Presented as late breaking results contribution
21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

On Minimum Time Control for Dynamical Transportation Using ADMM

Ryosuke Adachi * **Yuji Wakasa** * **Koichi Kobayashi** **

* *Graduate School of Sciences and Technology for Innovation, Yamaguchi University, Ube 755-8611, Japan (e-mail: {r-adachi,wakasa}@yamaguchi-u.ac.jp)*
** *Graduate School of Information Science and Technology, Hokkaido University, Sapporo 060-0814, Japan (e-mail: k-kobaya@ssi.ist.hokudai.ac.jp)*

Abstract: In this paper, a dynamical transportation problem over the graph is discussed. Over the graph, multiple agents transport their assets to goals. In transportation, all agents share capacities of nodes and edges. The dynamical transportation problems are formulated as an optimization problem such that the total transportation time is minimized by using finite optimal control problems. The optimization problem is dispersively solved by using the alternating direction method of multipliers.

Keywords: Dynamical Transportation, Distributed Optimization, Optimal Control

1. INTRODUCTION

Internet of Things (IoT) is a concept which means that anything can be connected via the Internet. In an IoT society, a large amount of information is collected from physical space into cyberspace. The collected information is then analyzed to create added value. Finally, these results are utilized in physical space to provide smarter social services, for example, smart grids and smart cities (Cassandras (2016)). An intelligent transport system (ITS) is one of these smart systems in which IoT technologies will improve transport efficiency and comfortableness. In the ITS, traffic conditions are updated in real-time, and drivers can make rational decisions based on that information. As a result, we can expect that traffic congestion is resolved.

Due to the above background, many works have focused on analysis and control of traffic flow of the transportation network, for example, Pham et al. (2019); Dotoli and Fanti (2006); De Nunzio et al. (2016); Daganzo (1994, 1995); Moore et al. (2004); Como et al. (2012); Coogan and Arcak (2014); Ba et al. (2015); Hilliges and Weidlich (1995); Davidsson et al. (2005). In these works, some transportation models, for example, Petri net and a cell transmission model (CTM), are proposed to represent the transportation dynamics. Based on these models, the past works have aimed to minimize the global cost of the whole network while satisfying some constraints such as speed limitation and traffic capacity.

However, these works do not consider the problem that multiple agents move their assets to different destinations in minimum time. In actual transportation systems, many agents have different destinations. One of the most common objectives of these agents is to move the assets to the destination in minimum time. In this case, the cooperation of all agents is difficult because the objectives of agents conflict. Therefore, to reconcile these agents, we require an aggregator. The aggregator leads all agents into minimizing global costs. In the process of minimizing global cost, Each agent communicates only with the aggregator. The aggregator is prohibited from providing the information of each agent to the other ones.

To satisfy the above specifications for the aggregator, we solve the dynamical transportation problem over graphs based on a distributed optimization algorithm. Following Pham et al. (2019), the dynamical transportation network in this work is modeled by mathematical graph and CTM. The dynamics of the assets of each node are defined by CTM. Increments and decrements of these amounts which are associated with each node are defined by in-flow and out-flow, respectively. To design a schedule such that the agent transports all assets to the destination in minimum time, we utilize a linear objective function. Then, the problems of this work result in finite-time optimal control problems. By introducing some new variables, these problems are transformed into separable problems in the sense of an alternating direction method of multipliers (ADMM).

2. PROBLEM FORMULATION

A mathematical model of the network is given by a connected directed graph $\mathcal{G} := (\mathcal{V}, \mathcal{E})$. The set $\mathcal{V} := \{1, \dots, n\}$ represents a node set whose elements are segments of a route. A set of edges are defined by $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, where $(i,j) \in \mathcal{E}$ indicates that each agent can transport own asset from i-th to j-th node. The cardinality of $\mathcal{E}$ is given by m. We define $\mathcal{O}_i := \{\ j\ : (i,j) \in \mathcal{E}\}$ as set of out-flow neighbors and $\mathcal{I}_i := \{\ j\ : (j,i) \in \mathcal{E}\}$ as set of in-flow neighbors.

