

# ADMM-Based Optimization of Distributed Energy Management Systems with Demand Response <sup>★</sup>

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**Abstract:** Design of distributed energy management systems composed of several agents is important for realizing smart cities. Demand response for saving the power consumption is also important. In this paper, we propose a design method of distributed energy management systems with real-time demand response. Here, we use ADMM (Alternating Direction Method of Multipliers). In the proposed method, demand response is performed in real-time, based on the difference between the planned demand and the actual value. Furthermore, utilizing the blockchain is also discussed.

**Keywords:** ADMM, demand response, distributed energy management systems.

## 1. INTRODUCTION

As one of control technologies for a smart city, design of distributed energy management systems (EMSs) is one of the important problems. A distributed EMS is composed of several agents such as factories and buildings (see, e.g., Miyamoto et al. (2016a,b)). By transactions between agents, the surplus power may be generated. As a result, the power traded with an external district can be controlled.

On the other hand, demand response (DR) is well known as one of the key technologies in EMSs. DR is defined as the changes in electricity usage of end-use consumers by changing the electricity price, and so on. There have been many results from several viewpoints. In e.g., Conejo et al. (2010); Miyazaki et al. (2019), the future demand is re-scheduled based on the error of the past planned demand and the past actual power consumption, based on model predictive control. For a distributed EMS, it is important to develop an optimization method for re-scheduling considering demand response. However, only few results have been obtained so far (see, e.g, Soares et al. (2013)).

In this paper, as an extension of Miyazaki et al. (2019), we consider both day-ahead scheduling and re-scheduling for a distributed EMS considering both electrical energy and thermal energy. The error of the past planned demand and the past actual value is distributed to the future demand. We suppose that the difference between the planned demand and the modified demand is compensated by DR. In both day-ahead scheduling and re-scheduling, we use ADMM (Alternating Direction Method of Multipliers), which is one of the powerful methods in distributed optimization (Boyd et al. (2011)). Furthermore, we also discuss the effectiveness of utilizing the Blockchain technology. The blockchain is compatible with distributed optimization (see, e.g., Munsing et al. (2017); Ogawa et al. (2019)). By a numerical example, we demonstrate the proposed method. We also discuss the adverse effect of tampering the past actual value, and we suggest the importance of introducing a blockchain.

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**Notation:** Let  $\mathcal{R}$  denote the set of real numbers. For the finite set  $\mathcal{A}$ , let  $|\mathcal{A}|$  denote the number of elements in  $\mathcal{A}$ . Let  $0_{m \times n}$  denote the  $m \times n$  zero matrix. For the vector  $x$ , let  $x^\top$  denote the transpose of  $x$ . For the vector  $x$ , let  $\|x\|_2$  denote the Euclidean norm of  $x$ . For the vector  $x$ , let  $x^{(i)}$  denote the  $i$ -th element of  $x$ .

## 2. EXCHANGE PROBLEM AND ADMM

In this section, after the exchange problem (EP) is explained, ADMM is explained.

Let  $\mathcal{I} = \{1, 2, \dots, n\}$  denote the set of agents. Let  $x_i, \mathcal{X}_i$ , and  $f_i : \mathcal{X}_i \rightarrow \mathcal{R}$  denote the decision variable vector, the domain of  $x_i$ , and the convex objective function, respectively. Let  $\mathcal{M}$  denote the finite set of markets. Let  $x_i^{(m_j)}$ ,  $j \in \{1, 2, \dots, |\mathcal{M}|\}$  denote the scalar decision variable for the agent  $i$  in the market  $m_j \in \mathcal{M}$ . The vector  $x_i^{\mathcal{M}}$  is defined by  $x_i^{\mathcal{M}} := [x_i^{(m_1)} x_i^{(m_2)} \dots x_i^{(m_{|\mathcal{M}|})}]^\top$ . Then, EP is given as follows: Find  $x_i$ ,  $i \in \mathcal{I}$  minimizing  $\sum_{i \in \mathcal{I}} f_i(x_i)$  subject to  $x_i \in \mathcal{X}_i$ ,  $i \in \mathcal{I}$  and

$$\sum_{i \in \mathcal{I}} x_i^{\mathcal{M}} = 0_{|\mathcal{M}| \times 1}. \quad (1)$$

In EP, a sum of objective functions for agents is minimized under the condition that demand and supply are balanced in all markets. For the market  $m \in \mathcal{M}$ , the agent  $i$  is called a supplier if  $x_i^{(m)} < 0$ , and the agent  $i$  is called a consumer if  $x_i^{(m)} > 0$ .

