

# Fast extremum-seeking control with self-tuning dilution rate amplitude for biogas production in anaerobic bioreactors <sup>★</sup>

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**Abstract:** The paper presents a new fast extremum-seeking control (FESC) strategy, based on the FESC proposed by Ramírez-Carmona et al. (2018). The objective of this new controller is to decrease the oscillation of the output/flowrate of biogas by updating the values of upper and lower bounds. In order to numerically validate the strategy, we consider the methanisation of organic matter with the objective to maximize the production of biogas in anaerobic bioreactors. Simulations show that the new approach reaches the optimal productivity regions with smaller variance if compared with the FESC proposed by Ramírez-Carmona et al. (2018). In consequence of that, the process reaches larger productivity, it produces 13% more biogas than the conventional fast extremum-seeking control.

*Keywords:* anaerobic digestion model, biogas, extremum-seeking, output feedback, productivity.

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## 1. INTRODUCTION

Anaerobic digestion processes are responsible for the conversion of organic matter to biogas, which is mainly composed of methane and carbon dioxide. In this process, a microorganism consortium degrades the organic matter in liquid phase by several interconnected biochemical reactions. Anaerobic digestion is not only a depollution tool, but also a strategy to produce renewable energy. One challenging aspect of this process is to find and maintain the process in a region of the maximum conversion of the substrate into biogas. This adversity is related to intrinsic characteristics of the process, since anaerobic biodegradation corresponds to a nonlinear time-varying system, with potentially unmodelled dynamics. To increase the complexity of the process operation, there are inhibition effects due to high concentrations of volatile fatty-acids and a lack of reliable sensors for some process concentrations.

Normally, we can classify the control strategies to maximize productivity in two groups: (i) model based control strategies as in Cougnon et al. (2011) or Caraman et al. (2017) and (ii) model free strategies as in Vargas et al. (2019) or Ramírez-Carmona et al. (2018). One of these strategies, which stands out, is the extremum-seeking control that is an adaptive control strategy. This controller is based on perturbing the process and observing its response, and is vastly used in the optimization of uncertain and varying systems. Extremum-seeking control (ESC) can be applied to many different systems, as in maximum

power point tracking of photovoltaic systems, optimization of bioethanol production and maximization of wind farm power (Reisi et al., 2013; Brunton et al., 2010; Dewasme et al., 2015; Ghaffari et al., 2014).

Note that in many cases, model-based strategies suffer from the time scale separation, required to guarantee stability properties for the closed-loop system. Some approaches try to overcome this issue with state measurement techniques and parameter estimation. In the model-free approaches, the only requirement is that the system is stable in open-loop. Even if it is not the case, adaptations can be done, as for example adding state feedback stabilizing controllers (Wang et al., 1999).

The main drawback of the ‘classical’ ESC is that the time of convergence to the maximum is much slower than the time constant of the system. In order to overcome this issue, the fast extremum-seeking control (FESC) was proposed by Ramírez-Carmona et al. (2018). It presents the optimization of biogas productivity based on the switching of the dilution rate. It uses a simple model with two states. Vargas et al. (2019) extended this approach to a model with four states (acidogenesis and methanisation) and Moreno (2019) presented a review for switching signals applied to optimization bioprocess.

The contribution of this paper is an algorithm to reduce output oscillations caused by the switching of the dilution rate by self-tuning the maximum and minimum values of the dilution rate bounds. The control strategy is able

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to track the optimal operating point, while reducing the amplitude variations in the output.

The paper has the following structure. Section 2 presents the Anaerobic Digestion Model 2 (AM2) as the used mathematical model. Section 3 proposes and explains the proposed fast extremum-seeking algorithm and Section 4 presents simulations of the algorithm using the AM2 model. Section 5 concludes the paper.

## 2. MATHEMATICAL MODEL

The Anaerobic Digestion Model (AM2) proposed by Bernard et al. (2001) is a macro model, which considers the methanisation of organic waste composed by two steps: acidogenesis and methanogenesis. The components are the acidogenic biomass  $X_1$ , the soluble substrate  $S_1$  (mainly simple carbohydrates), methanogenesis biomass  $X_2$ , and volatile fatty acids (VFA)  $S_2$ . Also, the methanisation conversion is considered fast enough such that  $CH_4$  can be measured instantaneously in the flowrate  $q_m$ .

