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Consistent Measures of Dependence in the Identification of Multi-input/Multi-output Systems and Applications

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Abstract: An approach to constructing linearized input/output mappings of multi-input/multi-output stochastic systems is presented. The approach is based on involving an information-theoretic measure of dependence of random vectors as a criterion of selection of input variables of the model. An application of the mathematical technique developed to a task of advanced automated process control system functions regarding nuclear power plants efficiency monitoring is considered.

Key words: Automated process control system, Information task, Measures of dependence, Multi-input/multi-output description, Mutual information, Symmetric Tsallis divergence, System identification, Nuclear power plant, Technical and economical indexes.

1. INTRODUCTION

In comparison to linear systems, for which numerous identification techniques based both on gathered dada and current observations are known (Ljung, 1999, Ljung and Söderström, 1983, Söderström and Stoica, 1989), the set of theoretical methods of non-linear systems identification is considerably restricted. Meanwhile, different algorithms are available, whose applicability varies from non-linear system identification problems based on their most general representation in the form of functional series (Carini et al., 2019, Jing Xingjian and Lang Ziqiang, 2015, Hung and Stark, 1977, Lu Lu et al., 2016, Prawin and Rama Mohan Rao, 2017. Schmidt et al., 2014, Wiener, 1958) to problems admitting use of methods developed for block-oriented systems, when linear and non-linear constituent components are pre-specified (Bianchi et al., 2020, Giri and Bai Er-Wei, 2010, Linwei Li and Xuemei Ren, 2018, Mzyk, 2014, Piroddi et al., 2012, Vörös, 2014, Yinggan Tang et al., 2014, Zhenghao Ding et al., 2019).

The solution to the problem of identifying systems is always based on the application of certain measures of the dependence of random values, whether we are talking about representing the systems under study in the form of an input-output relation or in the state space. Most often, such a measure is the conventional linear covariance or correlation, the use of which directly arises from the identification problem statement on the basis of the mean square criterion. Their main advantage is the ease of use, including the ability to construct explicit analytical expressions for determining the required characteristics, and the relative ease of constructing their estimates, involving those on the basis of observation of the dependent data. However, the main drawback of measures of dependence based on linear correlation is, as known, the possibility of their reversal to zero even if there exists a deterministic dependence between the pair of random values under study, with corresponding examples being available, e.g. in (Rajbman, 1981, Rényi, 1959).

It is to precisely to overcome this drawback that the use of more complex, nonlinear measures of dependence, such as the dispersion function, which is an analogue of the well known correlation ratio, maximum correlation, and mutual information (that is, generically, a divergence measure of two probabilistic distributions, when one of which is a joint probability distribution of two random values, while the second one is the product of the marginal distributions of these random values), is aimed to solve identification problems. Moreover, the last two measures are *consistent* (in accordance to the terminology, introduced in the fullness of time by A.N. Kolmogorov, one can also refer to the paper of Sarmanov, and Zakharov (1960)) measures of dependence, i.e., vanishing if and only if when the random values in this pair are stochastically independent. This, in the first place, is the appeal of using maximum correlation and mutual information in identification problems, especially in the case of non-linear systems, since conventional linear measures of dependence possess the pointed out drawbacks.

More frequently, when using consistent measures of dependence in identification problems, the case of bivariate probabilistic distributions is involved. In the paper, we will consider the problem of identifying a multidimensional (multi-input/multi-output, MIMO) nonlinear dynamic stochastic system. Moreover, the identification procedure is based on the use of such a measure of multiple dependence of vector valued random values as mutual information based on a corresponding symmetric Tsallis divergence. As well, an application of the mathematical apparatus developed to a task of automated process control system (APCS) functions regarding nuclear power plants (NPP) efficiency monitoring is considered.

2. A DISCRETE-TIME MIMO SYSTEMS IDENTIFICATION: AN INFORMATION THEORETIC APPROACH

2.1 Symmetric Tsallis Divergence and Mutual Information

In accordance to the statement of Ljung (2010) that the system identification as a science and *art* of constructing mathematical models by use of sample observations is a component of a versatile process, selecting the identification criterion under a system identification problem statement should be considered as a constituent part, requiring both accounting its relevance to available data and practical implementation suitability. In the present Section, an approach to the linearized multi-input/multi-output mapping identification of stochastic systems is constructed in accordance to information-theoretic criteria that are obtained on the basis of a symmetric divergence measure, built in turn, by use of Tsallis (Tsallis, 2009) entropy. Meanwhile, a parameterized description of the model of the system under study is applied accompanied with a technique of selecting system input variables to be included in the system model. A feature of the proposed approach is its basing on *consistent* measures of dependence of random vectors.