Let us consider the agents that have the assets in the nodes of $\mathcal{G}$. The set of agents are defined as $\mathcal{A} := \{1, \dots, N\}$. The

Presented as late breaking results contribution
21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

agent $i \in \mathcal{A}$ has the assets $x^i[k] := [x_1^i[k], \ldots, x_n^i[k]]^\top \in \mathbb{R}_+^n$, where $x_j^i[k] \in \mathbb{R}_+$ indicates an amount of the asset in node $j \in \mathcal{V}$ at time k.

Each agent transports own assets along the edges of $\mathcal{G}$. A transportation amount on the edge $(j, l) \in \mathcal{E}$ by the agent $i \in \mathcal{A}$ at time k is defined as $u_{(j,l)}^i[k] \in \mathbb{R}_+$. Based on this transportation amount, in-flow and out-flow in j-th node by the agent i at time k are respectively expressed by $f_j^i[k] = \sum_{l \in \mathcal{I}_j} u_{(l,j)}^i[k]$ and $g_j^i[k] = \sum_{l \in \mathcal{O}_j} u_{(j,l)}^i[k]$. The above is summarized in dynamics with respect to $x_j^i[k]$ as follow:

$$\begin{aligned} x_j^i[k+1] =& x_j^i[k] + f_j^i[k] - g_j^i[k] \\ =& x_j^i[k] + \sum_{l \in \mathcal{I}_j} u_{(l,j)}^i[k] - \sum_{l \in \mathcal{O}_j} u_{(j,l)}^i[k]. \end{aligned} \tag{1}$$

Because the out-flow is smaller than the amount of the asset which is stored in a source,

$$x_j^i[k] \geq g_j^i[k] = \sum_{l \in \mathcal{O}_j} u_{(j,l)}^i[k] \tag{2}$$

is required. From (1) and (2), the dynamics with respect to $x^i[k]$ is expressed by

$$\begin{aligned} x^i[k+1] &= x^i[k] + (B_{\text{in}} - B_{\text{out}}) u^i[k] \\ x^i[k] &\geq B_{\text{out}} u^i[k], \end{aligned} \tag{3}$$

where $B = B_{\text{in}} - B_{\text{out}}$ is an adjacency matrix of $\mathcal{G}$ and $B_{\text{in}} \geq 0$ and $B_{\text{out}} \geq 0$ are satisfied.

Let us consider capacities of the nodes and the edges on the graph $\mathcal{G}$. Assume that the node can not store the assets more than $b_x \in \mathbb{R}_+^n$, and the transportation amounts of the edges per one discrete-time are limited by $b_u \in \mathbb{R}_+^m$. These are the constraints with respect to all agents and can be formulated as:

$$\sum_{i=1}^N x^i[k] \leq b_x, \tag{4}$$

$$\sum_{i=1}^N u^i[k] \leq b_u. \tag{5}$$

For the system, initial conditions $x^i[k]$ and a set of collection nodes $\mathcal{V}_g^i \subseteq \mathcal{V}$ for all $i \in \mathcal{A}$ are given. The objective of the agent i is to transport the all assets to the collection nodes, which means that $\lim_{\tau \to \infty} x_j^i[k+\tau] = 0$ for all $j \in \mathcal{V} \setminus \mathcal{V}_g^i$. If the agent i accomplished this objective, we call "The agent i completed the transportation." In this paper, we find the transportation schedule which minimizes the time such that the all agents complete the transportation. To simplify the problem, we consider the case that $\mathcal{A} = \{1\}$. Then, the problem is summarized as follows.

Problem 1. For given $x^1[k]$, $\mathcal{G}$, $\mathcal{V}_g^1$, (4) and (5), find $u^1[k], \ldots, u^1[k+\tau]$ and minimum τ such that $x_j^1[k+\tau] = 0$ for all $j \in \mathcal{V} \setminus \mathcal{V}_g^1$.

