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Port-Hamiltonian system model identification of a micro-channel ⋆

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Abstract: In this paper we identify the parameters for a finite Port-Hamiltonian system (PHS) model of a micro-channel using ordinary least squares method. The PHS model is based on the interconnection of basic finite elements equivalent to capacitors, inductance and resistors. The interconnection of several basic element models can represent the micro-channel model behavior. The parameter capacitance depends on the geometry of the channel, but the inertance, the uncontrolled and the controlled resistance are difficult to measure. The PHS based Fluid-structure interconnected system can estimate the behavior of the process as well as Partial Differential Equations (PDE) based models. The resultant model can then be used to design a controller to have a desired level at any given position in the micro-channel.

Keywords: Port-Hamiltonian model, Fluid-structure interconnected system, micro-channel, model identification, ordinary least squares.

1. INTRODUCTION

Mora et al. (2018) deduced a vocal folds model based in a vocal fluid-structure interaction compared with a mechanical structure equivalent to a mass-spring-damper system. In Cisneros et al. (2019) a micro-channel model is developed based in the interconnection of basic elements also modeled as capacitors, inductance and resistors that represents tanks, pipes and gates respectively. The interconnection of several subsystems (tank-pipe-fluid resistance) generate a good approximation to the behavior of a micro-channel. In Cisneros et al. (2019) the micro-channel fluid capacitance $\mathrm{C_f}$ (equivalent to the electrical capacitance) was found from the geometric structure of the micro-channel. The inertance I (equivalent to the inductance in an electrical circuit) is more difficult to find because physically there is no pipe . The uncontrolled resistance $\mathrm{R_f}$ related with the friction of the the walls is also difficult to obtain. Finally, the controlled resistance R_{fu} can be deduced from the sluice gate model. In order to get an accurate model that represents the dynamics of a micro-channel experimental plant, a process of identification has to be done to obtain the model parameters that are difficult to measure.

Recent works apply identification methodologies to different applications such as, Wang et al. (2015) where a modified recursive least squares algorithm is used on sparse systems (most coefficients of the impulse responses are zero or non-zero). In Valarmathi and Guruprasath (2017) various linear methods such as state space (SS) model, autoregressive with exogenous terms (ARX) model and nonlinear methods like nonlinear autoregressive exogenous (NARX) model and Hammerstein- Wiener models are applied to a MIMO process. The validation techniques used are Mean Squared Error (MSE) and Final Prediction Error (FPE). In Raafiu and Darwito (2018) ARX and autoregressive moving average exogenous (ARMAX) are used to represent a four wheel mobile robot (FWMR). In Miller (2016) a state space model identification methodology is presented. A Chirp signal is used to identify a Hammerstein model for a nonlinear process in Burrascano et al. (2017). Fixed wing lateral dynamics of an Unmanned Aerial Vehicle (UAV) model are identified using least square error estimation technique in Ahmad et al. (2015). The model of a coupled tank nonlinear Multiple-Input Multiple-Output (MIMO) system is identified in Nath et al. (2017).

Fig. 1. Experimental setup micro-channel.

The contribution of this work is to find a PHS finite micro channel model from a basic element identification. We identify two dfferent type of subsystems. The model developed by Mora et al. (2018) is used to represent the behave of the vocal folds such a fluid-structure system. In this work a similar approach to Mora et al. (2018) is

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used to represent the micro-channel. The model obtained here can be used to estimate the process variables, and differently from Mora et al. (2018), also to synthesize a control law for a desired level in a section of the micro-channel experimental plant, located at the Systems Control Laboratory, Universidad de Concepción, Chile. This paper is organized as follows: In Section 2 the notation in this paper and the basic element PHS model of the micro-channel is shown. Section 3 presents the PHS model parameters identification equations. The experimental results are shown in Section 4 . Final remarks are presented in Section 5.

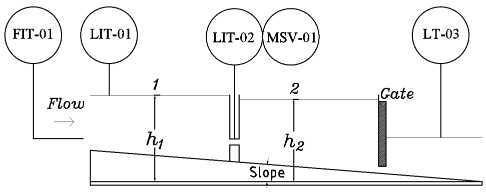


Fig. 2. Schematic representation of two subsystems

2. PRELIMINARIES

2.1 Notation

Constants are represented by non italic letters.

Table 1. Notation used in this paper.