A broad class of measures of dependence is constructed by use of corresponding measures of comparison of continuous probabilistic distributions, for instance, $f(\mathbf{z})$ and $g(\mathbf{z})$ of a k-dimensional random vector $\mathbf{Z}$, which are well known as divergence measures. Among these measures, Kullback-Leibler divergence

$$D_{KL}(g_1\|g_2) = -\int_{R^k} f(\mathbf{z})\ln\left(\frac{g(\mathbf{z})}{f(\mathbf{z})}\right)d\mathbf{z}$$

is, perhaps, the most widely known and applicable.

In turn, the divergence measures may be considered as a performance index within different theoretical and practical problems. In particular, Kullback-Leibler divergence leads to the corresponding expression for the mutual information $I\{\mathbf{Z}_1,\mathbf{Z}_2\}$ (relative differential entropy) of two random vectors $\mathbf{Z}_1$ and $\mathbf{Z}_2$ of the dimensions k_1 and k_2 correspondingly, when one of the probability distribution densities in $D_{KL}(f\|g)$, namely $f(\mathbf{z}) = f_{\mathbf{z}_1\mathbf{z}_2}(\mathbf{z}_1,\mathbf{z}_2)$, is the joint probability distribution density of these random vectors, while the second one, $g(\mathbf{z}) = g_{\mathbf{z}_1}(\mathbf{z}_1)g_{\mathbf{z}_2}(\mathbf{z}_2)$, is the product of the marginal probability distribution densities of the vectors $\mathbf{Z}_1$ and $\mathbf{Z}_2$.

Analogously, corresponding Kullback-Leibler divergence $D_{KL}\left(f_{\mathbf{z}_1\mathbf{z}_2}\|(g_{\mathbf{z}_1}g_{\mathbf{z}_2})\right)$ leads to the information theoretic performance index that can be considered as a basis of constructing a system identification criterion, defining thus an information theoretic approach to the system identification:

$$D_{KL}\left(f_{\mathbf{z}_1\mathbf{z}_2}\|(g_{\mathbf{z}_1}g_{\mathbf{z}_2})\right) = I\{\mathbf{Z}_1,\mathbf{Z}_2\} =$$
$$= -\int_{R^{k_1}}\int_{R^{k_2}} f_{\mathbf{z}_1\mathbf{z}_2}(\mathbf{z}_1,\mathbf{z}_2)\ln\frac{g_{\mathbf{z}_1}(\mathbf{z}_1)g_{\mathbf{z}_2}(\mathbf{z}_2)}{f_{\mathbf{z}_1\mathbf{z}_2}(\mathbf{z}_1,\mathbf{z}_2)}d\mathbf{z}_1 d\mathbf{z}_2 =$$
$$= \mathbf{E}\left(\ln\frac{f_{\mathbf{z}_1\mathbf{z}_2}(\mathbf{z}_1,\mathbf{z}_2)}{g_{\mathbf{z}_1}(\mathbf{z}_1)g_{\mathbf{z}_2}(\mathbf{z}_2)}\right),$$

where $\mathbf{E}(\cdot)$ stands for the mathematical expectation.

In turn, as well known, the mutual information (relative differential entropy) is expanded in the sum of corresponding differential entropies. Meanwhile, there exist more general approaches to define the entropy of a random value / vector. For a k-dimensional random vector $\mathbf{Z}$ having a probability distribution density (pdd) $f(\mathbf{z})$, Tsallis entropy of the order α (Tsallis, 2009) is defined as

$$T_\alpha(f) = \frac{1}{\alpha-1}\left(1-\int_{R^k}(f(\mathbf{z}))^\alpha d\mathbf{z}\right), \alpha > 0,\ \alpha \neq 1. \qquad (1)$$

Simultaneously, as α tends to the infinity, $T_\alpha(f)$ tends to the expression defining the conventional differential entropy that, thus, may be considered as the limit case of Tsallis entropy of "order 1".

From the computational point of view, especially under the necessity of estimating by use of sample data, Tsallis entropy is commonly recognized as more attractive than the differential entropy, since the latter involves "integral of logarithm" possessing certain computational complexity, while Tsallis entropy does not involve logarithm et all.

