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Distributed event-triggered consensus of multi-agent systems with input delay ⋆

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Abstract: This paper investigates distributed event-triggered consensus control for multi-agent systems with input delay. To deal with input delay, the original system is converted to a delay-free system via Artstein-Kwon-Pearson reduction transformation method. Distributed event-triggered protocols are designed to alleviate the communication burden of the agents. The system convergence is validated by using Lyapunov stability analysis and solving linear matrix inequality function. Furthermore, it is proved that the system does not display Zeno behavior under the proposed event-triggering function, and thus, consistent triggering is excluded from the system. A simulation example is given to demonstrate the effectiveness of the control algorithm.

Keywords: Event triggered control, Consensus control, Multi-agent systems, Input delay.

1. INTRODUCTION

In the past decade, cooperative control for multi-agent systems is broadly investigated in various areas, including unmanned aerial vehicles, mobile robots and spacecraft formation (Wang et al., 2019b,a; Nazari et al., 2016). In addition to its high autonomy, adaptivity and robustness, it outweighs the traditional single unit in application due to low cost and implementation simplicity. The objective of consensus control is to synchronize desired agent states by exchanging neighboring information in a communication network. Within rather large multi-agent systems, communication required by the traditional network poses great burden to the agents by expending excessive resources (Hu et al., 2016). Although frequent communication among the agents does guarantee precise and fast convergence of tracking errors, the bandwidth of communication network is constrained and the transmission process is time consuming (Cheng and Li, 2019). Therefore it is reasonable to adopt a novel communication method to alleviate the burden from the network.

Event-triggering approach provides an alternative communication method by designing a threshold for controller update. The agents can only receive the latest state of its neighbours when the function of tracking errors is greater than the threshold (Heemels et al., 2012). With proper design of the triggering function, continuous communication is avoided and convergence can be achieved in a short period of time. In previous works, event-triggered control has been used for high-order systems (Zhang et al., 2019) and general linear systems (Dimarogonas et al., 2012; Zhang et al., 2014; Garcia et al., 2014; Xu et al., 2015; Zhang et al., 2016), as well as systems with nonlinearity (Liu and Jia, 2018) and time delay (Liu et al., 2016; Mu et al., 2015).

Time delay mainly occurs during the communication process among the agents and is an ignorable factor in the control system. To deal with communication delay, the previous works proposed multiple Lyapunov functions to obtain a rather complex linear matrix inequality, eventually the system can achieve consensus by using robust control method. A different approach is Artstein model reduction method (Kwon and Pearson, 2015; Artstein, 1982), which is used to convert the original system into a delay free one. The principle of this method is to introduce a state predictor with an integral operator. By differentiating the equation, a delay-free system is obtained. In this way, the event-triggered controller design and stability proof can be greatly simplified. This method is used in (Wang et al., 2016) for multi-agent systems subject to Lipschitz nonlinearity and it is validated that the converted multi-agent systems can reach consensus with proper design of the control gain.

Motivated by the discussions above, this paper studies the distributed event-triggered control problem of multi-agent systems with input delay. First, Artstein model reduction method is used to convert the original system into a delay free system. Then event-triggered controller is designed to alleviate the communication burden among the agents and is proved to achieve consensus through Lyapunov stability

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analysis. It is further validated that the agents do not exhibit Zeno behavior under the proposed event-triggering sequence.

2. PRELIMINARIES

2.1 Graph Theory

The communication topology among agents is described by a graph $\mathcal{G}_n = \{\mathcal{N}, \mathcal{E}\}$, where $\mathcal{N} = \{n_1, \ldots, n_n\}$ is a finite nonempty set of nodes and $\mathcal{E} \subseteq N \times N$ is a set of edges. A node represents an agent, and each edge represents a connection between the agents. An edge $(i, j) \in \mathcal{E}$ in a directed graph means that agent n_j can receive information from agent n_i, but not vice versa. In an undirected graph, edge $(i, j) \in \mathcal{E}$ means the agents can exchange information with each other. The set of neighbouring agents of node i is denoted by $\mathcal{N}_i = \{n_j : n_j \in \mathcal{N} | (n_i, n_j) \in \mathcal{E}\}$.