Next, we consider the general problem. Due to the capacity of the nodes and edges, the minimization of the transportation time of each agent is not satisfied at the same time. Therefore, to reconcile the agent and realize the minimization of transportation time for all agents, we consider a distributed optimization problem. In the distributed optimization, the agent i calculates a candidate of the schedule $X^i[k]_t := [x^i[k+1]_t^\top, \ldots, x^i[k+\tau]_t^\top]^\top$ and $U^i[k]_t := [u^i[k]_t^\top, \ldots, u^i[k+\tau-1]_t^\top]^\top$. To satisfy constraints (4) and (5), we introduce an aggregator. The aggregator collects the candidate schedule and returns amounts of correction $Z^i[k]_t := [z^i[k]_t^\top, \ldots, z^i[k+\tau]_t^\top]^\top$. Based on the amounts of correction, each node updates its own schedule. The problem in which we find the agent's schedule from the iteration of the above is summarized as follows.

Problem 2. For given $x^i[k]$, $\mathcal{G}$, $\mathcal{V}_g^i$, (4) and (5), find minimum τ and the algorithm

$$\begin{aligned} &(X^i[k]_{t+1}, U^i[k]_{t+1}) = f_i(Z^i[k]_t), \\ &(Z^1[k]_{t+1}, \ldots, Z^N[k]_{t+1}) = g(X^1[k]_t, \ldots, X^N[k]_t, \\ &\qquad\qquad\qquad\qquad\qquad U^1[k]_t, \ldots, U^N[k]_t) \end{aligned}$$

such that $x_j^i[k+\tau]_t = 0$ for all $j \in \mathcal{V} \setminus \mathcal{V}_g^1$ as $t \to \infty$.

3. PROPOSED METHOD

3.1 Single-Agent Case

In this subsection, we consider the transportation problem when $N = 1$. To simplify the notation, we omit the index which indicates the agent. A predictive horizon for the finite-time optimal control is defined by T. Note that to obtain the solution of Problem 1, T should be sufficiently large because $T \geq \tau$ is required. The extended state $X[k]$ which include all $x[\bar{k}], \bar{k} \in \{k+1, \ldots, k+T\}$ are defined by

$$X[k] := [x[k+1]^\top, x[k+2]^\top, \ldots, x[k+T]^\top]^\top \in \mathbb{R}_+^{nT}.$$

In the above way, an extended input are defined by

$$U[k] = [u[k]^\top, u[k+1]^\top, \ldots, u[k+T-1]^\top]^\top \in \mathbb{R}_+^{mT}.$$

Following the dynamics (1), $X[k]$ and $U[k]$ are expressed by

$$\begin{aligned} X[k] &= \bar{A}X[k] + \bar{B}U[k] + X_0[k] \\ \bar{A}X[k] &- \bar{B}_{\text{out}} \geq X_0[k], \end{aligned} \tag{6}$$

where $\bar{A}$, $\bar{B}$, $\bar{B}_{\text{out}}$ and $X_0[k]$ are given by

$$\begin{aligned} \bar{A} &= \begin{pmatrix} 0 & \cdots & & 0 \\ I_n & & & \\ & \ddots & & \vdots \\ & & I_n & 0 \end{pmatrix} \\ \bar{B} &= I_T \otimes (B_{\text{in}} - B_{\text{out}}) \\ \bar{B}_{\text{out}} &= I_T \otimes B_{\text{out}} \\ X_0[k] &= [x[k]^\top, 0^\top, \ldots, 0^\top]^\top. \end{aligned}$$

To find $U[k]$ which is a solution of Problem 1, a linear objective function $V(X[k])$ is introduced as

$$V(X[k]) := \bar{c}^\top X[k] = \sum_{j=1}^T v(x[k+j]) \tag{7}$$

$$v(x[k+j]) = c^\top x[k+j] \tag{8}$$

where

$$\bar{c} = 1_T \otimes c\ ,\ [c]_i = \begin{cases} 0 & \text{if } i \in \mathcal{V}_g \\ 1 & \text{otherwise} \end{cases}.$$

A total amount of the asset $1_T^\top x[k+k']$ for all $k' \in \{1, \ldots, T\}$ is time-invariant. Therefore, $v(x[k+k']) = 0$

if and only if the agent completes the transportation, and we can obtain the following equation:

$$c^\top x[k+k'] - 1_T^\top x[k+k'] = \begin{cases} 0 & \text{if } x_i[k+k']=0 \text{ for all } i \in \mathcal{V}_g \\ \text{negative values} & \text{otherwise} \end{cases}.$$