Based on the definition of Tsallis entropy (1), in the paper of Chenyshov (2018) a symmetric Tsallis divergence $D_\alpha^T(f\|g)$ of the order α of probability distribution densities $f(\mathbf{z})$ and $g(\mathbf{z})$ has been constructed. It has the form

$$D_\alpha^T(f\|g) = \frac{1}{2\cdot|\alpha-1|}\int_{R^k}\left((f(\mathbf{z}))^{\alpha/2}-(g(\mathbf{z}))^{\alpha/2}\right)^2 d\mathbf{z}, \qquad (2)$$

and meets the following natural conditions.

1) $D_\alpha^T(f\|g) \geq 0$ for any $\alpha > 0$, $\alpha \neq 1$ and any probability distribution densities $f(\mathbf{z})$ and $g(\mathbf{z})$.

2) $D_\alpha^T(f\|g) = 0$ if and only if, when $f(\mathbf{z}) \equiv g(\mathbf{z})$.

3) $D_\alpha^T(f\|g) = D_\alpha^T(g\|f)$.

One should be noted that the third condition, the symmetry, is a considerable advantage in the comparison to Kullback-Leibler divergence that is not symmetric: $D_{KL}(f\|g) \neq D_{KL}(g\|f)$.

Again, since the above involved random vector $\mathbf{Z}$ can be considered as concatenation of two random vectors $\mathbf{Z} = (\mathbf{Z}_1^T\ \mathbf{Z}_2^T)^T$, where $\dim\mathbf{Z}_1 = k_1$ and $\dim\mathbf{Z}_2 = k_2$, its pdd $f(\mathbf{z})$ can accordingly be considered as the joint pdd $f_{\mathbf{Z}_1\mathbf{Z}_2}(\mathbf{z}_1,\ \mathbf{z}_2)$ of the random vectors $\mathbf{Z}_1$ and $\mathbf{Z}_2$. Simultaneously, if within these designations the pdd $g(\mathbf{z})$ is considered as product of two marginal probability distribution densities $g_{\mathbf{Z}_1}(\mathbf{z}_1)$ and $g_{\mathbf{Z}_2}(\mathbf{z}_2)$ of the random vectors $\mathbf{Z}_1$ и $\mathbf{Z}_2$ correspondingly, then symmetric Tsallis divergence (2),

$$D_\alpha^T(f\|g) = D_\alpha^T\left(f_{\mathbf{Z}_1\mathbf{Z}_2}\|g_{\mathbf{Z}_1}g_{\mathbf{Z}_2}\right),$$

acquires the sense of a measure of dependence of the random vectors $\mathbf{Z_1}$ and $\mathbf{Z_2}$. Accordingly, such a measure of dependence is natural to be referred as *symmetric* Tsallis mutual information $I_\alpha^T(\mathbf{Z_1},\ \mathbf{Z_2})$ of the order α of the random vectors $\mathbf{Z_1}$ and $\mathbf{Z_2}$:

$$I_\alpha^T(\mathbf{Z_1},\ \mathbf{Z_2}) = \frac{1}{2\cdot|\alpha-1|}\int_{R^{k_1}}\int_{R^{k_2}}(\delta_\alpha(\mathbf{z_1},\ \mathbf{z_2}))^2 d\mathbf{z_1}d\mathbf{z_2}\,,$$
$$\delta_\alpha(\mathbf{z_1},\ \mathbf{z_2}) = \tag{3}$$
$$= \left(f_{\mathbf{Z_1}\mathbf{Z_2}}(\mathbf{z_1},\ \mathbf{z_2})\right)^{\alpha/2} - \left(g_{\mathbf{Z_1}}(\mathbf{z_1})g_{\mathbf{Z_2}}(\mathbf{z_2})\right)^{\alpha/2}.$$

Meanwhile, selecting a particular magnitude of the order α in (1) (and, correspondingly, in (3)) is of importance, since the larger the order is, the more complicated the calculations become. Within the context, the magnitude of $\alpha = 2$, which quadratic symmetric Tsallis mutual information corresponds to,

$$I_2^T(\mathbf{Z_1},\ \mathbf{Z_2}) = \frac{1}{2}\int_{R^{k_1}}\int_{R^{k_2}}(\delta_2(\mathbf{z_1},\ \mathbf{z_2}))^2 d\mathbf{z_1}d\mathbf{z_2}\,,$$
$$\delta_2(\mathbf{z_1},\ \mathbf{z_2}) = f_{\mathbf{Z_1}\mathbf{Z_2}}(\mathbf{z_1},\ \mathbf{z_2}) - g_{\mathbf{Z_1}}(\mathbf{z_1})g_{\mathbf{Z_2}}(\mathbf{z_2}), \tag{4}$$

is commonly recognized as the most appropriate.