Define the adjacency matrix of $\mathcal{G}$ as $\mathcal{A} \in R^{N\times N}$, its element $a_{i,j}$ denotes the connection between two agents as: if a connection exists from agents n_j to n_i, $a_{i,j} = 1$; otherwise $a_{i,j} = 0$. The degree matrix $\mathcal{D}$ of $\mathcal{G}$ is defined as $\mathcal{D} = diag\{d_{1,1}, d_{2,2}, \ldots, d_{n,n}\}$, where $d_{i,i}$ is the degree of node n_i defined as $d_{i,i} = \sum_{j=1}^{n} a_{i,j}$. The Laplacian matrix $\mathcal{L} = [l_{i,j}]$ of $\mathcal{A}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, that is, $l_{i,i} = \sum_{j=1}^{n} a_{i,j}$ and $l_{i,j} = -a_{i,j}$ when $i \neq j$.

For multi-agent systems with a leader, the connection between followers and the leader is directed. There are only edges from the leader to followers. The connection matrix is defined as $\mathcal{F} = diag\{F_1, F_2, \ldots, F_n\} \in R^{n\times n}$, where $F_i > 0$ means the ith agent can receive information from the virtual leader, otherwise $F_i = 0$.

2.2 Some lemmas

Lemma 1. (Hong et al., 2016) The communication graph $\mathcal{G}$ for the agents is connected, $\mathcal{L}$ represents the Laplace matrix and $\mathcal{F}$ is the connection matrix. There exists at least one follower that has access to the leader, and thus at least one element $F_i > 0$ and matrix $\mathcal{L} + \mathcal{F}$ is strictly positive definite.

Lemma 2. (Ioannou and Sun, 1996) Given that $x(t)$ and $\dot{x}(t)$ are bounded, if function $\int_0^{+\infty} x^T(s)x(s)ds < +\infty$ is satisfied, then $x(t) \to 0$ as $t \to +\infty$.

Lemma 3. For any given $a, b \in R^n$, the following inequality holds

$$2a^T SQb \leq a^T SPSa + b^T Q^T P^{-1} Qb \tag{1}$$

where S, Q, P are matrices with $P > 0$, S and Q have appropriate dimensions.

3. PROBLEM STATEMENT

Consider a group of $N + 1$ agents consisting of a leader indexed by 0 and N followers, system dynamics are shown as follows.

$$\begin{cases} \dot{x}_0(t) = Ax_0(t) \\ \dot{x}_i(t) = Ax_i(t) + Bu_i(t - \tau) \end{cases}, \tag{2}$$

where for agent $i, (i = 1, 2, \cdots, N), x_i \in R^n$ is the state, $u_i \in R^q$ is the control input, and $\tau \geq 0$ is the input time delay. $A \in R^{n\times n}$ and $B \in R^{n\times q}$ are constant matrices. For the leader agent, its control input is zero $u_0 \equiv 0$.

Assumption 4. System matrix pair (A, B) is stabilizable.

To deal with the input delay, state $z_i(t)$ is used to predict the state of $x_i(t)$ at time $t + \tau$.

$$z(t) = x(t) + \int_t^{t+\tau} e^{A(t-s)} Bu(s - \tau)ds \tag{3}$$

Differentiate $z(t)$ versus time and we can get

$$\begin{aligned} \dot{z}_0(t) &= Az_0(t) \\ \dot{z}_i(t) &= Az_i(t) + e^{-A\tau} Bu_i(t) \\ &= Az_i(t) + Du_i(t) \end{aligned} \tag{4}$$

where $D = e^{-A\tau}B$. In this way, the original system is transformed into a delay-free system.

Remark 5. Given the condition that the original system pair (A, B) is controllable, it can be proved that the new matrices (A, D) is also stabilizable.

$$\begin{aligned} &Rank([D, AD, \cdots, A^{n-1}D]) \\ &= Rank(e^{-A\tau}[B, AB, \cdots, A^{n-1}B]) \\ &= Rank([B, AB, \cdots, A^{n-1}B]) \end{aligned} \tag{5}$$

where $Rank(e^{-A\tau}) = n$ is used.

