce conservativeness of the dwell-time condition, an adaptive data-driven extremum seeking (ESC) method is used to develop a time-varying dwell-time condition. Specifically, the ESC drives the evolution of the dwell-time condition to a less conservative dwell-time condition while simultaneously ensuring stability of the overall system. Simulations demonstrate a nearly threefold increase in a maximum dwell-time that results in significant changes to the behavior of an agent tasked with following a path outside a feedback region.

Keywords: Time-Varying Dwell-Time Conditions, Intermittent State Feedback, Switched System, Extremum Seeking Control (ESC)

1. INTRODUCTION

Factors such as task definition, complex operating environments, or sensor modality may result in temporary losses (i.e., intermittencies) in state feedback, thereby restricting the ability to utilize closed-loop feedback control. In efforts to relax continuous state feedback constraints, switched system approaches have been investigated in results such as Parikh et al. (2017), Jia and Liu (2015), and Mehta et al. (2008). Inherently, modes of operation where state feedback is unavailable may introduce instabilities. Hence, for dynamical systems governed by a family of subsystems with stable and unstable modes, a switched system approach can be leveraged to model and analyze the stability of the overall system (Goebel et al. (2012), Liberzon (2003), Zhai et al. (2001), and Branicky (1998)).

Similar to Chen et al. (2018) and Chen et al. (2019), development in this paper considers a single agent (extendable to multiple) system tasked with following a path that lies completely outside of a feedback region. To achieve this objective, the system switches between two modes of operation—one mode where the agent exits the feedback region and dead-reckons to follow the desired path, and another mode where the agent returns to the feedback region to regulate state estimate errors. In this context, the switched system is comprised of a single (or multiple) agent(s) equipped with local sensing capabilities subject to intermittent state information. Dwell-time conditions are developed to determine the duration a system must operate with feedback, and the duration the system can dead-reckon. In results such as Parikh et al. (2017), Chen et al. (2018), and Zegers et al. (2019), Lyapunov-based switching control designs inherently result in conservative dwell-time constraints. The practical implications of such conservative dwell-time conditions is that the agent is required to return to the feedback region more frequently than actually required, thereby reducing the amount of time the agent can follow the desired path in feedback-denied regions.

Extremum Seeking Control (ESC) is a feedback control method that exploits an unknown steady-state input-to-output mapping (i.e., response map) with a local (or global) extremum to achieve real-time optimization. Typical ESC methods include the use of a periodic perturbation injected in the feedback loop to explore the neighborhood around a setpoint to find the extremum. Various numerical-based extremum search algorithms have been developed (e.g., Brent's method), but the first perturbation-based stability proof was introduced in Krstic and Wang (2000). ESCs have since been an attractive model-free method in applications where the response of a system is governed by an unknown nonlinear model, and has a local (or global) extremum (Ariyur and Krstic (2003)). Due to its simplicity and adaptability, ESCs have been extensively utilized in various applications such as tuning gains of a PID controller (Killingsworth and Krstic (2006)), maximizing power output in rehabilitation robotics (Duenas et al. (2018)), and hybrid systems (Poveda and Teel (2017)). ESC is used in this paper as an online optimization method to regulate a state to the neighborhood of an unknown setpoint that reduces the conservativeness of the maximum dwell-time condition.

In this paper, a switched system approach is used to develop a time-varying dwell-time condition. The dwell-time condition development uses intermittent measurements of the dead-reckoning error to maximize the agent's duration in the feedback-denied region while simultaneously ensuring stability. Specifically, the framework introduced in Chen et al. (2018) is modified to include an ESC algorithm to generate an input that uses output measurements intermittently to increase the maximum dwell-time condition. Although ESC methods drive an input towards an unknown setpoint, the challenge in the context of this problem is to simultaneously ensure the evolution of the dwell-time conditions while maintaining stability of the tracking error objectives, i.e., if the agent remains in the feedback-denied region too long, it may restrict the agent's ability to navigate back to the feedback region. By virtue of ESC algorithms, the evolution of the input must exploit the neighborhood around a setpoint to find the extremum. Thus, to ensure stability of the system, an adjustable offset parameter is incorporated within the ESC algorithm.

2. SYSTEM MODEL

The dynamics of an agent can be modeled as

$$\dot{x}\left(t\right)=f\left(x\left(t\right),t\right)+v\left(x\left(t\right),t\right)+d\left(t\right), \tag{1}$$

where $x:\mathbb{R}_{\geq0}\to\mathbb{R}^{n}$ denotes the state of the agent, $f:\mathbb{R}^{n}\times\mathbb{R}_{\geq0}\to\mathbb{R}^{n}$ denotes the known drift dynamics, $d:\mathbb{R}_{\geq0}\to\mathbb{R}^{n}$ denotes unknown exogenous disturbances, and $v:\mathbb{R}^{n}\times\mathbb{R}_{\geq0}\to\mathbb{R}^{n}$ denotes the control input. The following assumptions are used in the subsequent development.