The two step reaction model is represented as follows:

$$\begin{cases} \dot{X}_1 = (\mu_1(S_1) - \alpha D)X_1 \\ \dot{S}_1 = D(S_1^{in} - S_1) - k_1\mu_1(S_1)X_1 \\ \dot{X}_2 = (\mu_2(S_2) - \alpha D)X_2 \\ \dot{S}_2 = D(S_2^{in} - S_2) + k_2\mu_1(S_1)X_1 - k_3\mu_2(S_2)X_2 \end{cases} \quad (1)$$

$$q_m = k_4\mu_2(S_2)X_2, \quad (2)$$

where  $\alpha \in [0, 1]$  is the factor that considers partial biomass retention in the reactor ( $\alpha = 1$  implies a perfectly mixed reactor). The specific reaction rates are Monod type for acidogenesis and Haldane type for methanogenesis

$$\mu_1(S_1) = \frac{\mu_{m1}S_1}{K_{S1} + S_1}, \quad (3)$$

$$\mu_2(S_2) = \frac{\mu_{m2}S_2}{K_{S2} + S_2 + \left(\frac{S_2}{K_{I2}}\right)^2}. \quad (4)$$

The control input  $D$  is the dilution rate, a quantity related to the amount of volume flowing in and out of the bioreactor, injecting substrate concentrations  $S_1^{in}$  and  $S_2^{in}$ . It can be verified that constant values of  $D$  along with constant parameters and input concentrations produce operation points with equilibrium states. The objective is to make the system achieve and maintain the equilibrium point in which the biogas production  $q_m$  is maximized.

Note that in practice this optimal operation point is unknown, since it depends on the reaction parameters and input concentrations. Furthermore, the biological reactions are known to be complex and influenced by environmental aspects such as temperature and pH, to mention few. These obstacles make an adaptive approach needed when dealing with the practical bioreactor operation.

Vargas et al. (2019) proposed a switching control law using high and low values of the dilution rate to achieve near optimal steady-state operation. However, switching between high and low values may produce high oscillations in the output and high demand from the actuators. The next section presents an algorithm whose main aspect is to

estimate the optimal dilution rate value and modify high and low values towards this optimal estimate.

## 3. CONTROLLER AND ALGORITHM

The new fast extremum-seeking algorithm is based on the algorithm proposed by Ramírez-Carmona et al. (2018), which considers values of the output discretely at  $t = kT_s$ , where  $T_s$  is the sample time. Given the data  $q_m[k]$ ,  $q_m[k-1]$  and  $D[k-1]$ , the dilution rate  $D[k]$  is modified as follows:

**Algorithm 1** FESC: Fast extremum-seeking algorithm proposed by Ramírez-Carmona et al. (2018).

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1: function FESC( $q_m[k]$ ,  $q_m[k-1]$ ,  $D[k]$ )
2:   if ( $q_m[k] - q_m[k-1]$ )  $\leq$  0 and  $D[k-1] = D_{hi}$  then
3:      $D[k] = D_{lo}$ 
4:   else if ( $q_m[k] - q_m[k-1]$ )  $\leq$  0 and  $D[k-1] = D_{lo}$  then
5:      $D[k] = D_{hi}$ 
6:   else
7:      $D[k] = D[k-1]$ 
8:   end if
9: end function

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Algorithm 1 is very easy to implement and has its mathematical proof of stability and its implementation in a simple bioreactor and a methanisation bioreactor explained in Ramírez-Carmona et al. (2018) and Vargas et al. (2019), respectively. The main concept is to switch between  $D_{hi}$  and  $D_{lo}$  every time that the biogas flowrate  $q_m[k]$  decreases, keeping the system in an optimal region. One point that is not straightforward is to obtain adequate values of  $D_{hi}$  and  $D_{lo}$ . Depending on the values, the process will not converge to the optimum or will have large oscillations around this point. The proposed Algorithm 2, denoted here AFESC (adaptive FESC), self-tunes the amplitude of the dilution rate bounds  $D_{hi}$  and  $D_{lo}$ , which results in smaller oscillations in the biogas production if compared with the FESC Algorithm 1.