Symbol	Description	Symbol	Description
V	Volume	R_{fu}	Controlled fluid resistance
A	Area	R_{fua}	Adjusted controlled fluid resistance
h	Height	$\mathrm{R_f}$	Uncontrolled fluid resistance
q	Flow	p_1	Pressure inside
t	Time	p_2	Pressure outside
g	Gravity	Π	Pressure momentum
$\mathrm{T_s}$	Sample period	$\mathrm{C_f}$	Fluid capacitance
ρ	Fluid density	B	Width of the channel
q_1	Input flow	α_g	Gatte flow coefficient
q_2	Output flow	γ	Model parameter
I	Fluid inertance	Φ	Regressors matrix
h_g	Opening height gate	Θ	Unknown coefficient vector
$\hat{\Theta}$	OLS estimate of Θ	k	Sample-indexing variable

2.2 PHS model of the micro-channel

The model that represents the interconnection of two groups of tanks, pipes and resistors developed in Cisneros et al. (2019) is:

$$\dot{V}_1 = q_1 - q_2 \tag{1}$$

$$\dot{\Pi}_1 = p_1 - (p_2 - p_{\Delta 1}) - \mathrm{R_{f1}} q_2 \tag{2}$$

$$\dot{V}_2 = q_2 - q_3 \tag{3}$$

$$\dot{\Pi}_2 = p_2 - p_3 - R_{fu} q_3 \tag{4}$$

$$\begin{bmatrix} \dot{V}_1 \\ \dot{\Pi}_1 \\ \dot{V}_2 \\ \dot{\Pi}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & -\mathrm{R_{f1}} & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -R_{fu} \end{bmatrix}}_{J(x)-R(x)} \underbrace{\begin{bmatrix} p_1 \\ q_2 \\ p_2 \\ q_3 \end{bmatrix}}_{\frac{\partial H}{\partial x}} + \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{g(x)} \underbrace{\begin{bmatrix} q_1 \\ p_{\Delta 1} \\ p_3 \end{bmatrix}}_{d(x)}$$
$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}}_{g^T(x)} \underbrace{\begin{bmatrix} p_1 \\ q_2 \\ p_2 \\ q_3 \end{bmatrix}}_{\frac{\partial H}{\partial x}} = \begin{bmatrix} p_1 \\ q_2 \\ -q_3 \end{bmatrix} \tag{5}$$

3. MODEL IDENTIFICATION

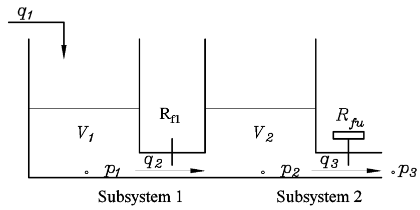


Fig. 3. Two subsystems schema

To adjust the theoretical PHS structure to the experimental setup, a parameter identification experiment has to be done.

To find the best fitting curve a mathematical procedure called Ordinary Least Squares (OLS) is used, given a set of points.

The experimental setup has three level sensors, one flow sensor and one micro speed velocimeter (MSV) developed in Alarcón et al. (2019), that is used to estimate the flow. The first sensor registers the input flow directly from the pump output, the second flow sensor is located in the middle of the section to be identified. The identification is divided in two subsystems. The first subsystem considers an estimated flow calculated from the speed measure as the output flow and it is the input flow for he second subsystem. The identification procedure considers the position of the gate as the control variable. Figure 2 indicates the sensors used in the identification and is left for a future work. The capacitance is a geometric parameter easy to verify and it is related with the longitudinal section of the channel. The slope of the micro-channel is not considered at this point in the identification. With $k = 1, 2, 3, ..., \mathrm{N}$ being N the total number of observations. The schematic representation of the identified subsystems is shown in Figure 3.

3.1 Subsystem one

The first subsystem considers one tank and one pipe. The output flow is estimated using the speed and the level measures.

$$\begin{aligned} \frac{\mathrm{I_1}}{\mathrm{T_s}}(q_2(k\mathrm{T_s} + \mathrm{T_s}) \; - q_2(k\mathrm{T_s})) &= \frac{\mathrm{A_1}}{\mathrm{C_{f1}}} h_1(k\mathrm{T_s}) \\ &- \frac{\mathrm{A_2}}{\mathrm{C_{f2}}} h_2(k\mathrm{T_s}) - \mathrm{Rf_1} q_2(k\mathrm{T_s}) \end{aligned} \tag{6}$$

Reordering:

$$\begin{aligned} h_1(k\mathrm{T_s}) = & \frac{\mathrm{C_{f1} I_1}}{\mathrm{A_1 T_s}}(q_2(k\mathrm{T_s} + \mathrm{T_s}) - q_2(k\mathrm{T_s})) \\ & + \frac{\mathrm{C_{f1} A_2}}{\mathrm{A_1 C_{f2}}} h_2(k\mathrm{T_s}) + \frac{\mathrm{C_{f1} Rf_1}}{\mathrm{A_1}} q_2(k\mathrm{T_s}) \end{aligned} \tag{7}$$

