Presented as late breaking results contribution
21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

On the Passivity of Clustered District Heating Subsystems ⋆

Juan E. Machado * **Michele Cucuzzella** *
Jacquelien M.A. Scherpen *

* *Faculty of Science and Engineering, University of Groningen, Nijenborgh 4, 9747 AG Groningen, the Netherlands (e-mail: { j.e.machado.martinez, m.cucuzzella, j.m.a.scherpen} @rug.nl).*

Abstract: We present in this brief note a nonlinear model of a cluster of district heating subsystems, each of which is conformed by a local producer, a storage tank, and a local consumer. We explicitly provide storage functions and adequately chosen outputs from which we establish, under certain assumptions, that such a system is shifted passive, opening the possibility for decentralized and distributed passivity-based control design.

Keywords: District Heating Systems (DHSs), Thermal Storage, Shifted Passivity

1. INTRODUCTION

1.1 Motivation

Heat represents an important proportion of the total energy demand in urbanized areas. Such heat demand is supplied through Heat Exchanger Networks (HENs), for the case of industrial facilities, and District Heating Systems (DHSs), for residential buildings. In both settings, the primary resources for heat production are usually fossil fuels, notably, natural gas. However, environmental concerns are pushing—as for electrical power systems—the gradual introduction of Renewable Energy Sources (RESs), *e.g.*, geothermal and solar thermal, for heat production. [1]

The use of RESs as primary resources for heat production introduce challenging constraints in the operation and control of DHSs. In this setting, the control system in charge of the balance between production and demand has to deal with the uncertainties associated to both the RESs and the heat demand profile: a heat production-demand imbalance may give rise to energetic inefficiency in the heat distribution network and poor quality of service to end-users. The introduction of (thermal) storage devices in heating networks has been shown to be an effective asset for a correct production-demand balance by allowing any excess of heat to be stored for later use, *e.g.*, during the peak of demand; nonetheless, a key feature of heating networks, particularly of DHSs, is that they can extend through large geographical areas, imposing severe constraints for the correct implementation of centralized controller architectures, which motivates the exploration of both decentralized and distributed control approaches.

⋆ This work was funded by Innovation Fund Denmark (IFD) and the Dutch Organization for Scientific Research (NWO) through TOP-UP, project number 91176.

[1] Other alternative primary resources are: residual heat from industries or data centers and, prominently, from electrical power production, in what is known as a Combined Heat and Power (CHP) scheme.

A promising perspective to simultaneously augment energy efficiency levels and increase operational flexibility is the interconnection between initially isolated DH subsystems, to form, through a network of distribution pipes, a *cluster* of DHSs, in a clear analogy with interconnected distributed power systems, *e.g.*, microgrids. In the present text we focus particularly on modeling and establishing passivity properties for such clustered DHSs.

1.2 Literature overview

In De Persis and Kallesoe (2011) is presented, along with a detailed modeling of the underlying hydraulic network of a DHS, a solution to the problem of practical end-user pressure regulation; these results were extended in De Persis et al. (2014) to achieve exact pressure regulation via decentralized PI controllers, and more recently in Scholten et al. (2017) to consider the case of constrained input variables. On the other hand, simultaneous temperature and storage tank volume regulation has been addressed in Scholten et al. (2015) for a DHS comprising a single producer, single storage tank, and many consumers. In Trip et al. (2019)—now for a cluster of DH subsystems—the problem of storage tank water volume regulation is addressed via distributed controllers, guaranteeing the minimization of a suitable cost function and simultaneously satisfying both input and flow transient constraints.

1.3 Contributions

As an extension, if modest, to the above described volume of research, we propose in this brief manuscript a *nonlinear* model of a cluster of district heating subsystems in which temperature and storage tank volume dynamics are simultaneously considered. Furthermore, we provide conditions to ensure shifted passivity with respect to explicitly identified storage function and passive output, which given the nonlinear nature of the model, proves to be a non trivial task.

Passivity concepts have shown to be instrumental for the distributed control of electrical DC microgrids, where features such as fair load sharing—which basically prevents the over-stressing of distributed generation units De Persis et al. (2018)—can be achieved whilst preserving system stability Cucuzzella et al. (2019). For, we believe that our contribution is particularly relevant to attain similar features in the operation of clustered district heating networks.

