Presented as late breaking results contribution
21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Numerical Evaluation on Frequency Domain Nonparametric Modeling for Stable/Unstable Systems with I/O Noise ⋆

Toshiharu Sugie * **Natchanon Khurewattanakul** **

* *Osaka University, Suita, Japan, (e-mail: sugie@jrl.eng.osaka-u.ac.jp)*
** *Kyoto University, Kyoto Japan, (email: natchanon49523@live.com)*

Abstract: Though most of the existing work focus on parametric modeling, non-parametric modeling methods have attracted a lot of attention these days, partly because it does not require the structural information such as system orders. From the viewpoint of control system analysis and synthesis, frequency responses are crucial in practice. Hence, the frequency domain models are important. Nevertheless, there are not so many reports which evaluate the effectiveness of nonparametric frequency modeling methods so far. Hence, this paper focuses on numerical evaluation of such modeling methods. First, the effectiveness of the periodic input (for identification) is demonstrated for a stable system in the presence of input noise as well as the output one. Second, it is shown that the frequency domain methods are effective for a closed loop system with unknown nonlinear stabilizing controller.

Keywords: system identification, linear systems, frequency domain, non-parametric models

1. INTRODUCTION

Most of the existing researches on system modeling have been focusing on parametric modeling (i.e. finding the model parameters see e.g., Ljung (2001),), with the assumption that the system structure (such as the order of the target system) is known. However, it is often difficult to determine the system structure in practice. One way to avoid this problem, which has been attracting a lot of attention in the recent years, is to adopt nonparametric modeling methods instead.

Nonparametric identification can be done in both time and frequency domains (see Pintelton and Schoukens (2001)). Though the time domain methods (see e.g., Pillonetto et al. (2018)) are much popular than the frequency domain ones, the later methods have various merits: (i) They directly handle the frequency responses which play an essential role in control system design and analysis. (ii) They can handle the input noise easily as well as the output noise, and the noise could be colored. (iii) The closed loop identification can be done by using the standard procedure, hence no advanced techniques are needed. This is in contrast to the parametric modeling methods (see e.g., Maruta and Sugie (2018)) . So far, some important progress (such as the method to cope with the leakage effects) has been made in frequency domain nonparametric modeling (see Pintelton et al. (1997), Schoukens et al. (2018)). However, despite of its importance, not so much numerical evidence has been shown how effective the frequency domain methods are. Hence they may be underestimated for many control engineers.

Based on the above observations, this paper tries to demonstrate their effectiveness through numerical examples. The authors are interested in whether the standard frequency domain approach is effective for some difficult cases. One is that the target system has both fast and slow modes with an unstable zeros. The other is that the system itself is unstable but it is stabilized by an unknown nonlinear controller. In addition, the colored noise may be added at the input/output channels.

2. NONPARAMETRIC FREQUENCY DOMAIN MODELLING

This section summarizes some basic tools (see Schoukens et al (2012)) for nonparametric frequency domain modeling.

Let $u(kT_s)(k = 0, 1, ..., N-1)$ be the time domain data with sampling time T_s. Its DFT(Discrete Fourier Transform) is defined by

$$U(\ell) = \frac{1}{N}\sum_{k=0}^{N-1} u(kT_s)e^{-j2\pi\frac{k\ell}{N}}$$

where ℓ is the frequency line index. The line index can be transformed into the frequency scale f by using: $f = \frac{l}{N}f_s$ with $f_s = \frac{1}{T_s}$ being the sampling frequency.

For periodic excitation, random multisine signals will be used. A multisine signal is the sum of harmonically related sines, i.e.,

$$u = \sum_{k=1}^{F} A\sin(2\pi k f_0 t + \phi_k).$$

where the amplitude A is constant and the phase ϕ_k will be randomized.

⋆ This work is partly supported by JSPS KAKENHI No. JP17H03281.

Presented as late breaking results contribution
21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

An alternative method to excite every frequency at the same time is to use random excitation, specifically Gaussian noises.

2.1 Calculating the Frequency Responses

Consider the open loop system shown in Fig. 1,

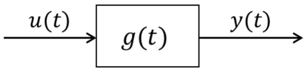

Fig. 1. Open loop system.

where $u(t)$, $y(t)$, $g(t)$ are the input, the output and the target system model, respectively, in the time domain. Define $U(k)$, $Y(k)$, $G(k)$ as the Discrete Fourier Transform of $u(t)$, $y(t)$, $g(t)$ with respect to the frequency index k. Ignoring the finite length measurement effects, the following relation holds:

$$Y(k) = G(k)U(k) \tag{1}$$

This relation is used to estimate $G(k)$ (FRF: Frequency Response Function) in the case of periodic inputs and if $U(k)$ does not become very small or equal to zero.