From which we have the following parameters:

$$\begin{aligned} h_1(k\mathrm{T_s}) = & \; \gamma_1(q_2(k\mathrm{T_s} + \mathrm{T_s}) - q_2(k\mathrm{T_s})) + \gamma_2 h_2(k\mathrm{T_s}) \\ & + \gamma_3 q_2(k\mathrm{T_s}) \end{aligned} \tag{8}$$

3.2 Subsystem two

In the second subsystem the output flow is estimated from the level measurements before and after the gate. To determine the fluid resistance value we consider the term $\alpha_g = 0.66$ (that is an empirical term), Hamroun (2009):

$$q_2 = \frac{\mathrm{B}h_2\sqrt{2}}{\sqrt{\rho\left(\frac{h_2^2}{\alpha_g^2 h_g^2} - 1\right)}}\sqrt{P_1 - P_2} \quad (9)$$

Considering the stationary state

$$R_{fu} = \frac{\Delta P}{q_2} \quad (10)$$

$$\frac{\mathrm{I}_2}{\mathrm{T_s}}(q_3(k\mathrm{T_s} + \mathrm{T_s}) - q_3(k\mathrm{T_s})) = \frac{\mathrm{A}_2}{\mathrm{C_{f2}}}h_2(k\mathrm{T_s}) - \mathrm{K}_1 h_3(k\mathrm{T_s}) - Rf(k\mathrm{T_s})q_3(k\mathrm{T_s}) \quad (11)$$

The final expression is:

$$h_2(k\mathrm{T_s}) = \frac{\mathrm{C_{f2}I_2}}{\mathrm{A_2T_s}}(q_3(k\mathrm{T_s} + \mathrm{T_s}) - q_3(k\mathrm{T_s})) + \frac{\mathrm{C_{f2}K_1}}{\mathrm{A_2}}h_3(k\mathrm{T_s}) + \frac{\mathrm{C_{f2}}Rf(k\mathrm{T_s})}{\mathrm{A_2}}q_2(k\mathrm{T_s}) \quad (12)$$

Replacing with the unknown parameters:

$$h_2(k\mathrm{T_s}) = \gamma_4(q_3(k\mathrm{T_s} + \mathrm{T_s}) - q_3(k\mathrm{T_s})) + \gamma_5 h_3(k\mathrm{T_s}) + \gamma_6 Rf(k\mathrm{T_s})q_2(k\mathrm{T_s}) \quad (13)$$

Table 2 resumes the unknown parameters:

Table 2. Model parameters to identify

γ_1	$\frac{\mathrm{C_{f1}I_1}}{\mathrm{A_1T_s}}$	γ_4	$\frac{\mathrm{C_{f2}I_2}}{\mathrm{A_2T_s}}$
γ_2	$\frac{\mathrm{C_{f1}A_2}}{\mathrm{A_1C_{f2}}}$	γ_5	$\frac{\mathrm{C_{f2}k_1}}{\mathrm{A_2}}$
γ_3	$\frac{\mathrm{C_{f1}Rf_1}}{\mathrm{A_1}}$	γ_6	$\frac{\mathrm{C_{f2}}}{\mathrm{A_2}}$

The parameters are estimated using (14), (15) and (16), see Norton (2009).

$$y^T = [h_1(T)\ \ldots\ h_1(NT)\ h_2(T)\ \ldots\ h_2(NT)] \quad (14)$$

$$\Theta^T = [\gamma_1\ \gamma_2\ \gamma_3\ \gamma_4\ \gamma_5\ \gamma_6\ \gamma_7] \quad (15)$$

$$\hat{\Theta} = (\Phi^T\Phi)^{-1}\Phi^T y \quad (16)$$

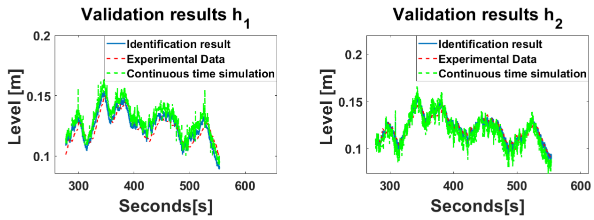
3.3 Two subsystems identification

In this experiment the input flow is $1.5l/s$. A seventh order Pseudo Random Binary Sequence (PRBS) with a period of ten minutes was applied in the gate position, varying between $0.6[cm]$ and $2.6[cm]$ from the channel floor, because these values cause a change in the micro-channel level. The PRBS design parameters such as the period and the order were taken from Alarcón et al. (2018) where an identification process was made to identify the parameters of a micro-channel. The first half of the experimental data was used to calculate the parameters , whilst the second half of data was used to validate the model. The sample period was $\mathrm{T_s} = 0.04s$. Table 3 indicates the parameters obtained from the identification method. The area and the capacitance are found directly from the geometry of the channel ($\mathrm{C_f} = \frac{\mathrm{A}}{\rho \mathrm{g}}$). The identification method gives the inertances, and the uncontrolled fluid resistance caused by the friction of the walls. R_{fu} is not really a parameter, since it obeys (10).

