

# Input-to-State Stability of Time-distributed Optimization in Real-time Model Predictive Control<sup>\*</sup>

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**Abstract:** This paper presents a general system theoretic framework for the analysis of Time-distributed Optimization (TDO) in the context of real-time Model Predictive Control (MPC). When implemented using TDO, an MPC controller maintains a running estimate of the solution of the underlying optimal control problem and improves it at each sampling instant, instead of computing high precision solutions. TDO can be conceptualized as the combination of truncation and warmstarting in online optimization and includes the real-time iteration scheme as a special case. This paper considers a broad class of optimization algorithm/MPC formulations combinations and derives sufficient conditions under which it is possible for TDO based MPC controllers to recover the stability and robustness properties of optimal MPC using a finite amount of computational power.

*Keywords:* Real-time optimization, Model predictive control, Real-time iterations, Input-to-state stability, Constrained control, Control of nonlinear systems

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