The control objective is for all the agents to achieve consensus under the proposed event-triggered protocol, that is $\lim_{t\to\infty} x_i(t) - x_j(t) = 0$, in terms of the converted system, $\lim_{t\to\infty} z_i(t) - z_j(t) = 0$.

4. EVENT-TRIGGERED CONTROLLER

4.1 Controller design

In this section, an event-triggered controller is designed for the agents to achieve consensus, meanwhile the communication frequency is reduced.

The tracking error is shown below.

$$\eta_i(t) = z_i(t) - z_0(t) \tag{6}$$

Differentiate $\eta_i(t)$ against time and we can obtain

$$\dot{\eta}_i(t) = A\eta_i(t) + Du_i(t) \tag{7}$$

Event-triggering error is designed as

$$\varepsilon_i(t) = z_i(t_k^i) - z_i(t) \tag{8}$$

where t_k^i is the kth event-triggered instant for agent i, and $z_i(t_k^i)$ is the system state at time instant t_k^i.

The distributed event-triggered control input is designed below.

$$u_i(t) = -K \sum_{j=1}^{n} a_{ij}(z_i(t_k^i) - z_j(t_{k'}^j)) + F_i(z_i(t_k^i) - z_0(t)) \tag{9}$$

where $K \in R^{q\times n}$ is the control matrix gain to be designed later, and $t_{k'}^j$ denotes a triggering time instant for agent j different with t_k^i.

With the tracking error $\eta(t)$ and event-triggering error $\varepsilon(t)$ defined above, the controller function can be rewritten as

$$u(t) = -(H \otimes K)(\eta(t) + \varepsilon(t)) \tag{10}$$

where $H = \mathcal{L} + \mathcal{F}$.

Since the agents i and j has their own event-triggering time t_k^i and $t_{k'}^j$, it is a distributed control protocol, enabling each agent to trigger individually. Thus the accuracy of leader tracking is enhanced and excess communication among the agents is reduced.

By substituting the control input, the tracking error dynamics is written as

$$\dot{\eta}(t) = (I_n \otimes A)\eta(t) - (H \otimes DK)(\eta(t) + \varepsilon(t)) \tag{11}$$

The control gain is chosen as

$$K = D^T P \tag{12}$$

where P is a positive definite matrix.

Initially, the event-triggering sequence for agent i is designed as

$$f_i(t) = \|\varepsilon_i(t)\| - \beta_1 \|\eta_i(t)\| \tag{13}$$

where β_1 satisfies the following inequality

$$\beta_1 < \frac{\lambda_{min}(H)\lambda_{min}(DD^T)}{\lambda_{max}(H)\lambda_{max}(DD^T)} \tag{14}$$

Theorem 6. Consider the converted multi-agent systems with the designed control algorithm and event-triggered sequence (13), the event-triggered tracking control objective can be achieved if there exists $W > 0$ satisfying the following Riccati inequality function

$$A^T W + WA - \alpha_1 < 0 \tag{15}$$

where $W = P^{-1}$ and

$$\alpha_1 = \lambda_{min}(H)\lambda_{min}(DD^T) - \beta_1 \lambda_{max}(H)\lambda_{max}(DD^T)$$

Proof

Consider the following Lyapunov function

$$V(t) = \eta^T(t)(I_n \otimes P)\eta(t) \tag{16}$$

where $P > 0$.