2.2 Local Multi-input/multi-output Models

Just $I_2^T(\mathbf{Z_1},\ \mathbf{Z_2})$ from (4) will be applied as an information theoretic criterion to construct an input/output model of a MIMO system. Namely, let a nonlinear discrete time dynamic system to be identified be characterized by *n*-dimensional output process $Y(t)$ and *m*-dimensional input process $U(t)$. To model the system behavior, a linearized system model is considered as a totality of local models of the form

$$\hat{Y}(t;\tau) = W(\tau)U(t-\tau),$$
$$t = 1,\ 2,\ \ldots;\quad \tau = 0, 1,\ \ldots, \tau_p < \infty. \tag{5}$$

In expression (5), $\hat{Y}(t;\tau)$ is the model output process, $W(\tau),\ \ \tau = 0,\ \ 1,\ \ldots\ ,\ \tau_p$ stand for a coefficient $n \times m$-matrices to be identified by use of sample observation of the system input and output processes, with $W(\tau) = \mathbf{0}_{n\times m}$ as $t < \tau$.

Within the present identification problem statement, information-theoretic measure of dependence (4) plays a dual role. As the first step, the model coefficient matrices in (5) are determined in accordance to the following information-theoretic criterion based on expression (4):

$$I_2^T\left(Y(t), \hat{Y}(t;\tau)\right) = I_2^T(Y(t), W(\tau)U(t-\tau)) \to \max_{W(\tau)},$$
$$\tau = 0, 1, \ldots, \tau_p.$$

2.3 Criterion of Involving Local Models and Constructing a Total Model

As the second step, quadratic symmetric Tsallis mutual information (4) is applied to make decisions on final involvement of local models (5) in accordance to the following procedure.

Let

$$W^*(\tau) = \underset{W(\tau)}{\operatorname{argmax}} I_2^T(Y(t),\ W(\tau)U(t-\tau)),$$
$$\tau = 0,\ \ 1,\ \ldots\ ,\ \tau_p. \tag{6}$$

Then, the decision on involving a corresponding local model (4) is to be made on accounting the magnitude of the corresponding measure of dependence $I_2^T(Y(t),\ W^*(\tau)U(t-\tau))$. Meanwhile, such a decision is to be made by use of a corresponding normalized magnitude rather than that of $I_2^T(Y(t),\ W^*(\tau)U(t-\tau))$ in (6) directly, with the normalization being understood in the sense of taking values in the unit interval, while symmetric Tsallis mutual information (3) takes its values in the whole positive semiaxis.

A corresponding normalization procedure is just constructing a mapping of the semiaxis in the unit interval, but selecting such a mapping is not straightforward, since, from one hand side, such a selection is to be justified and explained, and, from another hand side, there are infinitely many such mappings.

Nevertheless, there exists an approach to construct a normalization mapping, which can be considered as objective, since it is based purely on properties of measures of dependence of random values and vectors. Namely, in accordance to an axiom from the commonly recognized set of (Rényi, 1959) axioms for measures of dependence, if in the case of bivariate probabilistic distribution the joint probability distribution between two random values is Gaussian, then any measure of dependence between the random values is to coincide with the absolute value of the ordinary correlation coefficient between them.

Hence, applying the approach implies calculating symmetric Tsallis mutual information (4) for bivariate Gaussian distribution of two random values, say, x and y, with the correlation coefficient r_{xy} as a function in $|r_{xy}|$, with subsequent inverting this function. Then straightforward calculations yield the following equation for $|r_{xy}|$:

$$I_2^T(x,y) = \frac{1}{8\pi\sqrt{1-r_{xy}^2}} - \frac{1}{2\pi\sqrt{4-r_{xy}^2}} + \frac{1}{8\pi} = \varphi(|r_{xy}|). \tag{7}$$

As a measure of dependence of random values, the exact solution of equation (7) meets all axioms (Rényi, 1959) for measures of dependence with the exception of the axiom of invariance to one-to-one transformations of the random values.

At the same time, as can easily be seen, the exact analytical solution of equation (7) cannot be derived explicitly, so a suitable approximation of the function $\varphi(|r_{xy}|)$ in (7) is required to obtain an explicit analytical approximation of $\varphi^{-1}(|r_{xy}|)$. For this purpose, the addendum

$$\frac{1}{2\pi\sqrt{4-r_{xy}^2}}$$

in (7) can be substituted with a suitable constant, namely $(4\pi)^{-1}$, as an approximation. The entity of selecting such a constant is the aim to provide meeting the following conditions: vanishing $I_2^T(x,y)$ if the random values x and y

are independent; and tending $I_2^T(x, y)$ to the infinity if the random values x and y are deterministically dependent.