Assumption 1. The function $(x,t)\mapsto f(x,t)$ is continuous in t and globally Lipschitz in x, i.e., $\|f(a,t)-f(b,t)\|\leq L\|a-b\|$ $\forall a,b\in\mathbb{R}^{n}$, where $L\in\mathbb{R}_{\geq0}$ is the Lipschitz constant. ■

Assumption 2. The exogenous disturbances $d\left(\cdot\right)$ can be bounded as $\|d\left(t\right)\|\leq\overline{d}$ $\forall t\in\mathbb{R}_{\geq0}$, where $\overline{d}\in\mathbb{R}_{>0}$ is known. ■

3. STATE ESTIMATION AND CONTROL OBJECTIVE

A known region where feedback is available is denoted by a compact set $\mathcal{F}\subset\mathbb{R}^{n}$, and its complement is a region where feedback is unavailable. Feedback is available when the agent is in the feedback region (i.e., $x\left(t\right)\in\mathcal{F}$), and unavailable otherwise. The goal is for an agent to follow a desired path, denoted by $x_{d}:\mathbb{R}_{\geq0}\to\mathbb{R}^{n}$, that lies completely outside of the feedback region, i.e., $x_{d}\left(t\right)\notin\mathcal{F}$ for all $t\in\mathbb{R}_{\geq0}$. Since the desired path is outside of the feedback region, the state estimate, denoted by $\hat{x}:\mathbb{R}_{\geq0}\to\mathbb{R}^{n}$, is used to estimate the agent's state when the agent is outside of the feedback region. Consequently, open-loop state estimates may diverge from the true state, thus requiring the agent to intermittently return to the feedback region to ensure the estimation error remains bounded. Since the agent is required to cyclically traverse between feedback and feedback-denied regions, the control objective is to follow an auxiliary trajectory. An auxiliary trajectory, denoted by $x_{\sigma}:\mathbb{R}_{\geq0}\to\mathbb{R}^{n}$, must be designed such that the path traverses between the desired path and the feedback region. Given these objectives, three error signals are defined as

$$e\left(t\right)\triangleq x\left(t\right)-x_{\sigma}\left(t\right), \tag{2}$$

$$\hat{e}\left(t\right)\triangleq\hat{x}\left(t\right)-x_{\sigma}\left(t\right), \tag{3}$$

$$\tilde{e}\left(t\right)\triangleq x\left(t\right)-\hat{x}\left(t\right), \tag{4}$$

where $e:\mathbb{R}_{\geq0}\to\mathbb{R}^{n}$ denotes the error between the actual state and the auxiliary trajectory, $\hat{e}:\mathbb{R}_{\geq0}\to\mathbb{R}^{n}$ denotes the error between the state estimate and the auxiliary trajectory, and $\tilde{e}:\mathbb{R}_{\geq0}\to\mathbb{R}^{n}$ denotes the error between the actual state and the state estimate. When state feedback is available, the objective is to regulate all three error signals in (2)-(4). From (2)-(4), the following beneficial relations can be developed

$$\hat{e}\left(t\right)=e\left(t\right)-\tilde{e}\left(t\right), \tag{5}$$

$$e\left(t\right)=\hat{e}\left(t\right)+\tilde{e}\left(t\right). \tag{6}$$

The goal is to follow the desired path while ensuring the error signals in (2)-(4) remain bounded. Due to potential instabilities with open-loop state estimators, the challenge is to ensure the agent returns to the feedback region while simultaneously maximizing the time spent outside of the feedback region to enable the agent to follow the desired path (Chen et al. (2018)). When $x\left(t\right)\in\mathcal{F}$, a minimum dwell-time condition is required to ensure the norm of the errors is less than a user-defined threshold before exiting the feedback region. When $x\left(t\right)\notin\mathcal{F}$, a maximum dwell-time condition is established to ensure the norm of the errors do not exceed a user-defined threshold. The agent's capability to follow the desired path is restricted by the subsequently developed dwell-time conditions. To achieve the control objective while increasing robustness to intermittencies in state feedback, an ESC scheme is used to develop a time-varying dwell-time condition that increases the allowable duration outside of the feedback region.

4. CONTROLLER AND UPDATE LAW DESIGN

To facilitate the subsequent development and analysis, modes of the switched system (subsystems) when state feedback is available and unavailable are denoted by $\mathcal{P}_{a}\in\mathbb{N}$ and $\mathcal{P}_{u}\in\mathbb{N}$, respectively. The set of operating modes of the switched system is denoted by $\mathcal{P}\triangleq\{\mathcal{P}_{a},\mathcal{P}_{u}\}\subset\mathbb{N}$. Then a switching signal, denoted by $p:\mathbb{R}_{\geq0}\to\mathcal{P}$, indicates the active subsystem. The i^{th} instant when p switches from $\mathcal{P}_{u}$ to $\mathcal{P}_{a}$ is denoted by $t_{i}^{\mathcal{P}_{a}}\in\mathbb{R}_{\geq0}$ for all $i\in\mathbb{N}_{\geq0}$, i.e., the instant the agent enters the feedback region. For the complementary case, the i^{th} instant when p switches from $\mathcal{P}_{a}$ to $\mathcal{P}_{u}$ is denoted by $t_{i}^{\mathcal{P}_{u}}\in\mathbb{R}_{\geq0}$, i.e., the instant the agent exits the feedback region. Based on the switching instants, dwell-times of the i^{th} activation of the subsystems $\mathcal{P}_{a}$ and $\mathcal{P}_{u}$ are defined as $\Delta t_{i}^{\mathcal{P}_{a}}\triangleq t_{i}^{\mathcal{P}_{u}}-t_{i}^{\mathcal{P}_{a}}$ and $\Delta t_{i}^{\mathcal{P}_{u}}\triangleq t_{i+1}^{\mathcal{P}_{a}}-t_{i}^{\mathcal{P}_{u}}$, respectively. The following assumption is made about the initial operating mode of the system.