Alternatively, by calculating the cross-spectrum S_{YU} and the auto-spectrum S_{UU}, the FRF is obtained by

$$G(k) = \frac{S_{YU}(k)}{S_{UU}(k)}. \tag{2}$$

This one is used in the case of random inputs.

As for the closed loop system shown in Fig. 2, eq.(1) is used for periodic inputs, and

$$G(k) = \frac{S_{YR}(k)}{S_{UR}(k)} \tag{3}$$

is used for random inputs, where R is the DFT of the reference signal r.

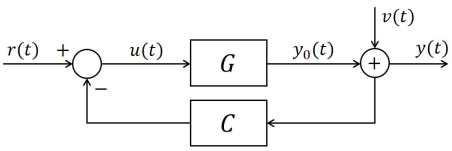

Fig. 2. Closed-loop system

A typical example of FRF estimation results are shown in Fig. 3. The measured FRF in the case of random excitation using the indirect method by eq.(3) (represented by pink line) and the measured FRF in the case of periodic excitation using the direct method by eq. (1) (represented by blue dot) have no bias, while bias is present when the direct method is used on random signal (represented by red line) due to the correlation between the input and the output noise. This example shows that the closed loop identification is straightforward in the frequency domain approach, which is not the case for most of the time domain approach.

3. SIMULATION RESULTS

In this section, the FRF of a couple of systems under various condition will be evaluated. The FRF will be estimated using the measured data $u(t) = u_0(t)+n_u(t)$ and $y(t) = y_0(t) + n_y(t)$ instead of its true value $u_0(t), y_0(t)$, where $n_u(t)$ and $n_y(t)$ denote the measurement noises.

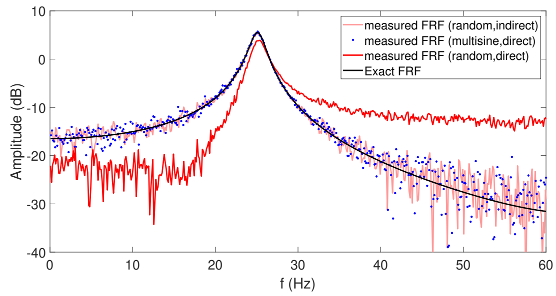

Fig. 3. Calculating the FRF under feedback condition with noise present

3.1 Open Loop System

Consider the system $\mathcal{S}_1$ depicted in Fig. 4, where G_1 is the transfer function of the target system, and N_u and N_y represent the noise filters driven by white noises n_u and n_y, repectively. These are given by

$$G_1 = \frac{9(s^2 + 5.6s + 25)(1 - 4s)}{(s^2 + 0.8s + 1)(s^2 + 0.9s + 225)} \tag{4}$$

$$N_u = \frac{121(s^2 + 8s + 64)(1 - 7s)}{16(s^2 + 0.1s + 0.25)(s^2 + 4.4s + 1936)} \tag{5}$$

$$N_y = \frac{49(s^2 + 0.9s + 225)}{225(s^2 + 1.4s + 49)(1 + 0.000001s)}. \tag{6}$$

The target system has two resonances at 1 rad/s and 15 rad/s, with the second resonance having less damping. The system also has an antiresonance at 5 rad/s, and an unstable zero at 0.25 rad/s. The noise filter N_y is set such that its antiresonance is at the same frequency as the second resonance of the system.

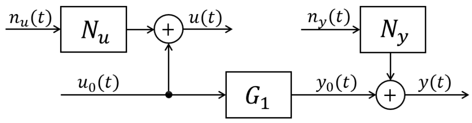

Fig. 4. Open loop system $\mathcal{S}_1$

The results when only output noise exists are shown in Figs. 5 to 7. We use M sets of subrecords and average the frequency domain data ($U(k)$, $Y(k)$) in order to reduce the noise effect. M ranges from 6 to 500. Each subrecord contains 10,000 data points with sampling frequency of 100 Hz. Figs. 5 and 6 show the estimation error of the gain plots and the FRF's phase plots, respectively. Here, the red and blue data signify the results using random excitation and periodic excitation, respectively. In Fig. 5, the true FRF plots are also shown by the lines in black for reference. Fig. 7 shows the comparison of estimation errors in the case of random excitation to evaluate the effectiveness of Local Polynomial Method (LPM, see Schoukens et al. (2018)) which is used to reduce the leakage error due to finite time length and the initial condition. The line in red represents the estimation error when LPM is not applied, and the line in blue shows the error with LPM. This figure shows that LPM reduces the estimation error.