In accordance to these considerations, finally the expression for the normalization of symmetric quadratic Tsallis mutual information (4) follows:

$$\iota_2^T(\mathbf{Z_1},\ \mathbf{Z_2}) = \sqrt{1 - \frac{1}{(8\pi \cdot I_2^T(\mathbf{Z_1}, \mathbf{Z_2}) + 1)^2}}. \qquad (8)$$

The dependence of $\iota_2^T(\mathbf{Z_1},\ \mathbf{Z_2})$ in (8) as a function in $I_2^T(\mathbf{Z_1},\ \mathbf{Z_2})$ is displayed in Fig. 1.

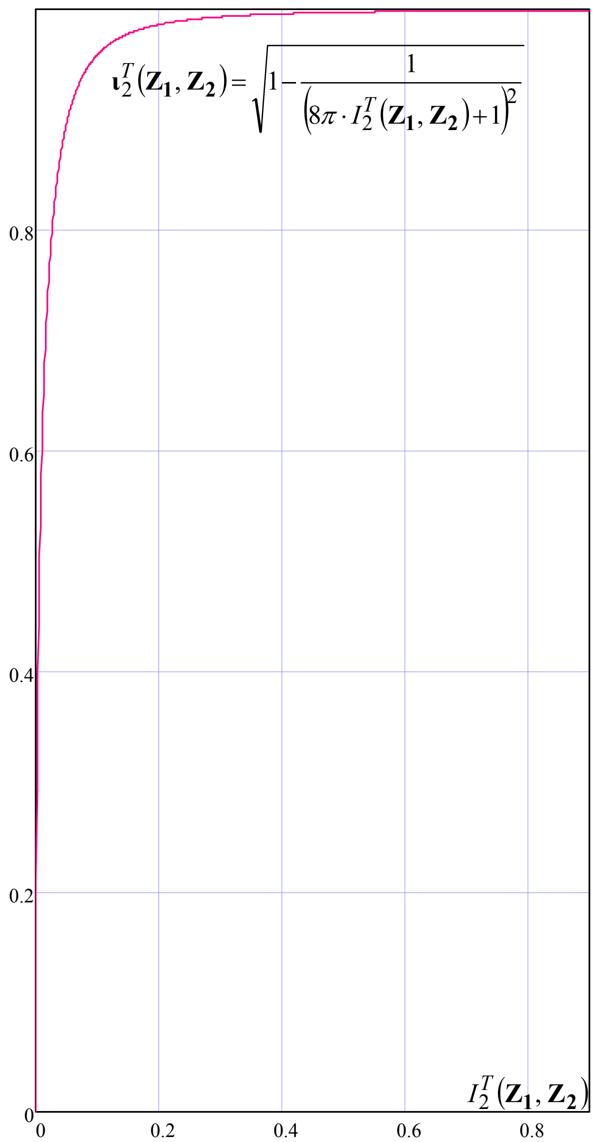


Fig. 1. The dependence of $\iota_2^T(\mathbf{Z_1},\ \mathbf{Z_2})$ as a function in $I_2^T(\mathbf{Z_1},\ \mathbf{Z_2})$.

Thus, selecting representative model input variables $U(t - \tau)$, $\tau = 0,\ 1,\ \dots,\ \tau_p$ is implemented by the researcher on the basis of verifying the condition

$$\iota_2^T(Y(t),\ W^*(\tau)U(t-\tau)) \geq \upsilon(\tau), \quad \tau = 0,\ 1,\ \dots,\ \tau_p, \qquad (9)$$

where $\upsilon(\tau)$, $\tau = 0,\ 1,\ \dots,\ \tau_p$ are certain threshold values preset by the researcher.

Meanwhile, within applying condition (9) in the light of assigning values υ_τ, $\tau = 0,\ 1,\ \dots,\ \tau_p$ it becomes particularly clear the importance of applying namely normalized values of (4), rather than their “direct” ones, since a value that could be recognized as rather small, and therefore could be neglected by the researcher, from the point of view of the whole positive semiaxis, reflects a considerable normalized value and corresponding significant quantitatively expressed dependence of input and output variables of model (4), what is evidently illustrated just by Fig. 1.