Assumption 3. The agent is initialized in a feedback region, i.e., $x\left(0\right)\in\mathcal{F}$ where $t=0$ corresponds to $t_{0}^{\mathcal{P}_{a}}$. ■

Based on the subsequent analysis, the state estimate update law, denoted by $\dot{\hat{x}}:\mathbb{R}_{\geq0}\to\mathbb{R}^{n}$, is designed as

$$\dot{\hat{x}}(t) \triangleq \begin{cases} f(\hat{x}(t), t) + v(x(t), t) \\ +v_r(\tilde{e}(t)), & t \in [t_i^{\mathcal{P}_a}, t_i^{\mathcal{P}_u}), \\ f(\hat{x}(t), t) + v(\hat{x}(t), t), & t \in [t_i^{\mathcal{P}_u}, t_{i+1}^{\mathcal{P}_a}), \end{cases} \tag{7}$$

where $\hat{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ denotes the state estimate, and $v_r : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ denotes the auxiliary input. The control and auxiliary inputs are designed as

$$v(x(t), t) \triangleq \begin{cases} \dot{x}_\sigma(t) - \overline{d}\mathrm{sgn}(e(t)) \\ -f(x(t), t) - ke(t), & t \in [t_i^{\mathcal{P}_a}, t_i^{\mathcal{P}_u}), \\ \dot{x}_\sigma(t) - f(\hat{x}(t), t) \\ -k\hat{e}(t), & t \in [t_i^{\mathcal{P}_u}, t_{i+1}^{\mathcal{P}_a}), \end{cases} \tag{8}$$

$$v_r(\tilde{e}(t), t) \triangleq k_{\tilde{e}}\tilde{e}(t) + \overline{d}\mathrm{sgn}(\tilde{e}(t)), \tag{9}$$

respectively, where $k, k_{\tilde{e}} \in \mathbb{R}_{>0}$ are adjustable parameters, and $\mathrm{sgn}(\cdot)$ denotes the signum function.

When the agent is in the feedback region (i.e., $t \in [t_i^{\mathcal{P}_a}, t_i^{\mathcal{P}_u})$), the controller inputs in (7)-(9) regulate the error signals in (2)-(4). When the agent exits the feedback region (i.e., $t \in [t_i^{\mathcal{P}_u}, t_{i+1}^{\mathcal{P}_a})$), the control inputs in (7)-(8) use open-loop state estimates (dead-reckoning) for navigation. Since open-loop estimates will drift over time, dead-reckoning may result in potential instabilities. Thus, a maximum dwell-time condition is developed to quantify the maximum duration the agent can dead-reckon while simultaneously ensuring the agent can return to the feedback region.

In Chen et al. (2018), the maximum dwell-time condition enforces the norm of the error to be bounded by a maximum error threshold, denoted by $e_M \in \mathbb{R}_{>0}$, before entering the feedback region. To guarantee re-entry to the feedback region, the selection of the maximum error threshold is dictated by the size of the feedback region, i.e., the larger the feedback region the larger e_M can be selected.[1] The maximum dwell-time condition in Chen et al. (2018) is conservative, i.e., experiments indicate that the agent can remain outside the feedback region for a longer duration while maintaining the specified maximum error threshold.

The main idea of this paper is to use intermittent state measurements in a sample-based feedback ESC algorithm to reduce the conservativeness of the maximum dwell-time condition, thereby maximizing the time an agent can follow the desired path outside the feedback region. Each re-entry into the feedback region provides a measurement of the accumulated dead-reckoning error, i.e., $e\left(t_{i+1}^{\mathcal{P}_a}\right)$. Specifically, the goal is to minimize the difference between the dead-reckoning error and the maximum allowable error e_M (minus an offset denoted as $\phi \in (0, e_M)$). To this end, an output function (i.e., a cost function), denoted by $y : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, of the system is defined as

$$y\left(t_{i+1}^{\mathcal{P}_a}\right) \triangleq k_y\left(e_M - \phi - \left\|e\left(t_{i+1}^{\mathcal{P}_a}\right)\right\|\right)^2, \tag{10}$$

where $k_y \in \mathbb{R}_{>0}$ is a user-defined parameter. The extremum of the output in (10) corresponds to $\left\|e\left(t_{i+1}^{\mathcal{P}_a}\right)\right\| = e_M - \phi$. As stated in the following assumption, an ESC algorithm can be used to develop an input parameter, denoted by $\Gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}$, that adjusts the dwell-time, and hence $e\left(t_{i+1}^{\mathcal{P}_a}\right)$, to minimize the cost in (10).