Multi-sine signals give better results, as expected. It is observed that the complexity of the system and the noise color do not affect the modeling process. In addition to this, the following points are also important:

(1) As seen in Fig. 5, the antiresonance characteristics of the noise is completely lost for random signals, due to the leakage error, but is kept intact for periodic signal. Therefore, periodic signals can be used in noise analysis.
(2) As seen in Fig. 7, even when using LPM, high leakage error still appears at the second resonance frequency. This is because the frequency resolution is not small enough to capture the sharp resonance peak.

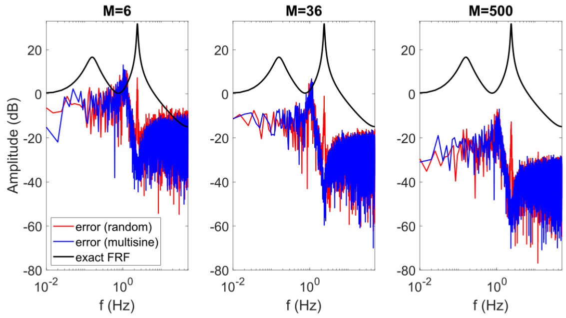

Fig. 5. Gain plots of the estimation errors of G_1 in open loop with output noise.

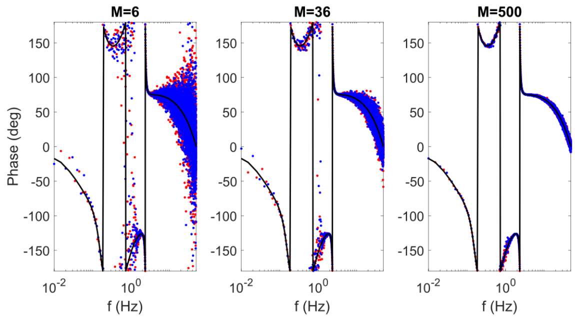

Fig. 6. Phase plots of estimated G_1 in open loop with output noise.

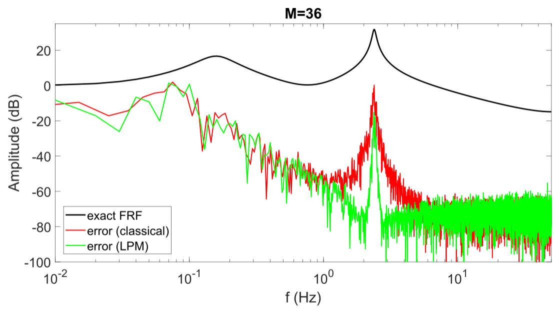

Fig. 7. Gain plots of the estimation errors of G_1 in open loop (with/without LPM)

We tested the case where both input and output noises exist. The estimated FRFs are shown in Fig. 8. The lines in red correspond to the random excitation and those in blue are related to mulit-sine excitation. The blue ones outperform the red ones again. Though the detail is omitted here, the LPM does not improve the performance very much in this case.

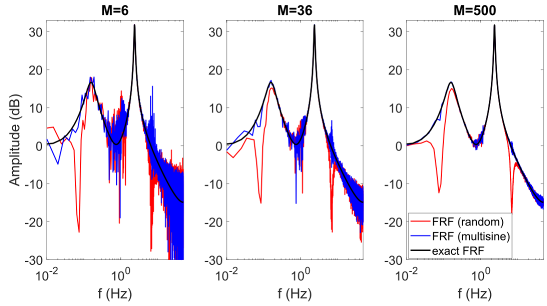

Fig. 8. Gain plots of the estimated FRF in open loop (with both input and output noises)

3.2 Closed Loop System

Consider the system $\mathcal{S}_2$ depicted in Fig. 9 with

$$G_2 = \frac{1}{s(s-1)}$$

$$C = \frac{7s+1}{s+4}$$

$$z(t) = y(t) + y^3(t)$$

where the noises n_u and n_y are white.