Therefore, by virtue of all the above considerations presented one can write the following expression for the total model output process $\hat{Y}(t)$ on the basis of local models (4):

$$\hat{Y}(t) = \left(\sum_{\tau=0}^{\tau_p} \eta_{\upsilon(\tau)}\right)^{-1} \sum_{\tau=0}^{\tau_p} \eta_{\upsilon(\tau)} W(\tau) U(t-\tau), \qquad (10)$$

where

$$\eta_{\upsilon(\tau)} = \begin{cases} 1, & \text{if } \iota_2^T(Y(t),\quad W^*(\tau)U(t-\tau)) \geq \upsilon_\tau, \\ 0, & \text{otherwise,} \end{cases}$$

$\tau = 0,\ 1,\ \dots,\ \tau_p$.

Evidently, model (10) has a sense if at least for a one of $\tau = 0,\ 1,\ \dots,\ \tau_p$ there is hold:

$$\eta_{\upsilon(\tau)} = 1. \qquad (11)$$

Otherwise, the system under study should be recognized as unidentifiable from the researcher decision making point of view. From the theoretical point of view, condition (11) is natural to be substituted simply with the following one

$$\tilde{\eta}_\tau = 1, \qquad (12)$$

where

$$\tilde{\eta}_\tau = \begin{cases} 1, & \text{if } I_2^T(Y(t),\quad U(t-\tau)) > 0 \\ 0, & \text{otherwise} \end{cases}$$

for at least one of $\tau = 0,\ 1,\ \dots,\ \tau_p$. Condition (12) means, thus, the condition of the theoretical identifiability of the system under study.

3. AN APPLICATION WITHIN AN INFORMATION TASK OF ADVANCED NPP APCS

The present Section presents the applicability of the technique proposed in the preceding Section within the information task “Calculation of technical and economical indexes” (“IT-TEI”). “IT-TEI” is a calculation program and is a part of the application software of the top unit-level system (Byvaikov et al., 2006, Zharko, 2006) of advanced NPP APCS. Meanwhile, the significance of the necessity of solving this task under any reactor power level mode can be illustrated by the fact that no novel NPP APCS will be accepted by the customer for operation until the “IT-TEI” task will be demonstrated to be able to perform under any reactor power level.

Due to the fact that there are no two identical NPP units, the problem arises for creating software oriented only to the specifics of a particular NPP unit, both in terms of hardware, as well as based on the available control points of technological parameters (Jharko, 2019). Calculation of TEI is the technological basis for the automated receipt of information characterizing the thermal efficiency of the unit and equipment involved as a part of it. Wherein, thermal efficiency (hereinafter referred to as simply “efficiency”) is understood as the efficiency of using the heat generated in the reactor by nuclear fuel to generate electricity.

The functional purpose of the “IT-TEI” information task is the implementation of processes for collecting, pre-processing information, determining and analyzing TEI for operational and reporting intervals, displaying and recording the results of calculation, preparation and reconfiguration of the software during operation.

The operational purpose of the information task “IT-TEI” is to provide operational, production and technical staff of the NPP with the operational and reporting information on the operating efficiency of the technological equipment.

“IT-TEI” is determined for the main equipment affecting the efficiency of the unit, as well as for the equipment whose status determines the operational mode of the NPP unit with the VVER-1000 reactor (pressurized water reactor (the water is both coolant and neutron moderator)) of 1000 megawatt power), the heat circuit diagram of which is shown in Fig. 2.