Next, the Lagrange function for EP is given by  $L(x, \alpha) = \sum_i f_i(x_i) + \alpha^\top \sum_i x_i^{\mathcal{M}}$ , where  $\alpha$  is a Lagrange multiplier, and corresponds to a shadow price in the market. For each agent, this Lagrange function can be decomposed to  $L_i(x_i, \alpha) = f_i(x_i) + \alpha^\top x_i^{\mathcal{M}}$ ,  $i \in \mathcal{I}$ . In the case of using ADMM for EP,  $x_i$  and  $\alpha$  are updated as follows:

$$x_i(k+1) := \arg \min_{x_i} \left( L_i(x_i, \alpha(k)) + \frac{\rho}{2} \|x_i^{\mathcal{M}} - \bar{x}_i^{\mathcal{M}}(k) + \bar{x}^{\mathcal{M}}(k)\|_2^2 \right), \quad i \in \mathcal{I},$$

$$\alpha(k+1) := \alpha(k) + \rho \bar{x}^{\mathcal{M}}(k+1) \quad (2)$$

where  $k \in \{0, 1, 2, \dots\}$  is the number of updates (turn),  $\rho$  is a penalty parameter, and  $\bar{x}^{\mathcal{M}}(k) = \sum_i x_i^{\mathcal{M}}(k)/n$ . See, e.g., Boyd et al. (2011); Miyamoto et al. (2016a) for further details.

In distributed optimization using ADMM, the whole system consists of an aggregator and  $n$  agents. The aggregator presents the shadow price  $\alpha$  and the mean value  $\bar{x}$  to each agent, and collects  $x_i(k+1)$  obtained by local optimization in each agent. In addition,  $\alpha$  is updated using  $x_i(k+1)$ . In each agent, the individual local optimization problem is solved. The penalty parameter  $\rho$  must be shared in the aggregator and agents. Fully-distributed ADMM has been proposed in e.g., Matsuda et al. (2019). However, it is desirable that an aggregator is introduced in the case of imposing equality constraints such as (1).

### 3. DISTRIBUTED ENERGY MANAGEMENT SYSTEMS

In this section, we formulate a distributed EMS. A mathematical model of a distributed EMS in this paper is based on Miyamoto et al. (2016a,b). Consider a special district that is composed of factory agents and building agents. In this section, we consider only a single period. A factory agent has energy conversion equipments such as boilers and turbines, and can sell excess energy to other agents. In a building agent, to satisfy its demand, energy from inside and outside of the district is purchased, and energy conversion equipments are operated. Here, there are two markets, i.e., an electricity market and a heat market.

First, we explain a factory agent. Suppose that a factory agent has a gas cogeneration system (GT) and a gas boiler (BA). The optimization problem for a factory agent is given as follows:

$$\begin{aligned} & \text{minimize} && \alpha_{BE}BE + \alpha_{BG}BG + \alpha_E SE_E + \alpha_H SH_H \quad (3) \\ & \text{subject to} && SE_E \leq 0, \quad SH_H \leq 0, \quad BE \geq 0 \end{aligned}$$

$$\begin{aligned} & BG_{GT} \geq 0, \quad BG_{BA} \geq 0 \\ & 0 \leq PE_{GT} \leq a_{GT_E} BG_{GT}^2 \\ & \quad + b_{GT_E} BG_{GT} + c_{GT_E} \quad (4) \end{aligned}$$

$$\begin{aligned} & 0 \leq PH_{GT} \leq a_{GT_H} BG_{GT}^2 \\ & \quad + b_{GT_H} BG_{GT} + c_{GT_H} \quad (5) \end{aligned}$$