The time derivation of $V(t)$ is as follows

$$\begin{aligned}\dot{V}(t) = &\eta^T(t)(I_n \otimes (A^T P + PA))\eta(t)\\ &- 2\eta^T(t)(H \otimes PDK)(\eta(t) + \varepsilon(t))\\ = &\eta^T(t)(I_n \otimes (A^T P + PA))\eta(t)\\ &- 2\eta^T(t)(H \otimes PDK)\eta(t) - 2\eta^T(t)(H \otimes PDK)\varepsilon(t)\end{aligned}$$

With lemma 3, the following inequality holds

$$\begin{aligned}-2\eta^T(t)(H \otimes PDK)\varepsilon(t) \leq &\eta^T(t)(H \otimes PDD^T P)\eta(t)\\ &+ \varepsilon^T(t)(H \otimes PDD^T P)\varepsilon(t)\end{aligned}$$

Therefore we can get

$$\begin{aligned}\dot{V}(t) \leq &\eta^T(t)(I_n \otimes (A^T P + PA))\eta(t)\\ &- \eta^T(t)(H \otimes PDD^T P)\eta(t) + \varepsilon^T(t)(H \otimes PDD^T P)\varepsilon(t)\\ = &\eta^T(t)(I_n \otimes (A^T P + PA) - H \otimes PDD^T P)\eta(t)\\ &+ \varepsilon^T(t)(H \otimes PDD^T P)\varepsilon(t)\end{aligned}$$

With the time-triggering sequence, it is further obtained that

$$\begin{aligned}\dot{V}(t) \leq &\eta^T(t)(I_n \otimes (A^T P + PA - H \otimes PDD^T P))\eta(t)\\ &+ \beta_1 \eta^T(t)(H \otimes PDD^T P)\eta(t)\\ \leq &\eta^T(t)(I_n \otimes (A^T P + PA))\eta(t)\\ &- \lambda_{min}(H)\eta^T(t)(I_n \otimes PDD^T P)\eta(t)\\ &+ \beta_1 \lambda_{max}(H)\eta^T(t)(I_n \otimes PDD^T P)\eta(t)\end{aligned}$$

We introduce a state transformation $W = P^{-1}$, W is also a positive definite matrix.

By multiplying P^{-1} on both sides of the function, it can be obtained that

$$\begin{aligned}\dot{V}(t) \leq &\eta^T(t)(I_n \otimes (A^T W + WA)))\eta(t)\\ &- \lambda_{min}(H)\eta^T(t)(I_n \otimes DD^T)\eta(t)\\ &+ \beta_1 \lambda_{max}(H)\eta^T(t)(I_n \otimes DD^T)\eta(t)\\ \leq &\eta^T(t)(I_n \otimes (A^T W + WA)))\eta(t)\\ &- \lambda_{min}(H)\lambda_{min}(DD^T)\eta^T(t)\eta(t)\\ &+ \beta_1 \lambda_{max}(H)\lambda_{max}(DD^T)\eta^T(t)\eta(t)\\ = &\eta^T(t)(I_n \otimes (A^T W + WA - \alpha_1)\eta(t)\end{aligned}$$

where α_1 satisfies

$$\alpha_1 = \lambda_{min}(H)\lambda_{min}(DD^T) - \beta_1 \lambda_{max}(H)\lambda_{max}(DD^T)$$

We have

$$\dot{V} \leq \eta^T(t)(I_n \otimes M)\eta(t) \tag{17}$$

where

$$M = A^T W + WA - \alpha_1 \tag{18}$$

Remark 7. Since all the followers demand the leader's state in order to decide the triggering time, the above mensioned event-triggering sequence is a centralized approach. Besides, when $\eta_i(t) = 0$ there is possibility that zeno behavior will occur in the system. Therefore it is not a proper triggering condition for the distributed control algorithm.

A distributed event-triggering sequence is given below.

$$f(t) = |\varepsilon(t)| - \beta_2(H \otimes I_n)|\eta(t)| - \mu e^{-\nu t} \tag{19}$$

where $y_i(t) = \sum_{i=1}^{N} a_{ij}(\eta_i(t) - \eta_j(t)) + F_i \eta_i(t)$, and β_2 satisfies the inequality below

$$\beta_2 < \frac{\lambda_{min}(H)\lambda_{min}(DD^T)}{\lambda_{max}(H^2)\lambda_{max}(DD^T)} \tag{20}$$

Therefore the event-triggering sequence can be written as

$$f(t) = \|\varepsilon(t)\| - \beta_2(H \otimes I_n)\|\eta(t)\| - \mu e^{-\nu t} \tag{21}$$

Under the novel event-triggering sequence, only the follower agents that has direct connection with the leader requires the leader's state, and thus is a distributed condition. Also, the extra term $\mu e^{-\nu t}$ can effectively prevent zeno behavior from happening.