From above discussion, to decrease the objective function (7), the agent should move the assets to the collection nodes. Therefore, we can find $U[k]$ which is a solution of Problem (9) from the following optimization problem:

$$\begin{aligned} \text{given} \quad & X_0[k] \\ \text{find} \quad & X[k], U[k] \\ \text{minimize} \quad & V(X[k]) \\ \text{subject to} \quad & X[k] = \bar{A}X[k] + \bar{B}U[k] + X_0[k] \\ & \bar{A}X[k] - \bar{B}_{\text{out}}U[k] \geq X_0[k] \\ & 0 \leq X[k] \leq \bar{b}_x \ , \ 0 \leq U[k] \leq \bar{b}_u \end{aligned} \tag{9}$$

where $\bar{b}_x$ and $\bar{b}_u$ is given by

$$\bar{b}_x = 1_T \otimes b_x \text{ and } \bar{b}_u = 1_T \otimes b_u.$$

The above is summarized by the following Theorem.

Theorem 1. From the solution of (9), we obtain $U[k]$ and the minimum τ such that

$$\sum_{i \in \mathcal{V}\setminus \mathrm{V}_g} x_i[k+\tau] = c^\top x[k+\tau] = 0.$$

Proof 1. An exact proof is omitted in this paper.

3.2 Multi-Agent Case

In this subsection, let us formulate the optimization problem in which agents transport their assets on the shared graph. Based on (7), the objective function which is optimized by the agent i is expressed by

$$V^i(X^i[k]) := \bar{c}_i^\top X^i[k]$$

$$\bar{c}_i = 1_T \otimes c_i \ , \ [c_i]_j = \begin{cases} 0 & \text{if } j \in \mathcal{V}_g^i \\ 1 & \text{otherwise} \end{cases}.$$

Based on $V^i(X^i[k])$, a global objective function is defined by

$$V(X^1[k], \ldots, X^N[k]) := \sum_{i=1}^N V^i(X^i[k]).$$

Therefore, the optimization problem for the agents to minimize a transportation time is formulated as follows:

$$\begin{aligned} \text{given} \quad & X_0^1[k], \ldots, X_0^N[k] & \text{(10a)} \\ \text{find} \quad & X^1[k], \ldots, X^N[k], U^1[k], \ldots, U^N[k] & \text{(10b)} \\ \text{minimize} \quad & V(X^1[k], \ldots, X^N[k]) & \text{(10c)} \\ \text{subject to} \quad & X^i[k] = \bar{A}X^i[k] + \bar{B}U^i[k] + X_0^i[k] & \text{(10d)} \\ & \bar{A}X^i[k] - \bar{B}_{\text{out}}U^i[k] \geq X_0^i[k] & \text{(10e)} \\ & 0 \leq X^i[k], \ 0 \leq U^i[k] & \text{(10f)} \\ & \sum_{i=1}^N X^i[k] \leq \bar{b}_x \ , \ \sum_{i=1}^N U^i[k] \leq \bar{b}_u & \text{(10g)} \end{aligned}$$

We dispersively solve (10) by using the ADMM. To utilize the ADMM, we transform (10) to a separable formulation. First, we introduce indicator functions to reduce the constrains from (10d) to (10g). Let us define the set $\mathcal{C}^i[k]$ which indicates the constrains (10d), (10e) and (10f) as

$$\begin{aligned} \mathcal{C}^i[k] := \{(X^i, U^i) \ : \ & X^i \geq 0 \ , \ U^i \geq 0, \\ & \bar{A}X^i - \bar{B}_{\text{out}}U^i \geq X_0^i[k] \\ & X^i = \bar{A}X^i + \bar{B}U^i + X_0^i[k]\}. \end{aligned}$$