$$\begin{aligned} & 0 \leq PH_{BA} \leq a_{BA} BG_{BA}^2 \\ & \quad + b_{BA} BG_{BA} + c_{BA} \quad (6) \end{aligned}$$

$$BE + PE_{GT} + SE_E = DE \quad (7)$$

$$PH_{GT} + PH_{BA} + SH_H = DH \quad (8)$$

$$BG = BG_{GT} + BG_{BA} \quad (9)$$

$$\underline{BG}_{GT} \leq BG_{GT} \leq \overline{BG}_{GT} \quad (10)$$

$$\underline{BG}_{BA} \leq BG_{BA} \leq \overline{BG}_{BA} \quad (11)$$

where the index for each factory agent is omitted. Meaning of decision variables is given as follows:

- $SE_E, SH_H$ : volumes of trading of electrical and thermal energy from inside of the district (if a factory is a supplier, then these are negative),
- $BE, BG$ : volumes of electrical and thermal energy purchasing from outside of the district,
- $BG_{GT}, BG_{BA}$ : input energy of each equipment,
- $PE_{GT}, PH_{GT}, PH_{BA}$ : volumes of electrical and thermal energy generated by each equipment.

Meaning of constants is given as follows:

- $\alpha_{BE}, \alpha_{BG}$ : unit price of electrical and thermal energy purchasing from outside of district,
- $\alpha_E, \alpha_H$ : unit price of electrical and thermal energy trading inside of district,
- $DE, DH$ : electrical and thermal demands,

- $a_{\bullet}, b_{\bullet}, c_{\bullet}$ : coefficients of input-output properties of equipments.

We remark here that  $x_i, x_i^M$ , and  $\alpha$  in Section 2 correspond to  $[SE_E \ SH_H \ BE \ BG \ BG_{GT} \ BG_{BA} \ PE_{GT} \ PH_{GT} \ PH_{BA}]^T$ ,  $[SE_E \ SH_H]^T$ , and  $[\alpha_E \ \alpha_H]^T$ , respectively. (3) represents the energy cost, (4)–(6) represent input-output properties of equipments (due to solver limitation, input-output properties are represented by inequalities). (7)–(9) represent energy balances. (10) and (11) represent constraints for input energy.

Next, we explain a building agent. Suppose that a building agent has a gas boiler (BA). The optimization problem for a factory agent is given as follows:

$$\begin{aligned} & \text{minimize} && \alpha_{BE}BE + \alpha_{BG}BG + \alpha_E BE_E + \alpha_H BH_H \\ & \text{subject to} && BE_E \geq 0, \quad BH_H \geq 0, \quad BE \geq 0, \quad BG_{BA} \geq 0 \\ & && 0 \leq PH_{BA} \leq a_{BA} BG_{BA}^2 + b_{BA} BG_{BA} + c_{BA} \\ & && BE + BE_E = DE, \quad PH_{BA} + BH_H = DH \\ & && BG = BG_{BA} \\ & && \underline{BG}_{BA} \leq BG_{BA} \leq \overline{BG}_{BA} \end{aligned}$$

where the index for each building agent is omitted. Meaning of decision variables is given as follows:

- $BE_E, BH_H$ : volumes of electrical and thermal energy purchasing from inside of the district (If a building agent is a consumer, these are positive).

Other decision variables and constants are the same as those of a factory agent. We remark here that  $x_i$  and  $x_i^M$  in Section 2 correspond to  $[BE_E \ BH_H \ BE \ BG \ BG_{BA} \ PH_{BA}]^T$  and  $[BE_E \ BH_H]^T$ , respectively.

Finally, since we consider two markets, the equality constraint (1) in Section 2 is given by the following two equality constraints:  $\sum_{i=1}^{N_F} SE_E^i + \sum_{i=1}^{N_B} BE_E^i = 0$  and  $\sum_{i=1}^{N_F} SH_H^i + \sum_{i=1}^{N_B} BH_H^i = 0$ , where  $N_F$  and  $N_B$  are the number of factory and building agents, respectively, and  $i$  is the index for agents.