Theorem 8. The distributed event-triggered tracking control objective can be achieved under the designed control algorithm and event-triggered condition (21) if there exists $W > 0$ satisfying the following Riccati inequality function

$$A^T W + WA - \alpha_2 < 0 \tag{22}$$

where

$$\alpha_2 = \lambda_{min}(H)\lambda_{min}(DD^T) - \beta_2 \lambda_{max}(H^2)\lambda_{max}(DD^T)$$

Proof

System stability is validated under the same Lyapunov function.

$$V(t) = \eta^T(t)(I_n \otimes P)\eta(t) \tag{23}$$

where $P > 0$.

Differentiate $V(t)$ against time, we have

$$\dot{V}(t) = \eta^T(t)(I_n \otimes (A^T P + PA))\eta(t) - 2\eta^T(t)(H \otimes PDK)\eta(t) - 2\eta^T(t)(H \otimes PDK)\varepsilon(t)$$

With lemma 3, we can get

$$\dot{V}(t) \leq \eta^T(t)(I_n \otimes (A^T P + PA) - H \otimes PDD^T P)\eta(t) + \varepsilon^T(t)(H \otimes PDD^T P)\varepsilon(t)$$

With the novel time-triggering sequence, it can be further obtained that

$$\begin{aligned}\dot{V}(t) \leq\ & \eta^T(t)(I_n \otimes (A^T P + PA))\eta(t) \\ & - \lambda_{min}(H)\eta^T(t)(I_n \otimes PDD^T P)\eta(t) \\ & + \beta_2\eta^T(t)(H^2 \otimes PDD^T P)\eta^T(t) + \mu e^{-\nu t} \\ \leq\ & \eta^T(t)(I_n \otimes (A^T P + PA))\eta(t) \\ & - \lambda_{min}(H)\eta^T(t)(I_n \otimes PDD^T P)\eta(t) \\ & + \beta_2\lambda_{max}(H^2)\eta^T(t)(I_n \otimes PDD^T P)\eta^T(t) + \mu e^{-\nu t} \\ \leq\ & \eta^T(t)(I_n \otimes (A^T P + PA))\eta(t) \\ & - (\lambda_{min}(H) - \beta_2\lambda_{max}(H^2))\eta^T(t)(I_n \otimes PDD^T P)\eta(t) \\ & + \mu e^{-\nu t}\end{aligned}$$

We introduce a state transformation $W = P^{-1}$, and thus W is also a positive definite matrix.

By multiplying P^{-1} on both sides of the function, it can be obtained that

$$\begin{aligned}\dot{V}(t) \leq\ & \eta^T(t)(I_n \otimes (A^T W + WA)))\eta(t) \\ & - (\lambda_{min}(H) - \beta_2\lambda_{max}(H^2))\eta^T(t)(I_n \otimes DD^T)\eta(t) \\ & + \mu e^{-\nu t} \\ \leq\ & \eta^T(t)(I_n \otimes (A^T W + WA)))\eta(t) \\ & - \lambda_{min}(H)\lambda_{min}(DD^T)\eta^T(t)\eta(t) \\ & + \beta_2\lambda_{max}(H^2)\lambda_{max}(DD^T)\eta^T(t)\eta(t) + \mu e^{-\nu t} \\ =\ & \eta^T(t)(I_n \otimes (A^T W + WA - \alpha_2)\eta(t) + \mu e^{-\nu t}\end{aligned}$$

where α_2 satisfies

$$\alpha_2 = \lambda_{min}(H)\lambda_{min}(DD^T) - \beta_2\lambda_{max}(H^2)\lambda_{max}(DD^T)$$