This is independent constraints among agents. By using $\mathcal{C}_i[k]$, the indicator function is defined by

$$h^i(X^i[k], U^i[k]) := \begin{cases} 0 & \text{if } (X^i[k], U^i[k]) \in \mathcal{C}^i[k] \\ \infty & \text{otherwise} \end{cases}$$

On the other hand, the indicator functions which are associated with (10g) are defined by

$$h_x(X^1[k], \ldots, X^N[k]) := \begin{cases} 0 & \text{if } \sum_{i=1}^N X^i[k] \leq \bar{b}_x \\ \infty & \text{otherwise} \end{cases},$$

$$h_u(U^1[k], \ldots, U^N[k]) := \begin{cases} 0 & \text{if } \sum_{i=1}^N U^i[k] \leq \bar{b}_x \\ \infty & \text{otherwise} \end{cases}.$$

To solve (10) by using the ADMM, we introduce slack variables $Z_x^i[k]$ and $Z_u^i[k]$ such that

$$X^i[k] = Z_x^i[k] \ , \ U^i[k] = Z_u^i[k].$$

Based on the indicator functions and the slack variables, (10) can be transformed to the following problem:

$$\begin{aligned} \text{given} \quad & X_0^1[k], \ldots, X_0^N[k] \\ \text{find} \quad & \bar{X}[k], \bar{U}[k], \bar{Z}_x[k], \bar{Z}_u[k] \\ \text{minimize} \quad & \bar{V}(\bar{X}[k], \bar{U}[k], \bar{Z}_x[k], \bar{Z}_u[k]) \\ \text{subject to} \quad & \bar{X}[k] = \bar{Z}_x[k] \ , \ \bar{U}[k] = \bar{Z}_u[k] \end{aligned} \tag{11}$$

where $\bar{X}[k], \bar{U}[k], \bar{Z}_x[k], \bar{Z}_u[k]$ and $\bar{V}$ are given by

$$\begin{aligned} \bar{X}[k] &:= [X^1[k]^\top, \ldots, X^N[k]^\top]^\top, \\ \bar{U}[k] &:= [U^1[k]^\top, \ldots, Y^N[k]^\top]^\top, \\ \bar{Z}_x[k] &:= [Z_x^1[k]^\top, \ldots, Z_x^N[k]^\top]^\top, \\ \bar{Z}_u[k] &:= [Z_u^1[k]^\top, \ldots, Z_u^N[k]^\top]^\top, \end{aligned}$$

and

$$\begin{aligned} & \bar{V}(\bar{X}[k], \bar{U}[k], \bar{Z}_x[k], \bar{Z}_u[k]) \\ & := \sum_{i=1}^N (V^i(X^i[k]) + h^i(X^i[k], U^i[k])) \\ & + h_x(Z_x^1[k], \ldots, Z_x^N[k]) \\ & + h_u(Z_u^1[k], \ldots, Z_u^N[k]). \end{aligned}$$

To solve the optimization problems by using the ADMM, an augmented Lagrangian function is required. Let us define $Y_x^i[k] \in \mathbb{R}^{nT}$ and $Y_u^i[k] \in \mathbb{R}^{mT}$ as dual variables. By using $Y_x^i[k]$ and $Y_u^i[k]$, $\bar{Y}_x[k]$ and $\bar{Y}_u[k]$ are defined by

$$\begin{aligned} \bar{Y}_x[k] &:= [Y_x^1[k]^\top, \ldots, Y_x^N[k]^\top]^\top, \\ \bar{Y}_u[k] &:= [Y_u^1[k]^\top, \ldots, Y_u^N[k]^\top]^\top. \end{aligned}$$

The augmented Lagrangian function which is associated with (11) is given by

$$\begin{aligned} & L_\rho(\bar{X}[k], \bar{U}[k], \bar{Z}_x[k], \bar{Z}_u[k], \bar{Y}_x[k], \bar{Y}_u[k]) := \\ & \sum_{i=1}^N L_\rho^i(X^i[k], U^i[k], Z_x^i[k], Z_u^i[k], Y_x^i[k], Y_u^i[k]) \\ & + h_x(\bar{Z}_x[k]) + h_u(\bar{Z}_u[k]), \end{aligned} \tag{12}$$

where augmented Lagrangian function of the agent i is expressed by

$$\begin{aligned} &L_\rho^i(X^i[k], U^i[k], Z_x^i[k], Z_u^i[k], Y_x^i[k], Y_u^i[k]) \\ &:= V^i(X^i[k]) \\ &+ \frac{\rho}{2}\|X^i[k] - Z_x^i[k] + Y_x^i[k]\|^2 \\ &+ \frac{\rho}{2}\|U^i[k] - Z_u^i[k] + Y_u^i[k]\|^2 \end{aligned} \tag{13}$$

and $\rho > 0$ is a penalty parameter.