## 4. PROPOSED METHOD

### 4.1 Outline

First, we explain the outline of the proposed method. We suppose that hourly electrical and thermal demands planned in the previous day are given. Then, the optimization problem is solved every hour. Since the planned demand and the actual consumption are different, the difference between these values must be compensated in the future. In this paper, we consider realizing this compensation by DR. Based on the difference occurred at the current time, we modify the demand in the future. By this method, the hourly demand is changed, and it is expected that the total consumption in one day is almost the same as the total demand in one day.

### 4.2 Proposed Procedure

Let  $DE^i(t)$  and  $DH^i(t)$ ,  $i = 1, 2, \dots, N_F + N_B$ ,  $t = 0, 1, 2, \dots, 23$  denote hourly electrical and thermal demands planned in the previous day, respectively. We define  $DE_{\text{total}}^i := \sum_{t=0}^{23} DE^i(t)$  and  $DH_{\text{total}}^i := \sum_{t=0}^{23} DH^i(t)$ . Let  $DE_a^i(t)$  and  $DH_a^i(t)$ ,  $i = 1, 2, \dots, N_F + N_B$ ,  $t = 0, 1, 2, \dots, 23$  denote hourly electrical consumption and hourly thermal consumption, respectively. We also define the error between the planned demand and

the actual consumption as follows:  $e_E^i(t) := DE^i(t) - DE_a^i(t)$  and  $e_H^i(t) := DH^i(t) - DH_a^i(t)$ . The scalar  $l(t)$  is defined by  $l(0) = l(1) = \dots = l(23 - l) = l$ ,  $l(23 - l + m) = l - m$ ,  $m = 1, 2, \dots, l$ , where  $l \in [0, 23]$  is a given integer. In addition,  $\gamma_j(t) \geq 0$ ,  $j = 0, 1, 2, \dots, l(t)$  are given parameters that satisfy  $\sum_{j=0}^{l(t)} \gamma_j(t) = 1$ . In the case of  $t = 23$ , two conditions:  $l(23) = 0$  and  $\gamma_0(23) = 1$  hold.

Under these preparations, we propose the procedure for optimization using DR as follows.

### Proposed Procedure:

**Step 0:** Give  $DE^i(t)$ ,  $DH^i(t)$ ,  $l(t)$ , and  $\gamma_j(t)$ ,  $t = 0, 1, 2, \dots, 23$ . Set  $t = 0$ .

**Step 1:** Solve the optimization problem EP using ADMM.

**Step 2:** Apply the computation result to each agent. Collect  $DE_a^i(t)$  and  $DH_a^i(t)$ .

**Step 3:** Modify  $DE^i(t+1+j)$  and  $DH^i(t+1+j)$ ,  $j = 0, 1, \dots, l(t)$  to

$$DE^i(t+1+j) := DE^i(t+1+j) + \gamma_j(t)e_E^i(t), \quad (12)$$

$$DH^i(t+1+j) := DH^i(t+1+j) + \gamma_j(t)e_H^i(t). \quad (13)$$

**Step 4:** Set  $t := t+1$ . If  $t = 24$ , then the procedure is terminated. Otherwise, return to Step 1.

In the above procedure, the errors  $e_E^i(t)$  and  $e_H^i(t)$  are distributed to the future demand depending on  $l(t)$  and  $\gamma_j(t)$  given in advance. Using this procedure, the following relations are achieved:

$$\sum_{t=0}^{23} DE_a^i(t) \approx DE_{\text{total}}^i, \quad \sum_{t=0}^{23} DH_a^i(t) \approx DH_{\text{total}}^i. \quad (14)$$

### 4.3 Discussion on Implementation Using Blockchain

When EP is solved using ADMM, utilizing the Blockchain technology provides some benefits. The blockchain is one of the open and distributed ledgers. In the blockchain, a peer-to-peer network, which adheres to a protocol for inter-node communication and validates new blocks, manages typically. The blockchain has been used in a smart grid (see, e.g., Noor et al. (2018)), and is compatible with a distributed EMS.

Using the blockchain, information managed by the aggregator in ADMM is shared by all agents in the safe form that tamper is difficult. Hence, we do not need the aggregator. On the other hand, the computation time is increased by introducing a blockchain (see Ogawa et al. (2019) for further details). It is necessary to consider the trade-off between the safety and the computation time.