The objective of Algorithm 2 is to maximize the biogas flowrate given the data  $q_m[k]$ ,  $q_m[k-1]$  and  $D[k-1]$ . Every instant that the biogas flowrate decays (line 2), an estimate of the optimum dilution rate  $\hat{D}^*[k]$  is computed based on the previous areas and periods of  $D_{hi}$  and  $D_{lo}$  (line 3), i.e. an average of past values of  $D[k]$ . Line 4 tests if the new estimate of the optimum  $\hat{D}^*[k]$  is inside the optimum region defined by the previous estimate  $\hat{D}^*[k-1]$  and the parameters  $\rho_1 < 1$  and  $\rho_2 > 1$ . If  $\hat{D}^*[k]$  is in the defined region,  $D_{hi}$  and  $D_{lo}$  start to converge to  $\hat{D}^*[k]$  with the rate  $\rho_3$  and  $\rho_4$ , computed in lines 5 and 6. If the value  $\hat{D}^*[k]$  is out of the bounds (line 7), the limits are recomputed to converge to the original  $D_{hi}[0]$  and  $D_{lo}[0]$  with rate  $\rho_5$  and  $\rho_6$  (lines 8 and 9). From lines 11 to 15, the procedure is exactly the same as the algorithm proposed by Ramírez-Carmona et al. (2018). Every instant that the biogas flowrate decays, the dilution rate changes from  $D_{hi}$  to  $D_{lo}$  and viceversa, in order to track the optimum value of the biogas production.

As the process is time varying, if the  $q_m[k]$  keeps more than  $N = 3$  switching events without decreasing,  $D_{hi}$  and  $D_{lo}$  converge with a rate of  $\rho_7$  and  $\rho_8$  to  $D_{hi}[0]$  or  $D_{lo}[0]$  (lines 19 to 27).

**Algorithm 2** AFESC: Fast extremum-seeking with self-tuning amplitude

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1: function AFESC( $q_m[k], q_m[k-1], D[k]$ )
2:   if ( $q_m[k] - q_m[k-1] < 0$ ) then
3:      $\hat{D}^*[k] = \frac{(A_{hi} + A_{lo})}{(T_{hi} + T_{lo})}$ 
4:     if  $\rho_1 \hat{D}^*[k-1] \leq \hat{D}^*[k] \leq \rho_2 \hat{D}^*[k-1]$  then
5:        $D_{hi}[k] = (D_{hi}[k-1] + \rho_3 \hat{D}^*[k])/2$ 
6:        $D_{lo}[k] = (D_{lo}[k-1] + \rho_4 \hat{D}^*[k])/2$ 
7:     else
8:        $D_{hi}[k] = (1 - \rho_5)D_{hi}[k-1] + \rho_5 D_{hi}[0]$ 
9:        $D_{lo}[k] = (1 - \rho_6)D_{lo}[k-1] + \rho_6 D_{lo}[0]$ 
10:    end if
11:    if  $D[k-1] = D_{hi}[k-1]$  then
12:       $D[k] = D_{hi}[k]$ 
13:    else if  $D[k-1] = D_{lo}[k-1]$  then
14:       $D[k] = D_{lo}[k]$ 
15:    end if
16:  else
17:     $D[k] = D[k-1]$ 
18:     $\hat{D}^*[k] = \hat{D}^*[k-1]$ 
19:    counter = counter + 1
20:    if counter=N then
21:      reset counter
22:      if  $D[k] = D_{hi}[k-1]$  then
23:         $D_{hi}[k] = (1 - \rho_7)D_{hi}[k-1] + \rho_7 D_{hi}[0]$ 
24:      else
25:         $D_{lo}[k] = (1 - \rho_8)D_{lo}[k-1] + \rho_8 D_{lo}[0]$ 
26:      end if
27:    end if
28:  end if
29: end function

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Table 1. Parameters for the AM2 model (left) and for the new FESC algorithm (right).