Based on the augmented Lagrangian function, we divide an ADMM based iteration algorithm to find a solution of (11). Candidate solutions of $X^i[k]$, $U^i[k]$, $Z_x^i[k]$, $Z_u^i[k]$, $Y_x^i[k]$ and $Y_u^i[k]$ are defined by $X^i[k]_t$, $U^i[k]_t$, $Z_x^i[k]_t$, $Z_u^i[k]_t$, $Y_x^i[k]_t$ and $Y_u^i[k]_t$, respectively. Due to the formulation of (12), each agent can update the state $X^i[k]_t$, the input $U^i[k]_t$ and the dual variables $Y_x^i[k]$ and $Y_u^i[k]$. On the other hand, the slack variables $Z_x^i[k]_t$ and $Z_u^i[k]_t$ should be updated by the aggregator.

Each agent receives $Z_x^i[k]_t$ and $Z_u^i[k]_t$ from the aggregator, and updates $X^i[k]_t$ and $U^i[k]_t$ as

$$\begin{aligned} &(X^i[k]_{t+1}, U^i[k]_{t+1}) = \\ &\arg\min_{X,U} L_\rho^i(X, U, Z_x^i[k]_t, Z_u^i[k]_t, Y_x^i[k]_t, Y_u^i[k]_t). \end{aligned} \tag{14}$$

Therefore, the update law (14) results in the following quadratic programming:

$$\begin{aligned} \text{given} \quad & X_0^i[k], Z_x^i[k]_t, Z_u^i[k]_t, Y_x^i[k]_t, Y_u^i[k]_t, \rho \\ \text{find} \quad & X^i[k]_{t+1}, U^i[k]_{t+1} \\ \text{minimize} \quad & \bar{c}_i^\top X^i[k]_{t+1} + \frac{\rho}{2}\|X^i[k]_{t+1} - Z_x^i[k]_t + Y_x^i[k]_t\|^2 \\ & + \frac{\rho}{2}\|U^i[k]_{t+1} - Z_u^i[k]_t + Y_u^i[k]_t\|^2 \\ \text{subject to} \quad & X^i[k]_{t+1} = \bar{A}X^i[k]_{t+1} + \bar{B}U^i[k]_{t+1} + X_0^i[k] \\ & \bar{A}X^i[k]_{t+1} - \bar{B}_{\text{out}}U^i[k]_{t+1} \geq X_0^i[k] \\ & 0 \leq X^i[k]_{t+1}, \ 0 \leq U^i[k]_{t+1} \end{aligned} \tag{15}$$

The update law of the dual variables are given by

$$\begin{aligned} Y_x^i[k]_{t+1} &= Y_x^i[k]_t + X^i[k]_{t+1} - Z_x^i[k]_t, \\ Y_u^i[k]_{t+1} &= Y_u^i[k]_t + U^i[k]_{t+1} - Z_u^i[k]_t. \end{aligned} \tag{16}$$

The aggregator collects $X[k]_{t+1}$, $U[k]_{t+1}$, $Y_x[k]_{t+1}$ and $Y_u[k]_{t+1}$ from the all agents. From these values, the slack variables are updated as follows:

$$\begin{aligned} &(Z_x[k]_{t+1}, Z_u[k]_{t+1}) = \\ &\arg\min_{Z_x, Z_u} L_\rho^i(X[k]_{t+1}, U[k]_{t+1}, Z_x, Z_u, \\ &\qquad Y_x[k]_{t+1}, Y_u[k]_{t+1}) \end{aligned} \tag{17}$$