## 5. NUMERICAL EXAMPLE

In this section, a numerical example is presented. We consider solving the optimization problem EP for the EMS in Section 3 with real-time demand response. Consider the EMS that is composed of two factory agents (F1, F2) and three building agents (B1, B2, B3). The unit energy prices from outside from the district is given by  $\alpha_{BE} = 10.39[10^3\text{JPY/MWh}]$  and  $\alpha_{BG} = 2.86[10^3\text{JPY}/10^2\text{m}^3]$ , respectively. Table 1 shows the parameters of each agent. Fig. 1 and Fig. 2 show hourly electrical

Table. 1. Parameters.

	F1	F2	B1	B2	B3
$a_{GT_E}$ [-]	-0.001	-0.002	-	-	-
$b_{GT_E}$ [-]	0.52	0.51	-	-	-
$c_{GT_E}$ [-]	-2.0	-2.5	-	-	-
$a_{GT_H}$ [-]	-0.001	-0.007	-	-	-
$b_{GT_H}$ [-]	0.78	1.3	-	-	-
$c_{GT_H}$ [-]	-3.3	-6.0	-	-	-
$\overline{BG}_{GT}$ [ $10^2\text{m}^3$ ]	46.4	27.5	-	-	-
$\underline{BG}_{GT}$ [ $10^2\text{m}^3$ ]	5.83	5.55	-	-	-
$a_{BA}$ [-]	-0.4	-0.4	-0.5	-0.45	-0.4
$b_{BA}$ [-]	5.1	4.95	5.0	4.9	4.95
$c_{BA}$ [-]	-1.0	-1.0	-0.5	-0.5	-1.0
$\overline{BG}_{GT}$ [ $10^2\text{m}^3$ ]	2.75	1.36	1.84	1.63	2.18
$\underline{BG}_{GT}$ [ $10^2\text{m}^3$ ]	0.405	0.23	0.12	0.14	0.18

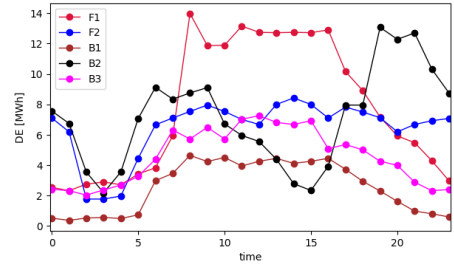


Fig. 1. Electrical demand planned in the previous day.

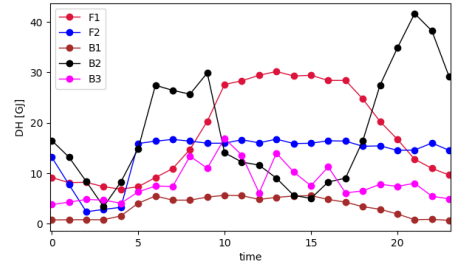


Fig. 2. Thermal demand planned in the previous day.

demand and hourly thermal demand planned in the previous day, respectively. The parameters and energy demands are generated based on the references Miyamoto et al. (2016a,b). In computation, we use Python/CVXPY.

The parameter  $\rho$  in ADMM is set to 0.1. If both  $\sum_{i \in \mathcal{I}} x_i^M(k) < \varepsilon$  and  $\rho(k+1)(x^M(k+1) - x^M(k)) < \varepsilon$  are satisfied, then the computation procedure is terminated. In this example, we set  $\varepsilon = 0.005$ . In addition, the parameter  $l$  in the proposed method is set to 0. The initial values of  $\alpha_E$  and  $\alpha_H$  are given by zero.

We explain the computation results. In this numerical example, we define  $DE_a^i(t)$  and  $DH_a^i(t)$  as  $DE_a^i(t) := DE^i(t) + v(t)$  and  $DH_a^i(t) := DH^i(t) + w(t)$ , respectively, where  $v(t)$  and  $w(t)$  are noises. First, we validate the effectiveness of the proposed method. Fig. 3 and Fig. 4 show hourly electrical consumption  $DE_a^i(t)$  and hourly thermal consumption  $DH_a^i(t)$  with and without the proposed method, respectively. Table 2 and Table 3 show the total demand and consumption of electrical energy and thermal energy in one day. From these results, we see that two relations (14) are achieved by using the proposed method.