We have

$$\dot{V}(t) \leq \eta^T(t)(I_n \otimes M)\eta(t) + \mu e^{-\nu t} \tag{24}$$

where

$$M = A^T W + WA - \alpha_2 \tag{25}$$

Since solution exists for inequality $M \leq 0$, we define $Q = -M$, resulting in

$$\dot{V}(t) \leq -\lambda_{min}(Q)\eta^T(t)\eta(t) + \mu e^{-\nu t} \tag{26}$$

It can be inferred that

$$0 \leq V(t) \leq \mu \int_0^t e^{-\nu s} ds \tag{27}$$

Therefore $V(t)$ is bounded, which means η and $\dot{\eta}$ is bounded.

Besides, we can get

$$V(\infty) - V(0) \leq -\int_0^\infty \lambda_{min}(Q)\eta^T(t)\eta(t) + \frac{\mu}{\nu} \tag{28}$$

It can be rewritten as

$$\int_0^\infty \eta^T(t)\eta(t) \leq \frac{1}{\lambda_{min}(Q)}[V(\infty) - V(0) + \frac{\mu}{\nu}] \tag{29}$$

Through Lemma 2, it is obtained that $\eta(t) \to 0$ as $t \to \infty$, and therefore the system reaches consensus.

4.2 Zeno behavior

As the information exchange timing among the agents is decided through event-triggered function, and it further influnces the control input, it is vital that consistent triggering is excluded from the control system. In other words, Zeno behavior does not exist in the system. It is proved by computing the triggering interval, a positive time interval does not lead to Zeno behavior. In the following subsection, the proof is done under the novel event-triggering condition.

Theorem 9. With the designed distributed event-triggering sequence, the multi-agent systems does not exhibit Zeno behavior.

Proof

For any agent $i, i \in N$, its current triggering time is taken as t_k^i. In this section, we will prove that the following triggering time instant t_{k+1}^i is strictly larger than t_k^i, that is $t_{k+1}^i - t_k^i > 0$.

Firstly, by computing the right-hand Dini derivative of $\varepsilon_i(t)$ in $[t_k^i, t_{k+1}^i)$, it can be derived that

$$\begin{aligned}\frac{d}{dt}\|\varepsilon_i(t)\| &\leq \|\dot{\varepsilon}_i(t)\| = \| - \dot{Z}_i(t)\| \\ &= \| - (AZ_i(t) + Bu_i(t))\| \\ &= \| - A(Z_i(t_k^i) - \varepsilon_i(t)) + Bu_i(t)\| \\ &\leq \|A\varepsilon_i(t)\| + m_k^i\end{aligned}$$

where $m_k^i = max_{t\in[t_k^i, t_{k+1}^i)}\|AZ_i(t_k^i) + Bu_i(t) + \phi_i(t)\|$.

We can obtain that

$$\|\varepsilon_i(t)\| \leq \frac{m_k^i}{\|A\|}e^{\|A\|(t-t_k^i)-1} \tag{30}$$

A sufficient condition for $f_i(t) < 0$ is given below

$$\|\varepsilon_i(t)\| \leq \rho_i\|Z_i(t_k^i)\| \tag{31}$$

where $\rho_i = \sqrt{\frac{\beta_2}{2+2\beta_2^2}}$.

Let $\theta_k^i = \rho\|Z_i(t_k^i)\|$, we can get

$$\|\varepsilon_i(t_{k+1}^i)\| = \theta_k^i \leq \frac{m_k^i}{\|A\|}(e^{\|A\|(t_{k+1}^i - t_k^i)-1}) \tag{32}$$

Then we can obtain

$$\tau_k^i = t_{k+1}^i - t_k^i \leq \frac{1}{\|A\|}\ln(\|A\|\frac{\theta_k^i}{m_k^i} + 1) \tag{33}$$

To ensure the event-triggering time interval is strictly positive, two cases are considered below.

Case 1: $Z_i(t_k^i) \neq 0$

Given the condition $Z_i(t_k^i) \neq 0$, we have $\theta_k^i > 0$, and therefore $\tau_k^i > 0$.