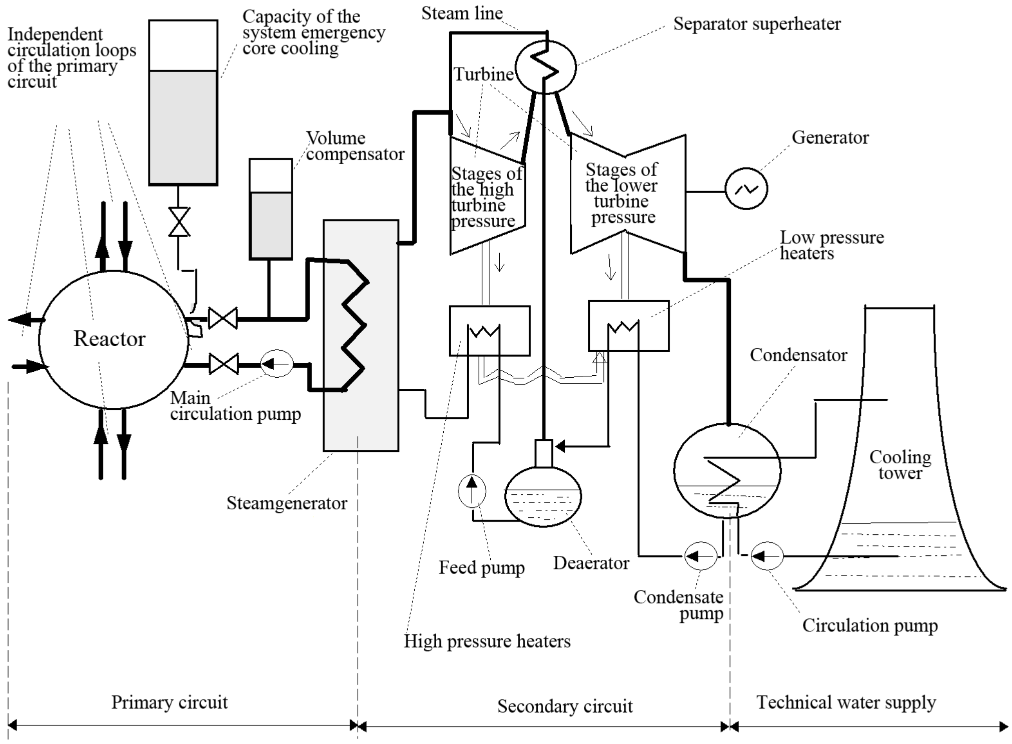


Fig. 2. Heat circuit diagram of NPP unit with reactor VVER-1000.

The purpose of the TEI calculation is to provide information for the most economical use of the equipment, forecasting its maintenance and repair, as well as to compile a report about the efficiency of the unit.

The problem is solved for the main equipment affecting the efficiency of the power unit, as well as for the equipment whose status determines the operational mode of the unit.

The first group includes equipment, the status of which is evaluated on the basis of the results of a set of tasks. It includes a turbine unit, including the turbine itself, high pressure heaters, low pressure heaters, separator steamoverheater, condenser; deaerator, etc.

The second group includes equipment (pumps, valves), which changes the operational mode of the technological systems of the first and second circuit depending on their status, and thereby changes the TEI calculation algorithm accordingly. This equipment includes feed and condensate pumps, fittings on the feed and condensate lines, main circulation pumps, etc.

The output data of “IT-TEI” is displayed on automated workstations of the unit’s operational shift managers.

The software of the information task “IT-TEI” according to the type of tasks being solved is subdivided and consists of the following major parts:

- providing proper calculation, analysis, and displaying TEI calculation results on the automated workstations (operates in real time);
- providing the formation of report forms (operates in interactive mode);
- providing service functions (operates in interactive mode).

TEI calculation is carried out with the help of a mutually coordinated set of functional tasks, which includes:

- preliminary calculations during the operational interval;
- calculation of TEI during the operational interval;
- sorting information by different periodic intervals;
- TEI calculation at different intervals;
- displaying the results of the TEI calculation.

All estimated technical and economic indexes are divided into three main groups:

- actual indexes that characterize the efficiency levels of the equipment during operating conditions;
- regulatory indexes that characterize the estimated level of the equipment efficiency;
- indexes of changes in the efficiency of the power unit due to deviation of the actual indexes from the regulatory ones.

The initial time period (sampling period) is the length of time between the start of two consecutive sensor polling cycles. The duration of the sampling period is assumed to be the same as the other functions of the control system. The most rational duration of the sampling period is 1 min for calculating TEI.

The following time periods for which the calculation was performed are provided for calculating TEI:

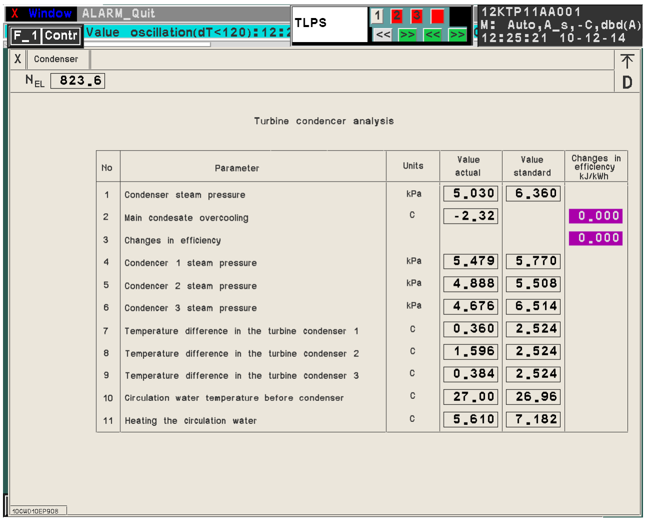
- an operational period of 15 minutes;
- shift – a period equal in duration to one work shift;
- day – a time period equal to 24 hours;
- month – a period equal to the number of hours in a calendar month.