Assumption 4. Given the dynamic model in (1), with the state estimate in (7), and controller in (8) and (9), there exists an ESC algorithm[2] such that each discrete measurement of $e\left(t_{i+1}^{\mathcal{P}_a}\right)$ produces $\Gamma\left(t_{i+1}^{\mathcal{P}_a}\right)$ that converges to a neighborhood of an unknown optimal setpoint $\Gamma^* \in \mathbb{R}$ that minimizes (10). ∎

5. SWITCHED SYSTEM STABILITY ANALYSIS

Taking the time derivatives of (2)-(4) and substituting the agent's dynamics, state estimate update law, control input, and auxiliary control input in (1), (7)-(9), respectively, yields (Chen et al. (2018))

$$\dot{e}(t) = -ke(t) - \overline{d}\mathrm{sgn}(e(t)) + d(t),\ t \in [t_i^{\mathcal{P}_a}, t_i^{\mathcal{P}_u}), \tag{11}$$

$$\dot{\hat{e}}(t) = -k\hat{e}(t),\ t \in [t_i^{\mathcal{P}_u}, t_{i+1}^{\mathcal{P}_a}), \tag{12}$$

$$\dot{\tilde{e}}(t) = \begin{cases} f(x(t), t) - f(\hat{x}(t), t) + d(t) \\ -k_{\tilde{e}}\tilde{e}(t) - \overline{d}\mathrm{sgn}(\tilde{e}(t)), & t \in [t_i^{\mathcal{P}_a}, t_i^{\mathcal{P}_u}), \\ f(x(t), t) - f(\hat{x}(t), t) + d(t), & t \in [t_i^{\mathcal{P}_u}, t_{i+1}^{\mathcal{P}_a}). \end{cases} \tag{13}$$

To analyze the switched system, candidate Lyapunov functions are defined as

$$V_e(e(t)) \triangleq \frac{1}{2}e^T(t)e(t), \tag{14}$$

$$V_{\hat{e}}(\hat{e}(t)) \triangleq \frac{1}{2}\hat{e}^T(t)\hat{e}(t), \tag{15}$$

$$V_{\tilde{e}}(\tilde{e}(t)) \triangleq \frac{1}{2}\tilde{e}^T(t)\tilde{e}(t), \tag{16}$$

where $V_e : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$, $V_{\hat{e}} : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$, and $V_{\tilde{e}} : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$. Taking the time derivatives of (14)-(16), using Assumption 1, and substituting (11)-(13) yields

$$\dot{V}_e(e(t)) \leq -k\|e(t)\|^2,\ t \in [t_i^{\mathcal{P}_a}, t_i^{\mathcal{P}_u}), \tag{17}$$

$$\dot{V}_{\hat{e}}(\hat{e}(t)) \leq -k\|\hat{e}(t)\|^2,\ t \in [t_i^{\mathcal{P}_u}, t_{i+1}^{\mathcal{P}_a}), \tag{18}$$

$$\dot{V}_{\tilde{e}}(\tilde{e}(t)) \leq \begin{cases} -\lambda_a\|\tilde{e}(t)\|^2, & t \in [t_i^{\mathcal{P}_a}, t_i^{\mathcal{P}_u}), \\ \lambda_u\|\tilde{e}(t)\|^2 + \frac{\overline{d}^2}{2}, & t \in [t_i^{\mathcal{P}_u}, t_{i+1}^{\mathcal{P}_a}), \end{cases} \tag{19}$$

respectively, where $\lambda_a, \lambda_u \in \mathbb{R}_{>0}$ are known positive constants defined as

$$\lambda_a \triangleq k_{\tilde{e}} - L, \tag{20}$$

$$\lambda_u \triangleq L + \frac{1}{2}, \tag{21}$$

respectively, provided $k_{\tilde{e}} > L$. Using the Comparison Lemma in Khalil (2002) and definitions of the candidate

[1] This paper considers circular feedback regions with a known radius. Therefore, the selection of the maximum error threshold must be selected less than or equal to the radius of the feedback region to guarantee re-entry.

[2] See Krstic and Wang (2000), Choi et al. (2002), and Guay (2014) for potential ESC schemes.

Lyapunov functions in (14)-(16), it follows from (17)-(19) that

$$\|e(t)\| \leq \left\|e\left(t_i^{\mathcal{P}_a}\right)\right\| \mathrm{e}^{-k(t-t_i^a)},\ t \in [t_i^{\mathcal{P}_a}, t_i^{\mathcal{P}_u}), \tag{22}$$

$$\|\hat{e}(t)\| \leq \left\|\hat{e}\left(t_i^{\mathcal{P}_u}\right)\right\| \mathrm{e}^{-k\left(t-t_i^{\mathcal{P}_u}\right)},\ t \in [t_i^{\mathcal{P}_u}, t_{i+1}^{\mathcal{P}_a}), \tag{23}$$

$$\|\tilde{e}(t)\| \leq \begin{cases} \left\|\tilde{e}\left(t_i^{\mathcal{P}_a}\right)\right\| \mathrm{e}^{-\lambda_a\left(t-t_i^{\mathcal{P}_a}\right)}, & t \in [t_i^{\mathcal{P}_a}, t_i^{\mathcal{P}_u}), \\ \left[C_{\tilde{e}} \mathrm{e}^{2\lambda_u\left(t-t_i^{\mathcal{P}_u}\right)} - \dfrac{1}{2\lambda_u}\overline{d}^2\right]^{1/2}, & t \in [t_i^{\mathcal{P}_u}, t_{i+1}^{\mathcal{P}_a}), \end{cases} \tag{24}$$

respectively, where $C_{\tilde{e}} : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$ is defined as

$$C_{\tilde{e}}\left(t_i^{\mathcal{P}_u}\right) \triangleq \left\|\tilde{e}\left(t_i^{\mathcal{P}_u}\right)\right\|^2 + \frac{\overline{d}^2}{2\lambda_u}. \tag{25}$$