Case 2: $Z_i(t_k^i) = 0$ as $t \to \infty$, it can be inferred that $Z_i(t) = 0$ and $\dot{Z}_i(t) = 0$

Because $\|\varepsilon_i(t)\| \leq \rho_i \|Z_i(t_k^i)\|$, we can obtain

$$\lim_{t\to\infty} \|Z_i(t)\| - \|Z_i(t_k^i)\| \leq \rho_i \|Z_i(t_k^i)\| \tag{34}$$

and

$$\lim_{t\to\infty} \frac{\|Z_i(t_k^i)\|}{\|Z_i(t)\|} \geq \frac{1}{\rho_i + 1} \tag{35}$$

The definition of m^ik can be rewritten as

$$\begin{aligned} m^ik &\leq \|A\|\|Z_i(t_k^i)\| + max_{t\in[t_k^i, t_{k+1}^i)} \|Bu_i(t) + \phi_i(t)\| \\ &= \|A\|\|Z_i(t_k^i)\| + \|AZ_i(t_*^i)\| \end{aligned}$$

where $t_*^i \in [t_k^i, t_{k+1}^i)$

With the definition of θ_k^i, we have

$$\begin{aligned} \lim_{t\to\infty} \frac{\theta_k^i}{m^ik} &\geq \lim_{t\to\infty} \frac{\rho_i \|Z_i(t_k^i)\|}{\|A\|\|Z_i(t_k^i)\| + \|AZ_i(t_*^i)\|} \\ &\geq \lim_{t\to\infty} \frac{\rho_i}{(1+\rho_i)\|A\|} \geq 0 \end{aligned} \tag{36}$$

Therefore

$$\begin{aligned} \lim_{t\to\infty} t_k^i &= \lim_{t\to\infty} (t_{k+1}^i - t_k^i) \\ &\geq \frac{1}{\|A\|} \ln(\|A\| \frac{\theta_k^i}{m_k^i} + 1) \\ &\geq \frac{1}{\|A\|} \ln(\frac{\rho_i}{2+\rho_i} + 1) > 0 \end{aligned} \tag{37}$$

From the two cases analysed above, we can come to the conclusion that event-triggering interval τ_k^i is strictly positive, as a result any agent i does not exhibit Zeno behavior, the proof is completed.

5. SIMULATION

In this section, an example is given to demonstrate the effectiveness of the proposed event-triggered controller.

The multi-agent systems is consisted of one leader indexed by 0 and four followers indexed from 1 to 4, the communication topology is shown in figure 1, it is specified by the following Laplacian matrix

$$\mathcal{L} = \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

and the leader accessibility matrix is $\mathcal{F} = diag\{1, 0, 0, 0\}$. Its eigenvalues $\lambda_{min}(H) = 0.1808$, $\lambda_{max}(H) = 2.3803$.

The dynamics of the agents are described with the following matrices

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} B = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

In simulation, the time delay constant is $\tau = 0.4s$ and simulation step is $0.01s$, and thus converted matrix D is

$$D = \begin{bmatrix} 1.1158 & 0.8499 \\ 0.0711 & 0.7264 \end{bmatrix}$$

The initial values of the agents are set as $x_0 = [-1, 1]^T$, $x_1 = [-11, 1]^T$, $x_2 = [9, -3]^T$, $x_3 = [-15, 5]^T$, $x_4 = [12, 7]^T$.