Depending on the period at which the indexes were calculated, they are called operational, shift, daily and monthly, respectively. In addition to these indexes, it has been provided to obtain integral indexes on an accrual basis (progressive total) from the beginning of the month until as demanded within the period of this month.

More than 400 analog and discrete signals are used as input variables for the "IT-TEI" task, and, even if at least one of these signals is unauthentic, it becomes impossible to obtain a authentic calculation of the unit's technical and economic parameters in full and there is no operational conclusive information on the task on the workstation monitors of shift engineers for reactor and turbine control. As a result, only those output signals for which there is a full set of necessary authentic input signals would be calculated and displayed on the monitors. The task of calculating technical and economic indexes is strongly connected and individual input signals (both discrete as well as analog) are involved in the calculation of a sufficiently large number of output signals and their unauthentic nature (for example, consumption of chemically demineralized water in turbine condensers) can lead to exclusion from the calculation of up to 30 % of the output signals of the task associated with this unauthentic signal. Fig. 3 shows an example of output data results to a video frame of an incomplete calculation (fields painted in crimson show the value fields for which there was no complete set of authentic input signals).

At the same time, during set up and commissioning it was necessary for at least certain partial data information output of the tasks in order to:

- diagnose both equipment malfunction and equipment inconsistencies with ranges of analog signal changes;
- determining the cost-effectiveness of equipment in use.



No	Parameter	Units	Value actual	Value standard	Changes in efficiency kJ/kWh
1	Condenser steam pressure	kPa	5.030	6.360	
2	Main condesate overcooling	C	-2.32		0.000
3	Changes in efficiency				0.000
4	Condencer 1 steam pressure	kPa	5.479	5.770	
5	Condencer 2 steam pressure	kPa	4.888	5.508	
6	Condencer 3 steam pressure	kPa	4.676	6.514	
7	Temperature difference in the turbine condenser 1	C	0.360	2.524	
8	Temperature difference in the turbine condenser 2	C	1.596	2.524	
9	Temperature difference in the turbine condenser 3	C	0.384	2.524	
10	Circulation water temperature before condenser	C	27.00	26.96	
11	Heating the circulation water	C	5.610	7.182	

Fig. 3. Example of output of incomplete calculation of "IT-TEI" problem to a video frame.

Consequently, the task arose of decomposing the problem into input signals and establishing the influence of these signals on the output signals (see Fig. 4).

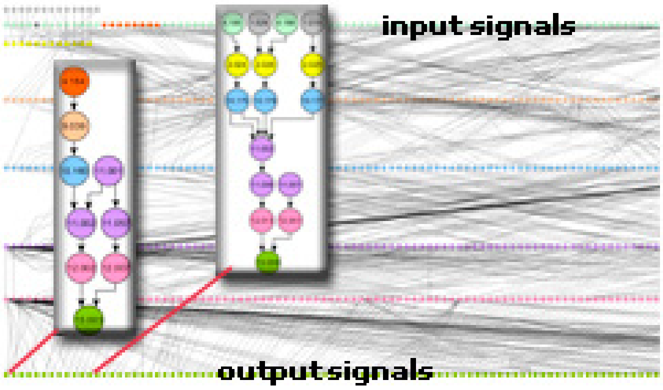


Fig. 4. Example of calculation decomposition.

To implement such a decomposition, applying the technique proposed, which is based on involving a consistent measure of dependence, quadratic symmetric Tsallis mutual information, between input and output signals is just the way to the required solution. Namely, applying of a model of the form defined by expression (10) enables one to elicit the available impact of input signals in output ones in a maximally flexible and comprehensive, but, simultaneously, a unified and concise manner due to the procedure of model (10) construction. Meanwhile, several type models are constructed in accordance to a requirement of the representation/displaying needs, namely ones calculated for different sampling periods in accordance to the time periods mentioned above, namely: operational period, shift, day, and month. Thus, applying the technique proposed in Section 2 enables one to exclude the case of elimination of essential interconnections between input and output signals due the properties of quadratic symmetric Tsallis mutual information involved in constructing model (10).

4. CONCLUSIONS

In the paper, a problem of identifying a multidimensional nonlinear dynamic stochastic system has been considered. Within the approach proposed, the identification procedure is based on the use of such a measure of dependence of vector-valued random processes as symmetric Tsallis mutual information constructed in (Chernyshov, 2018) on the basis of the Tsallis entropy definition (Tsallis, 2009).

From an industrial application point of view, model of the (10) class is involved as an algorithmic tool within an information task of advanced NPP APCS intended to monitor the power unit efficiency under the conditions, when direct calculations based on real physical data cannot be implemented just due to the reasons that these data are recognized as unauthentic.

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