Next, we comment about the effects of tampering and advantages of implementing the proposed method using a blockchain. We suppose here that (12) and (13) are tampered as follows:

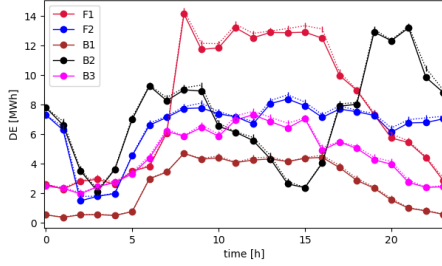


Fig. 3. Electrical consumption. Solid line: Using the proposed method. Dotted line: Not using the proposed method.

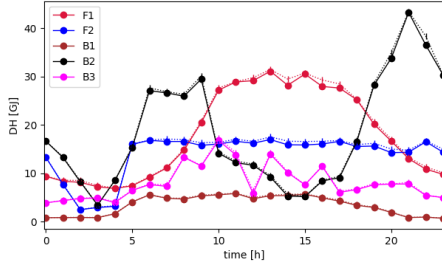


Fig. 4. Thermal consumption. Solid line: Using the proposed method. Dotted line: Not using the proposed method.

Table. 2. The total demand and consumption of electrical energy in one day.

	F1	F2	B1	B2	B3
$DE_{total}^i$	185.9	154.4	61.2	170.5	109.5
$\sum_{t=0}^{23} DE_a^i(t)$ with DR	185.9	154.6	61.2	170.8	109.7
$\sum_{t=0}^{23} DE_a^i(t)$ without DR	190.4	158.6	62.6	174.6	112.8

Table. 3. The total demand and consumption of thermal energy in one day.

	F1	F2	B1	B2	B3
$DH_{total}^i$	428.0	331.1	81.0	436.3	193.1
$\sum_{t=0}^{23} DH_a^i(t)$ with DR	428.4	331.5	81.0	437.5	193.1
$\sum_{t=0}^{23} DH_a^i(t)$ without DR	437.5	340.2	83.2	446.0	197.4

$$DE^i(t+1+j) := DE^i(t+1+j) - \gamma_j(t)e_E^i(t),$$

$$DH^i(t+1+j) := DH^i(t+1+j) - \gamma_j(t)e_H^i(t).$$

Fig. 5 and Fig. 6 show hourly electrical consumption  $DE_a^i(t)$  and hourly thermal consumption  $DH_a^i(t)$  in the normal case and in the case of tampering. From these figures, we see that tampering may cause excessive energy consumption. Using the blockchain, we can prevent such cases of tampering.

## 6. CONCLUSION

In this paper, we considered an ADMM-optimization method for a distributed EMS considering both electrical energy and thermal energy. We supposed that the difference between the planned demand and the modified demand is compensated by DR. The effectiveness of the proposed method was presented by a numerical example.

In future work, we will consider utilizing a blockchain based on our previous method (Ogawa et al. (2019)).

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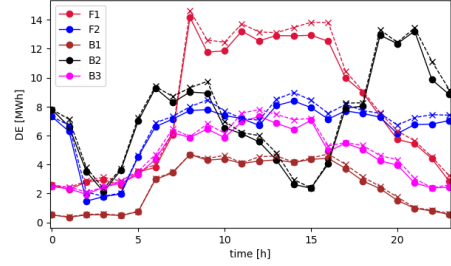


Fig. 5. Electrical consumption. Solid line: The normal case. Dashed line: The case of tampering.

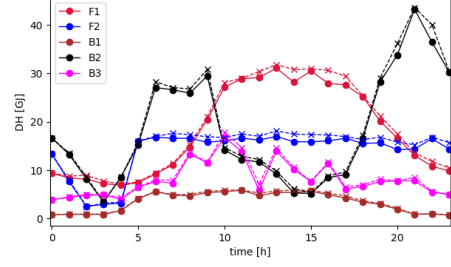


Fig. 6. Thermal consumption. Solid line: The normal case. Dashed line: The case of tampering.

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