Parameter	Value	Parameter	Value
$k_1$	12.1	$\rho_1$	0.9
$k_2$	181.2 mmol/g	$\rho_2$	1.1
$k_3$	181.2 mmol/g	$\rho_3$	1.1
$k_4$	1 mmol/g	$\rho_4$	0.9
$\alpha$	0.5	$\rho_5$	0.1
$\mu_{m1}$	1.25 d <sup>-1</sup>	$\rho_6$	0.2
$\mu_{m2}$	0.85 d <sup>-1</sup>	$\rho_7$	0.2
$K_{S1}$	7.65 g/L	$\rho_8$	0.2
$K_{S2}$	18 mmol/L	$N$	3
$S_1^{in}$	10 g/L	$T_s$	1 d
$S_2^{in}$	10 mmol/L		

#### 4. SIMULATION AND RESULTS

In order to test the algorithm in a realistic scenario, a routine to simulate a varying optimal equilibrium point is set as follows. From day 0 to day 70 the inhibition parameter  $K_{I2}$  is set to 5. From day 70 to day 110 this parameter is changed as a ramp from 5 to 3.5 with a slope of  $-0.037$ , remaining constant until day 150. From 150 to 205 it follows a ramp from 3.5 to 8 with a slope of  $+0.09$ , remaining constant with  $K_{I2} = 8$  until day 250. The initial conditions considered are  $X_1(0) = 0.5$ ,  $X_2(0) = 0.5$ ,  $S_1(0) = 10$ ,  $S_2(0) = 10$ ,  $D_{hi}[0] = 0.75$ ,  $D_{lo}[0] = 0.02$ . Also, the same parameters and conditions are used in a simulation for Algorithm 1, in order to compare both results.

The system and control parameters used in the simulation are listed on Table 1. The maximal attainable values for  $q_m$  in steady state for the  $K_{I2}(t)$  considered were numerically

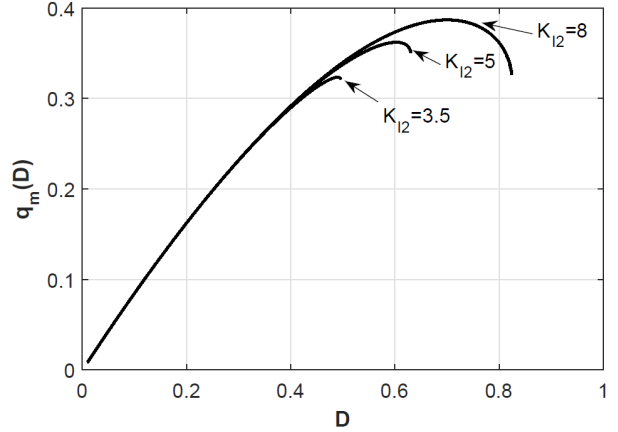


Fig. 1. Equilibrium map for the three values of  $K_{I2} = 3.5, 5$  and  $8$ .

computed based on model (2) and are presented in the equilibrium map on Figure 1.

Figure 2 presents the simulation results for the new algorithm: in the first column of plots for the AFESC Algorithm 2 and in the second column for the FESC Algorithm 1. The top row of plots present the biogas flowrate (output) and the bottom row the dilution rate (input). Both simulations use the same parameters and values of  $D_{hi}[0]$  and  $D_{lo}[0]$ . Analyzing the first column, as the simulation begins, the oscillations produced by the switching are noticeable, but as soon as the estimates are sufficiently constant –in the region defined by line 4 in Algorithm 2–, the upper and lower bounds converge to the optimal estimated dilution rate  $\hat{D}^*$ , which reduces the oscillations in the process output  $q_m$ . Note that until day 15, Figures 2(a) and 2(b) have exactly the same behavior and the same happens to Figures 2(c) and 2(d). When  $D_{hi}$  and  $D_{lo}$  are the same for both strategies, the dynamic behaviors are the same. The parameter  $K_{I2}$  changes as previously explained, which shows that the values of  $D_{hi}[k]$  and  $D_{lo}[k]$  follow the optimal estimate  $\hat{D}^*$  (dashed line). In contrast to that, in the second column of plots of Figure 2, a large oscillation occurs in the flowrate values with constant  $D_{hi}$  and  $D_{lo}$  as proposed in Algorithm 1. Note that both strategies find the optimum region of biogas production, but the variance in the biogas flowrates are quite different. The smaller variability in the output results in higher productivity, in this case with the improvement of 13%, from a volume of 76.93 to 86.91 L of biogas.

#### 5. CONCLUSION

This study presents a variation of the fast extremum-seeking control (FESC), proposed by Ramírez-Carmona et al. (2018). In this new approach the upper and lower bounds of the switching input are updated based on an estimation of the optimum dilution rate  $\hat{D}^*$ . This results in a flowrate of biogas with smaller variation and improves even more the productivity of the process. As future studies the authors are interested in finding a systematic way to compute the algorithm parameters and a simple method to find the values for  $D_{hi}[0]$  and  $D_{lo}[0]$ , as to mathematically prove the stability of this new algorithm.