Using (5) and (6), it follows from (22)-(24) that

$$\|\hat{e}(t)\| \leq C_{\hat{e}}\left(t_i^{\mathcal{P}_a}\right) \mathrm{e}^{-\min\{k,\lambda_a\}\left(t-t_i^{\mathcal{P}_a}\right)}, \quad t \in [t_i^{\mathcal{P}_a}, t_i^{\mathcal{P}_u}), \tag{26}$$

$$\|e(t)\| \leq \left[C_{\tilde{e}}\left(t_i^{\mathcal{P}_u}\right) \mathrm{e}^{2\lambda_u\left(t-t_i^{\mathcal{P}_u}\right)} - \frac{\overline{d}^2}{2\lambda_u}\right]^{\frac{1}{2}} + \left\|\hat{e}\left(t_i^{\mathcal{P}_u}\right)\right\|, \quad t \in [t_i^{\mathcal{P}_u}, t_{i+1}^{\mathcal{P}_a}), \tag{27}$$

where $C_{\hat{e}} : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$ is defined as

$$C_{\hat{e}}\left(t_i^{\mathcal{P}_a}\right) \triangleq \left\|e\left(t_i^{\mathcal{P}_a}\right)\right\| + \left\|\tilde{e}\left(t_i^{\mathcal{P}_a}\right)\right\|. \tag{28}$$

When the agent is in the feedback region, the inequality in (26) provides a bound on the convergence rate of the normalized error between the state estimate and the auxiliary trajectory. When the agent is outside of the feedback region, the inequality in (27) provides a bound on the growth rate of the normalized error between the true state and auxiliary trajectory. Using (26) and (27), dwell-time conditions are developed in the following theorem to ensure stability of the switched system, despite the potentially unstable subsystem.

Theorem 1. The composite error system trajectories of the switched system generated by the family of subsystems described by (11)-(13), with a piece-wise constant, right continuous switching signal $t \mapsto p(t) \in \mathcal{P}$ are globally uniformly ultimately bounded, provided Assumptions 1-4 hold and the switching signal satisfies the minimum feedback availability dwell-time condition

$$\Delta t_i^{\mathcal{P}_a} \geq -\frac{1}{\min\{k, \lambda_a\}} \ln\left(\min\left\{\frac{\hat{e}_T}{C_{\hat{e}}\left(t_i^{\mathcal{P}_a}\right)}, 1\right\}\right), \tag{29}$$

and the maximum loss of feedback dwell-time condition

$$\Delta t_i^{\mathcal{P}_u} \leq \frac{1}{2\lambda_u} \ln\left(\frac{\left(\Gamma\left(t_{i+1}^{\mathcal{P}_a}\right)^2 - \left\|\hat{e}\left(t_i^{\mathcal{P}_u}\right)\right\|\right)^2 + \frac{\overline{d}^2}{2\lambda_u}}{C_{\tilde{e}}\left(t_i^{\mathcal{P}_u}\right)}\right), \tag{30}$$

where λ_a, λ_u, $C_{\tilde{e}}\left(t_i^{\mathcal{P}_u}\right)$, and $C_{\hat{e}}\left(t_i^{\mathcal{P}_u}\right)$ are previously defined in (20), (21), (23), and (24), respectively, $\hat{e}_T \in (0, e_M)$ is a user-defined threshold, e_M is a user-defined threshold defined in (10), and $\Gamma\left(t_{i+1}^{\mathcal{P}_a}\right)$ is generated by the ESC scheme.

Proof. When the agent is inside the feedback region (i.e., $t \in [t_i^{\mathcal{P}_a}, t_i^{\mathcal{P}_u})$), the objective is to ensure the norm of the error in (26) is less than a user-defined threshold before exiting the feedback region, i.e., $\left\|\hat{e}\left(t_i^{\mathcal{P}_u}\right)\right\| \leq \hat{e}_T$. To determine the minimum dwell-time condition, (26) is used to develop the following constraint

$$\left\|\hat{e}\left(t_i^{\mathcal{P}_u}\right)\right\| \leq C_{\hat{e}}\left(t_i^{\mathcal{P}_a}\right) \mathrm{e}^{-\min\{k,\lambda_a\}\Delta t_i^{\mathcal{P}_a}} \leq \hat{e}_T. \tag{31}$$

Solving (31) for $\Delta t_i^{\mathcal{P}_a}$ yields the minimum dwell-time condition in (29).