1.4 Organization of the manuscript

The organization of the document is as follows. In Section 2 we present, after reviewing its main components, a nonlinear model of a cluster of DH subsystems. Our main result, which provides conditions for *shifted passivity* of the system model, appears in Section 3. The manuscript is concluded with Section 4 where a number of final remarks and future perspectives are given.

1.5 Notation

$\mathbb{R}$ denotes the set of real numbers. For a vector $x \in \mathbb{R}^n$, x_i denotes its i-th component. We also write vectors as $\mathrm{col}(x_i)_i^n = [x_1 \ x_2 \ \cdots \ x_n]^\top$. An $m \times n$ matrix with all-zero entries is written as $0_{m\times n}$. An n-vector of ones is written as 1_n, whereas the identity matrix of size n is represented by I_n. $\Lambda(x_i)_{i=1}^n$ is a diagonal matrix with elements x_i in its main diagonal and $\mathrm{block.diag}(A_i)_{i=1}^n$ as a block-diagonal matrix for which the block-diagonal submatrices are A_i.

2. SYSTEM MODEL

We consider a *cluster* of n district heating subsystems that are interconnected by a distribution piping network. Each node of the cluster represents a district heating subsystem itself, which comprises a local producer, a consumer and a stratified storage tank. The topology of each node is depicted in Fig. 1, where we have highlighted with colors and arrows the temperatures and directions of the water flow; in the same figure we show the variables associated with the thermal layer of the system, the physical meaning of which are explained in Table 1. [2] On the other hand, the hydraulic layer of the nodes and the distribution piping network is illustrated, for a two node district heating system, in Fig. 2; the meaning of the hydraulic layer's variables is presented in Table 2.

2.1 Conforming elements

We describe in detail the basic elements conforming each district heating subsystem. The thermal layer is modeled after Scholten et al. (2015) (see also Scholten (2017)), whereas the hydraulic layer is partially modeled after De Persis and Kallesoe (2011) (see also De Persis et al. (2014), Scholten et al. (2017)).

Producer: The producer can directly deliver a thermal power injection P_i^p into the network through a heat exchanger. The heat exchanger is assumed to maintain a constant volume of water V_i^p in its interior, moreover, the water flow through it is denoted by q_i^p, with an inflow and outflow temperature of $T_i^{S_c}$ and T_i^p, respectively.

[2] Please see Remark 2 for an explanation about the units of the power injection/extraction.

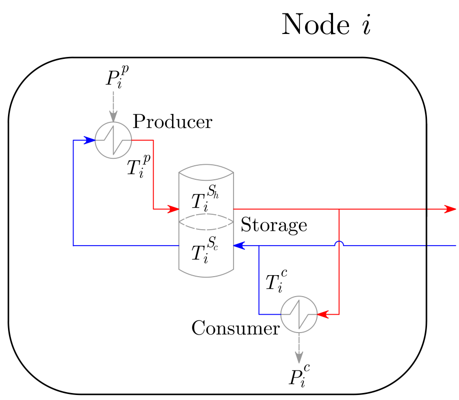

Fig. 1. Topology of a node in the district heating cluster; *cf.* (Scholten et al., 2015, Fig. 1), (Trip et al., 2019, Fig. 3b).

Storage Tank: The tank stores a mixture of hot and cold water. The device has four valves, two at the top and two at the bottom, which are used as inlets or outlets of hot or cold water.

Assumption 1. The volumes of hot and cold water in the storage tank are separated via a thermocline, with the hot water on top and the cold water at the bottom, and without heat exchange between them. Moreover, the storage tank is perfectly isolated and consequently does not have thermal losses through its walls. In addition, the capacity of the storage tank is denoted by $V_i^{\max}$ and is assumed to be always completely filled with water, *i.e.*, the following holds.

$$V_i^{S_h} + V_i^{S_c} = V_i^{\max} \text{ for all } t \geq 0 \tag{1a}$$

$$V_i^{S_h}(0), V_i^{S_c}(0) \in [V_i^{\min}, V_i^{\max} - V_i^{\min}] =: \mathcal{I}_i, \tag{1b}$$

where $0 < 2V_i^{\min} \leq V_i^{\max} < \infty$.