Table 3. Parameters obtained

γ_1	11.66	$\mathrm{A_1}$	0.1914
γ_2	0.78	$\mathrm{A_2}$	0.1914
γ_3	22.52	$\mathrm{K_1}$	9.8
γ_4	0.003	$\mathrm{C_{f1}}$	0.0195
γ_5	1.0814	$\mathrm{C_{f2}}$	0.0195
γ_6	0.9921	$\mathrm{Rf_1}$	22.52
		$\mathrm{I_1}$	4.6
		$\mathrm{I_2}$	0.0294

In Figure 4 we show the comparison between the experimental data and a simulation based on the identification results.



(a) Error h_1 (b) Error h_2

Fig. 4. Model validation using h_1 and h_2.

The MSE of each measure is shown in the Table 4.

Table 4. Mean squared error (MSE)

h_1	0.2059	h_2	1.2338

4. EXPERIMENTAL ESTIMATION

4.1 Three subsystems experiment

An estimation experiment was done using the parameters found previously. Two subsystems (tank, pipe, uncontrolled fluid resistance) and one subsystem (tank, pipe, controlled fluid resistance) interconnected in series were modelled, see Figure 5. Each subsystem has a length of $1.32m$. Figure 6 shows the model response to changes of different gate positions.

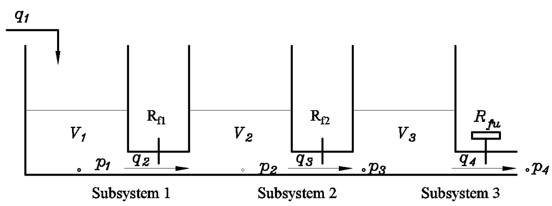


Fig. 5. Three subsystems schema

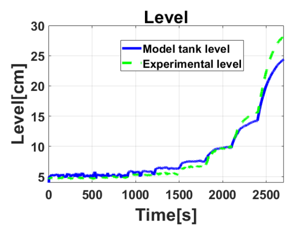


Fig. 6. Model estimation using h_1

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4.2 Fluid resistance adjustment

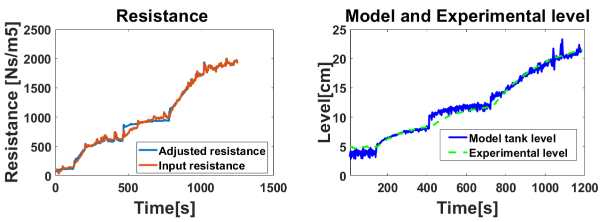
In order to improve the estimation results of the model, further the controlled fluid resistance was adjusted. The desired fluid resistance R_{fua} was compared with the fluid resistance R_{fu} obtained using (10). The method used to adjust the fluid resistance values was OLS. The structure of the equation for $R_{fua}$is a seventh order polynomial. Figure 7 shows the input fluid resistance. To validate it, an experiment was done.

Table 5 shows the adjusted resistance parameters.

$$\begin{aligned} R_{fua} = {} & \gamma_{R8}(R_{fu})^7 + \gamma_{R7}(R_{fu})^6 + \gamma_{R6}(R_{fu})^5 \\ & + \gamma_{R5}(R_{fu})^4 + \gamma_{R4}(R_{fu})^3 \gamma_{R3}(R_{fu})^2 \\ & + \gamma_{R2}(R_{fu}) + \gamma_{R1} \end{aligned} \tag{17}$$

Table 5. Adjusted resistance parameters

γ_{R1}	5.7e-18
γ_{R2}	-5.3e-14
γ_{R3}	1.6e-10
γ_{R4}	-2.1e-07
γ_{R5}	1.4e-04
γ_{R6}	-0.04
γ_{R7}	6.44
γ_{R8}	-2.6e02



(a) Fluid resistance

(b) Level with adjusted resistance

Fig. 7. Fluid resistance comparison and level with R_{fu} adjusted

The MSE obtained in the level comparison with and without the adjusted fluid resistance can be seen in Table 6.

Table 6. Mean squared error (MSE)

Without adjustment	1.3646
With adjustment	1.0692

5. CONCLUSION

In this paper the parameters model of a micro-channel were identified. The capacitance was calculated based on the geometric characteristics of the channel. The inertance and the uncontrolled resistance were identified. The resulting model was implemented using Matlab/Simulink. The estimation using three basic elements show some error between the estimated height and the experimental data. To improve the estimation the fluid resistance was further adjusted. In this work three subsystems were connected and future research will aim to interconnect more subsystems to better approximate the process and its control and analize the shortcomings of other model identification methods.

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