When the agent is outside the feedback region (i.e., $t \in [t_i^{\mathcal{P}_u}, t_{i+1}^{\mathcal{P}_a})$), the objective is to ensure the norm of the error in (27) is upper bounded by a user-defined threshold upon entering the feedback region, i.e., $\left\|e\left(t_{i+1}^{\mathcal{P}_a}\right)\right\| \leq e_M$. To incorporate the ESC scheme into the maximum dwell-time condition, (27) is used to develop the following constraint

$$\left\|e\left(t_{i+1}^{\mathcal{P}_a}\right)\right\| \leq \Gamma\left(t_{i+1}^{\mathcal{P}_a}\right)^2, \tag{32}$$

where $\Gamma\left(t_{i+1}^{\mathcal{P}_a}\right)$ is generated by the ESC scheme. Substituting (27) into the constraint in (32), the maximum dwell-time condition in (30) is the solution $\Delta t_i^{\mathcal{P}_u}$ to $\left\|\hat{e}\left(t_i^{\mathcal{P}_u}\right)\right\| + \left[C_{\tilde{e}}\left(t_i^{\mathcal{P}_u}\right) \mathrm{e}^{2\lambda_u \Delta t_i^{\mathcal{P}_u}} - \frac{\overline{d}^2}{\lambda_u}\right]^{1/2} \leq \Gamma\left(t_{i+1}^{\mathcal{P}_a}\right)^2$. Each instant the agent re-enters the feedback region, (10) is computed and used to update the ESC input and generate a new $\Gamma\left(t_{i+1}^{\mathcal{P}_a}\right)$. By Assumption 4, the ESC input $\Gamma\left(t_{i+1}^{\mathcal{P}_a}\right)$ converges to a neighborhood of the unknown setpoint Γ^*. Since Γ^* minimizes the output in (10), as $\Gamma\left(t_{i+1}^{\mathcal{P}_a}\right)$ approaches the neighborhood of Γ^*, the output approaches the neighborhood of the extremum, i.e., $\left\|e\left(t_{i+1}^{\mathcal{P}_a}\right)\right\| \to N_\rho(e_M - \phi)$ and $N_\rho(\cdot)$ denotes a neighborhood of size $\rho > 0$. To ensure $\left\|e\left(t_{i+1}^{\mathcal{P}_a}\right)\right\| \leq e_M$ is satisfied, the offset parameter in (10) is selected as $\phi > \rho$, where it is assumed $\rho \in (0, e_M)$ is known. ■

Remark 1. The minimum dwell-time condition can be eliminated using reset maps (cf., Chen et al. (2018) see Section IV A).

Remark 2. In Chen et al. (2018), when the agent is outside of the feedback region, (27) is bounded by a constant and results in the following constraint $\left\|\hat{e}\left(t_i^{\mathcal{P}_u}\right)\right\| + \left[C_{\tilde{e}}\left(t_i^{\mathcal{P}_u}\right) \mathrm{e}^{2\lambda_u \Delta t_i^{\mathcal{P}_u}} - \frac{\overline{d}^2}{\lambda_u}\right]^{1/2} \leq e_M$, and compared to (32), yields the more conservative maximum dwell-time condition $\Delta t_i^{\mathcal{P}_u} = \frac{1}{2\lambda_u} \ln\left(\frac{\left(e_M - \left\|\hat{e}\left(t_i^{\mathcal{P}_u}\right)\right\|\right)^2 + \frac{\overline{d}^2}{2\lambda_u}}{C_{\tilde{e}}\left(t_i^{\mathcal{P}_u}\right)}\right)$.

6. SIMULATION

A simulation is performed to illustrate the performance of the controller and the adaptability of the dwell-time condition. Based on (1), the dynamics are selected as $f(x,t) = Ax$ where $A = 0.5I_3$, I_3 represents a 3×3 identity matrix, and the disturbance $d(t)$ is drawn from a uniform distribution between $[0,\ 0.06]$ meters per second. The initial condition for the states and estimates are selected as $x(0) = \left[0.1\text{m},\ 0.0\text{m},\ \frac{\pi}{2}\ \text{rad}\right]$ and $\hat{x}(0) = \left[0.15\text{m},\ 0.05\text{m},\ \frac{\pi}{2}\ \text{rad}\right]$. The desired maximum and lower threshold for the error signals are selected as $e_M = 0.9$ and $\hat{e}_T = 0.01$ meters, respectively. The control input and auxiliary input gains are selected as $k = 1I_3$ and $k_{\tilde{e}} = 5I_3$ respectively. The offset parameter and gain parameter for the output function is selected as $\phi = 0.25$ meters and $k_y = 1000$, respectively. Based on Assumption 4, a discretized perturbation-based ESC scheme from Krstic and Wang (2000) [3], is used as