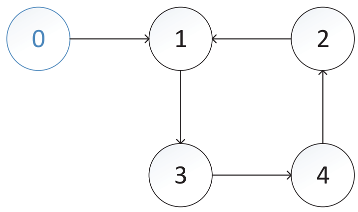

Fig. 1. Communication topology

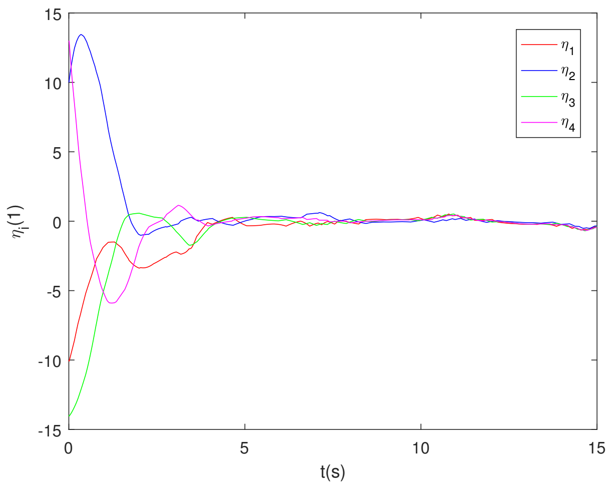

Fig. 2. Tracking error $\eta_i(1)$

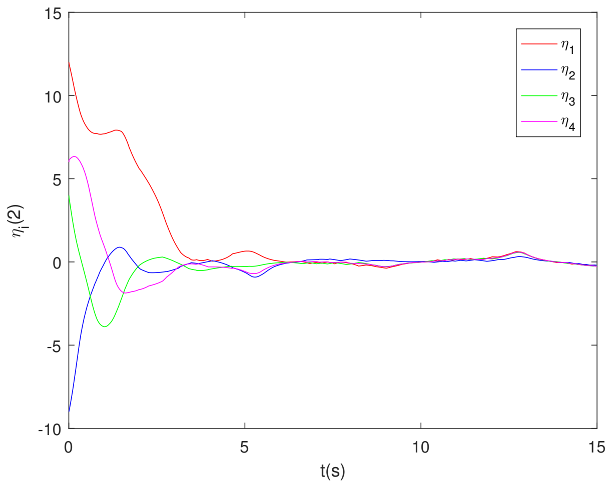

Fig. 3. Tracking error $\eta_i(2)$

By solving the Riccati inequality function, we can obtain

$$K = \begin{bmatrix} 0.5478 & 0.0349 \\ 0.4173 & 0.3566 \end{bmatrix}$$

and $\alpha_2 = 0.041$, $\beta_2 = 0.003$, $\mu = 2$, $\nu = 0.5$.

The tracking error of the event-triggered controller is shown in figures 2 and 3, it can be seen that despite the decreased communication times, the tracking errors can reach convergence after 10s. Although the system can only converge to small threshold, it is acceptable compared to the amount of resources saved.

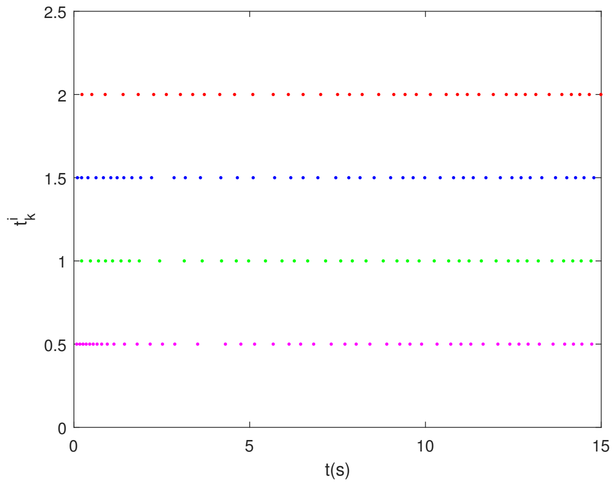

Fig. 4. Event-triggering time instant of the agents

The event-triggering time instants of the followers are shown in figure 4, it can be seen that the triggering time differ with each agent, by using the designed event-triggered methodology, information exchange frequency of the agents is lowered and communication burden is alleviated.

6. CONCLUSION

This paper investigates the event-triggered control of multi-agent systems with input delay. For systems with input delay, Artstein-Kwon-Pearson reduction transformation method is used for converting the original system into delay-free system. With the converted system, event-triggered controller is designed to alleviate communication burden among the agents and eventually achieve consensus with the leader. Besides, it is proved that the event-triggered control system does not exhibit Zeno behavior. Simulation results show that communication of the agents is reduced while achieving consensus within a short period of time.

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