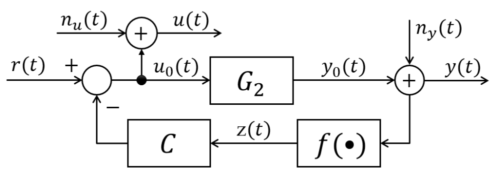

Fig. 9. Closed loop system $\mathcal{S}_2$.

The target system G_2 is an unstable system, and it is stabilized by some unknown nonlinear controller. Fig. 10 shows the gain plots of estimation errors when only the output noise exists. The line in red corresponds to random excitation, and the line in blue shows the results of multi-sine excitation. The line in black represents the true FRF for reference. Because of the measurement noises (Signal to Noise Ratio with an average of 31 dB), it can be seen from Fig. 10 that the error levels are quite similar at the higher frequency (say 2Hz or higher) in both case (random or periodic excitation). However, at the lower frequency, the inherent error caused by random excitation has stronger effect than the output noise. Hence periodic excitation outperforms clearly.

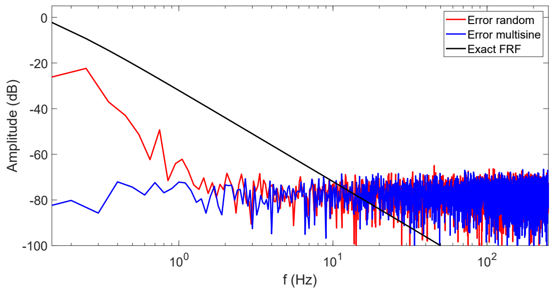

Fig. 10. Gain plots of estimation errors of G_2 in closed loop with output noise.

Fig. 11 shows the gain plots of estimation errors when both input and output noises exist. Though the difference between two cases (random/periodic excitation) become smaller, the periodic excitation still outperforms in this case.

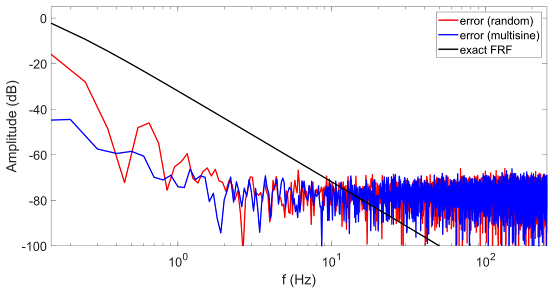

Fig. 11. Gain plots of estimation errors of G_2 in closed loop with input/output noise.

4. CONCLUSION

The nonparametric frequency domain modeling is shown to be quite effective, especially when used with periodic excitation. It provides us a good quality of FRF estimation even for an unstable system that is stabilized with nonlinear controller, and for open/closed systems in the presence of I/O colored noises. Also no special techniques are not necessary. Hence, much more attention should be paid to this frequency domain approach. On the other hand, it may be difficult in many practical cases to use multi-sine signals, and it often requires huge number of data (e.g, more than a million) in order to reduce the noise effects. One important future topic is that to develop a method which overcomes these weak points.

REFERENCES

Ljung, L. (2001). *System Identification; Theory for the user*, Second Edition, Pretice Hall

Pintelton, R. and Schoukens, J. (2001). *System identification; A frequency domain approach*, IEEE Press.

Pillonetto, G., Dinuzzo, F., Chen T., De Nicolao, G. and Ljung, L. (2014). Kernel methods in system identification, machine learning and function estimation: A survey, *Automatica*, **50**-3, pp. 657-682

Maruta, I. and Sugie, T. (2018). Stabilized prediction error method for closed loop identification of unstable systems Prep. of 18th IFAC Sympo. on System Identification, pp. 479-485

Schoukens, J., Godfrey, K., and Schoukens, M. (2018) Nonparametric data-driven modeling of linear systems: Estimating the frequency response and impulse response function, *IEEE Control Syst. Mag.*, vol. 38, pp. 49-88

Schoukens, J., Pintelon, R., and Rolain, Y. (2012) *Mastering System Identification in 100 Exercises.* Hoboken, NJ: Wiley

Pintelon, R., Schoukens, J., and Vandersteen, G. (1997). Frequency domain system identification using arbitrary signals, *IEEE Trans. Autom. Control* vol. 42, no. 12, pp. 1717 - 1720