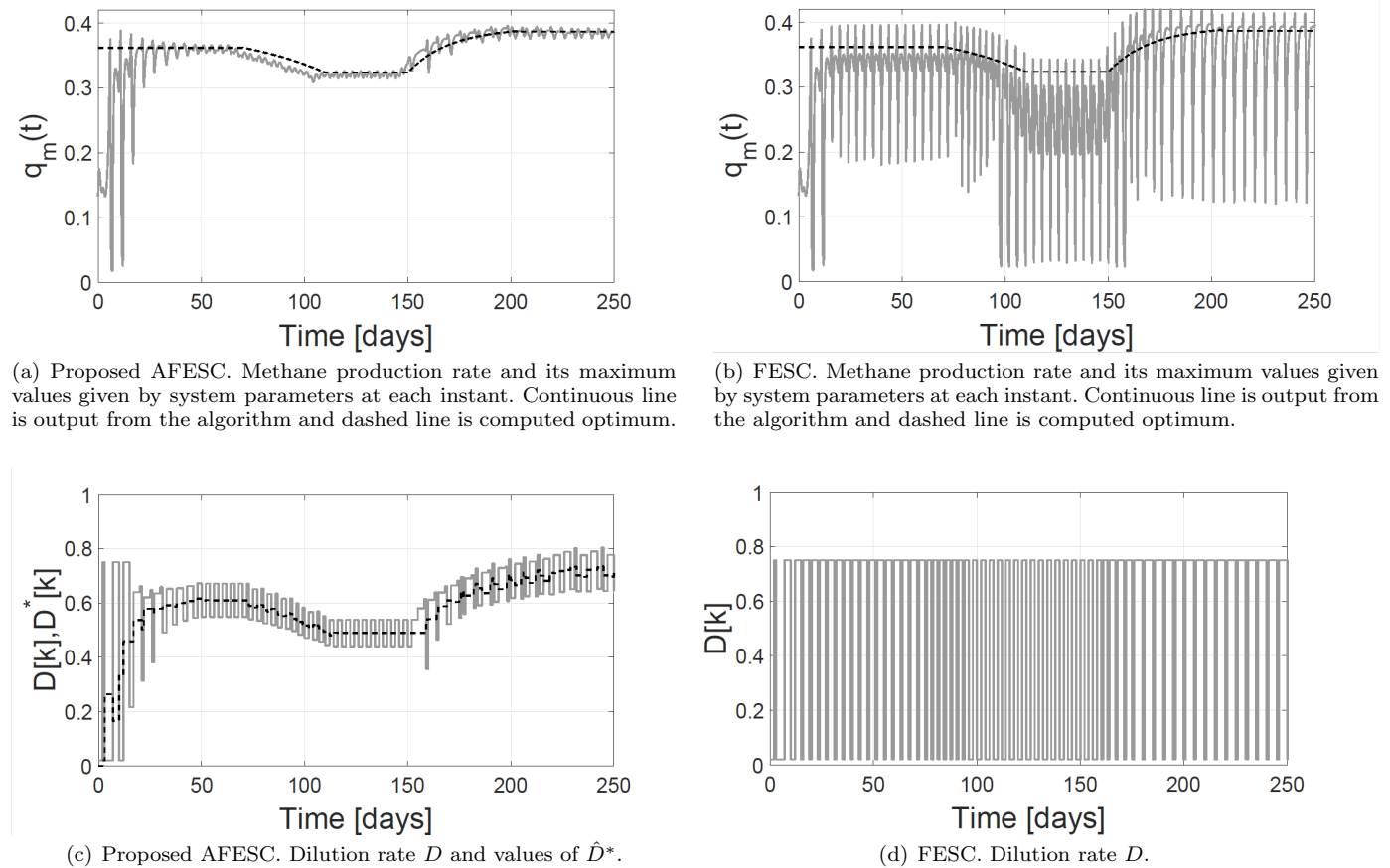


Fig. 2. Simulation comparison between the proposed AFESC and the FESC by Vargas et al. (2019).

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