Therefore, the update law (17) results in the following quadratic programming:

$$\begin{aligned} \text{given} \quad & \bar{X}[k]_{t+1}, \bar{U}[k]_{t+1}, \bar{Y}_x[k]_{t+1}, \bar{Y}_u[k]_{t+1} \\ \text{find} \quad & \bar{Z}_x[k]_{t+1}, \bar{Z}_u[k]_{t+1} \\ \text{minimize} \quad & \|\bar{X}[k]_{t+1} - \bar{Z}_x[k]_{t+1} + \bar{Y}_x[k]_{t+1}\|^2 \\ & + \|\bar{U}[k]_{t+1} - \bar{Z}_u[k]_{t+1} + \bar{Y}_u[k]_{t+1}\|^2 \\ \text{subject to} \quad & \sum_{i=1}^N Z_x^i[k]_{t+1} \leq \bar{b}_x \ , \ \sum_{i=1}^N Z_u^i[k]_{t+1} \leq \bar{b}_u \end{aligned} \tag{18}$$

By introducing the dual and slack variables, the dynamical transportation problems with minimum time (10) is transformed into the problem that we can solve by using the ADMM (11). Threfore, from the iteration of (15), (16) and (18), we can obtain the optimal solution $X_*^i[k]$ and $U_*^i[k]$ of (10) as

$$\lim_{t\to\infty} X^i[k]_t = X_*^i[k] \text{ and } \lim_{t\to\infty} U^i[k]_t = U_*^i[k].$$

4. CONCLUSION

In this paper, a dynamical transportation over a graph has been solved as a finite-time optimal control problem. To minimize the transportation times, a linear objective function has been utilized. When multi-agents transport the asset on the shared graph, the optimization problem has been dispersively solved by the ADMM.

REFERENCES

Ba, Q., Savla, K., and Como, G. (2015). Distributed optimal equilibrium selection for traffic flow over networks. In *2015 54th IEEE Conference on Decision and Control (CDC)*, 6942–6947. IEEE.

Cassandras, C.G. (2016). Smart cities as cyber-physical social systems. *Engineering*, 2(2), 156–158.

Como, G., Savla, K., Acemoglu, D., Dahleh, M.A., and Frazzoli, E. (2012). Robust distributed routing in dynamical networks–part ii: Strong resilience, equilibrium selection and cascaded failures. *IEEE Transactions on Automatic Control*, 58(2), 333–348.

Coogan, S. and Arcak, M. (2014). Dynamical properties of a compartmental model for traffic networks. In *2014 American Control Conference*, 2511–2516. IEEE.

Daganzo, C.F. (1994). The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research Part B: Methodological*, 28(4), 269–287.

Daganzo, C.F. (1995). The cell transmission model, part ii: network traffic. *Transportation Research Part B: Methodological*, 29(2), 79–93.

Davidsson, P., Henesey, L., Ramstedt, L., Törnquist, J., and Wernstedt, F. (2005). An analysis of agent-based approaches to transport logistics. *Transportation Research part C: emerging technologies*, 13(4), 255–271.

De Nunzio, G., De Wit, C.C., Moulin, P., and Di Domenico, D. (2016). Eco-driving in urban traffic networks using traffic signals information. *International Journal of Robust and Nonlinear Control*, 26(6), 1307–1324.

Dotoli, M. and Fanti, M.P. (2006). An urban traffic network model via coloured timed petri nets. *Control Engineering Practice*, 14(10), 1213–1229.

Hilliges, M. and Weidlich, W. (1995). A phenomenological model for dynamic traffic flow in networks. *Transportation Research Part B: Methodological*, 29(6), 407–431.

Moore, D., Leonard, J., Rus, D., and Teller, S. (2004). Robust distributed network localization with noisy range measurements. In *Proceedings of the 2nd international conference on Embedded networked sensor systems*, 50–61. ACM.

Pham, V.H., Sakurama, K., and Ahn, H.S. (2019). A decentralized control strategy for urban traffic network. In *2019 12th Asian Control Conference (ASCC)*, 1–6. IEEE.