$$\begin{aligned}
\Gamma\left(t_{i+1}^{\mathcal{P}_a}\right) &\triangleq \hat{\Gamma}\left(t_{i+1}^{\mathcal{P}_a}\right) + a\sin(wi),\\
\hat{\Gamma}\left(t_{i+1}^{\mathcal{P}_a}\right) &\triangleq \hat{\Gamma}\left(t_{i}^{\mathcal{P}_a}\right) + \Delta\hat{\Gamma}\left(t_{i+1}^{\mathcal{P}_a}\right),\\
\Delta\hat{\Gamma}\left(t_{i+1}^{\mathcal{P}_a}\right) &\triangleq k\xi\left(t_{i+1}^{\mathcal{P}_a}\right),\\
\xi\left(t_{i+1}^{\mathcal{P}_a}\right) &\triangleq \xi\left(t_{i}^{\mathcal{P}_a}\right) + \Delta\xi\left(t_{i+1}^{\mathcal{P}_a}\right),\\
\Delta\xi\left(t_{i+1}^{\mathcal{P}_a}\right) &\triangleq \omega_l\left(y\left(t_{i+1}^{\mathcal{P}_a}\right) - \eta\left(t_{i+1}^{\mathcal{P}_a}\right)\right)a\sin(wi)\\
&\quad -\omega_l\xi\left(t_{i+1}^{\mathcal{P}_a}\right),\\
\eta\left(t_{i+1}^{\mathcal{P}_a}\right) &\triangleq \eta\left(t_{i}^{\mathcal{P}_a}\right) + \Delta\eta\left(t_{i+1}^{\mathcal{P}_a}\right),\\
\Delta\eta\left(t_{i+1}^{\mathcal{P}_a}\right) &\triangleq -\omega_h\eta\left(t_{i+1}^{\mathcal{P}_a}\right) + \omega_h y\left(t_{i+1}^{\mathcal{P}_a}\right),
\end{aligned}$$

where $a,\ w,\ k,\ w_l,\ w_h \in \mathbb{R}_{>0}$ are adjustable parameters, $\hat{\Gamma},\ \xi,\ \eta\ :\ \mathbb{R}_{\geq 0} \to \mathbb{R}$ are auxiliary signals, $y\left(t_{i+1}^{\mathcal{P}_a}\right)$ is the intermittent measurements given in (10). The ESC parameters are $k = 0.005$, $a = 0.01$, $w = 0.02$, $w_h = 0.0012$, and $w_l = 0.002$.

The desired path x_d is selected as a circular path with a radius of 2 meters centered at the origin. The boundary of the feedback region is selected as a circle with a 1 meter radius about the origin. Following the framework of Chen et al. (2019), a smootherstep function, as defined in Ebert (2003) is used to design the auxiliary trajectory $x_\sigma(t)$.

Fig. 1 illustrates the agent's planar trajectory. When the agent is inside the region with state feedback, the estimation error exponentially converges. When the agent is outside the feedback region, the error exhibits divergence.

In Fig. 2, the error $\|e(t)\|$ is shown to converge to the extremum point of 0.7 meters while simultaneously remaining bounded by the maximal error of 0.9 meters for all time. In Fig. 3, the maximum dwell-time condition is illustrated. Specifically, the impact of using ESC to adjust the dwell-time condition is the maximum time to dead-reckon increases by a factor of 2.8 from 5.34 seconds to 14.93 seconds.

[3] Other ESC methods could also be explored to potentially yield different performance.

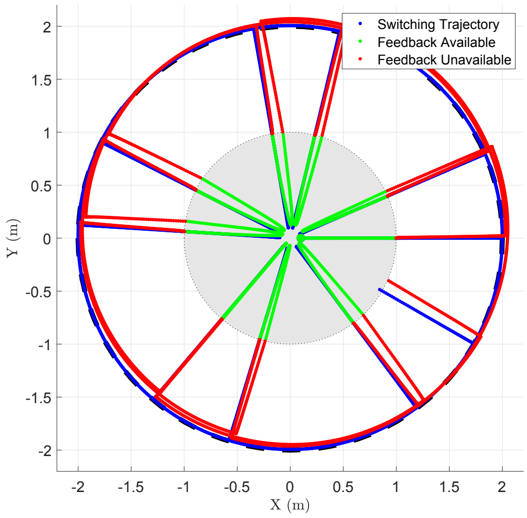

Fig. 1. Simulation result for 60 seconds. Both the system state $x(t)$ and auxiliary trajectory $x_\sigma(t)$ are initialized in the feedback region (gray shaded region). When $x(t) \in \mathcal{F}$, the state estimate $\hat{x}(t)$ exponentially converges to $x_\sigma(t)$. When $x(t) \notin \mathcal{F}$, the state $x(t)$ gradually diverges from $x_\sigma(t)$ due instabilities. Before the maximum dwell-time is reached, $x(t)$ re-enters the feedback region to prevent the error $\|e(t)\|$ from exceeding a user-defined threshold.

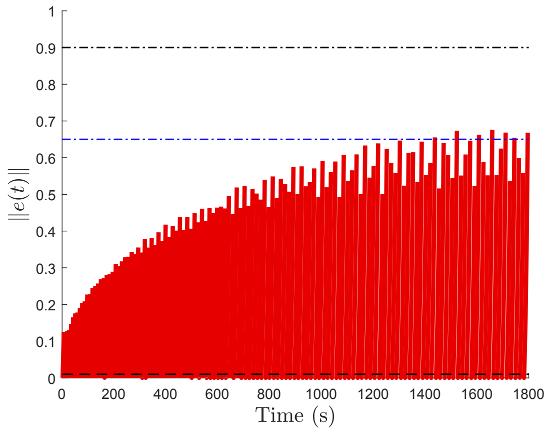

Fig. 2. Evolution of error $\|e(t)\|$ when $x(t) \notin \mathcal{F}$. The black (top) dashed line represents the maximum allowable error e_M and the blue (bottom) dashed line represents the extremum point of the output function that the ESC minimizes. The error increases to the extremum point while remaining bounded by e_M for all time.

7. CONCLUSION

A Lyapunov-based, switched system approach is used to develop a switching control design for path following in systems subject to intermittencies in state information. Maximum and minimum dwell-times are developed to provide sufficient stability conditions for the switched system. ESC is used to develop dwell-time conditions that reduce the conservativeness as more information is learned from intermittent measurements of the state. The dwell-

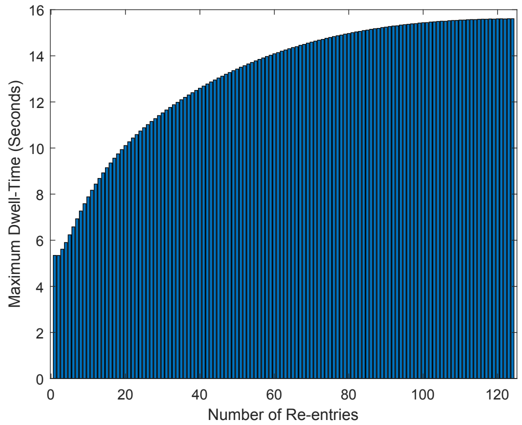

Fig. 3. Evolution of the dwell-time condition as the agent receives intermittent output measurements upon re-entry of the feedback available region.

time conditions allow the desired path to be completely outside of the feedback region, and an auxiliary trajectory is designed guarantee the agent re-enters the feedback region before the error growth exceeds a defined bound. The simulation indicates a factor of 2.8 increase in the maximum dwell-time as a result of the ESC method.

ACKNOWLEDGEMENTS

This research is supported in part by AFOSR award numbers FA9550-19-1-0169, FA9550-18-1-0109, and FA9550-18-1-0246, and NEEC award number N00174-18-1-0003. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect views of sponsoring agencies.

REFERENCES

Ariyur, K.B. and Krstic, M. (2003). *Real-Time Optimization by Extremum-Seeking Control*. Wiley.

Branicky, M. (1998). Multiple Lyapunov functions and other analysis tools for switched and hybrid systems. *IEEE Trans. Autom. Control*, 43, 475–482.

Chen, H.Y., Bell, Z., Deptula, P., and Dixon, W.E. (2019). A switched systems approach to path following with intermittent state feedback. *IEEE Trans. Robot.*, 35(3), 725–733.

Chen, H.Y., Bell, Z.I., Deptula, P., and Dixon, W.E. (2018). A switched systems framework for path following with intermittent state feedback. *IEEE Control Syst. Lett.*, 2(4), 749–754.

Choi, J., Krstic, M., Ariyur, K.B., and Lee, J.S. (2002). Extremum seeking control for discrete-time systems. *IEEE Transactions on Automatic Control*, 47, 318–323.

Duenas, V., Cousin, C.A., Rouse, C., and Dixon, W.E. (2018). Extremum seeking control for power tracking via functional electrical stimulation. In *Proc. IFAC Conf. Cyber. Phys. Hum. Syst.*, 164–169.

Ebert, D.S. (2003). *Texturing & modeling: a procedural approach*. Morgan Kaufmann.

Goebel, R.G., Sanfelice, R., and Teel, A.R. (2012). *Hybrid Dynamical Systems*. Princeton University Press.

Guay, M. (2014). A time-varying extremum-seeking control approach for discrete-time systems. *Journal of Process Control*, 24, 98 – 112.

Jia, B. and Liu, S. (2015). Switched visual servo control of nonholonomic mobile robots with field-of-view constraints based on homography. *Control Theory Technol.*, 13(4), 311–320.

Khalil, H.K. (2002). *Nonlinear Systems*. Prentice Hall, Upper Saddle River, NJ, 3 edition.

Killingsworth, N.J. and Krstic, M. (2006). PID tuning using extremum seeking: online, model-free performance optimization. *IEEE Control Syst. Mag.*, 26(1), 70–79.

Krstic, M. and Wang, H.H. (2000). Stability of extremum seeking feedback for general nonlinear dynamic systems. *Automatica*, 36(4), 595–601.

Liberzon, D. (2003). *Switching in Systems and Control*. Birkhauser.

Mehta, S.S., Hu, G., Dani, A.P., and Dixon, W.E. (2008). Multi-reference visual servo control of an unmanned ground vehicle. In *Proc. AIAA Guid. Navig. Control Conf.* Honolulu, Hawaii.

Parikh, A., Cheng, T.H., Chen, H.Y., and Dixon, W.E. (2017). A switched systems framework for guaranteed convergence of image-based observers with intermittent measurements. *IEEE Trans. Robot.*, 33(2), 266–280.

Poveda, J.I. and Teel, A.R. (2017). A framework for a class of hybrid extremum seeking controllers with dynamic inclusions. *Automatica*, 76, 113–126.

Zegers, F., Chen, H.Y., Deptula, P., and Dixon, W.E. (2019). A switched systems approach to consensus of a distributed multi-agent system with intermittent communication. In *Proc. Am. Control Conf.*, 2372–2377.

Zhai, G., Hu, B., Yasuda, K., and Michel, A.N. (2001). Stability analysis of switched systems with stable and unstable subsystems: An average dwell time approach. *Int. J. Syst. Sci.*, 32(8), 1055–1061.