The volumes, temperatures of the hot and cold layers of the storage tank are denoted by $V_i^{S_h}$, T^{S_h} and $V_i^{S_c}$, T^{S_c}, respectively.

Consumer: Analogously to a producer, a consumer can directly extract thermal energy from the system at rate

Table 1. Symbols used in the thermal layer of the system.

Symbol	Description	Unit
T_i^p	Temperature of the producer	$^\circ$C
P_i^p	Power injection of the producer	$\frac{\mathrm{m}^3\,^\circ\mathrm{C}}{\mathrm{s}}$
$T_i^{S_h}$	Temperature of the hot layer of the storage tank	$^\circ$C
$T_i^{S_c}$	Temperature of the cold layer of the storage tank	$^\circ$C
T_i^c	Temperature of the consumer	$^\circ$C
P_i^c	Power demand of the consumer	$\frac{\mathrm{m}^3\,^\circ\mathrm{C}}{\mathrm{s}}$

Table 2. Symbols used in the hydraulic layer of the system.

Symbol	Description	Unit
q_i^p	Flow of the producer	m^3/s
$V_i^{S_h}$	Volume of the hot layer of the storage tank	m^3
$V_i^{S_c}$	Volume of the cold layer of the storage tank	m^3
q_i^c	Flow of the consumer	m^3/s
$q_k^{\tau,h}$	Flow of hot water through the k-th distribution pipe	m^3/s
$q_k^{\tau,c}$	Flow of cold water through the k-th distribution pipe	m^3/s

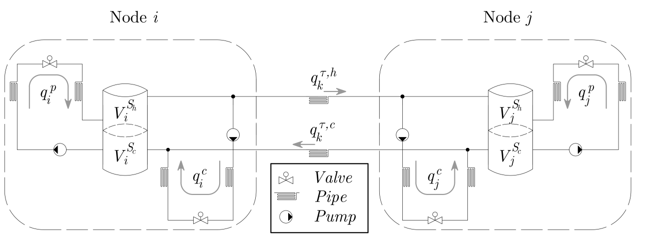

Fig. 2. Hydraulic layer of a two node district heating system. Although valves and pumps are included in this diagram, their effect is not considered our model.

P_i^c via a heat exchanger. The flow through the exchanger is denoted by q_i^c and has an outflow temperature of T_i^c; the exchanger is also assumed to maintain a constant volume of water V_i^c in its interior.

Remark 1. The power demand P_i^c of the consumer is regarded in our manuscript as a *constant* disturbance. This contrasts with the more complex model taken in Scholten et al. (2015), where P_i^c as a linear combination of constant and sinusoidal signals.

2.2 Aggregated Model

The topology of the network is described by a connected, directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, ..., n\}$ is the set of nodes of the cluster and $\mathcal{E} = \{1, 2, ..., m\}$ is the set of edges connecting the nodes, which physically represent hydraulic pipelines. The topology is codified through its corresponding node-to-edge incidende matrix $\mathcal{B} \in \mathbb{R}^{n\times m}$, defined by

$$\mathcal{B}_{ik} = \begin{cases} +1 & \text{if the edge } k \text{ targets node } i, \\ \text{-1} & \text{if the edge } k \text{ originates in node } i, \\ 0 & \text{otherwise.} \end{cases}$$

The interconnection between the nodes of the cluster is established through a piping network as we depict in Fig. 2 for a two-node district heating system: each link consists of two independent pipes for hot and cold water flow, respectively; water flow through the pipes is denoted by $q_k^{\tau,h}$ and $q_k^{\tau,c}$, $k \in \mathcal{E}$.

Assumption 2. Every flow $q_k^{\tau,h}$ and $q_k^{\tau,c}$, $k \in \mathcal{E}$, is an independent control input. However, we consider that if $q_k^{\tau,h}$ is the hot water flow from i to j, then $q_k^{\tau,c} \equiv q_k^{\tau,h}$ would be the cold water flow from j to i. In addition, the flow directions are taken according to edges' directions, which are codified by the incidence matrix.

Then we can formulate, on the one hand, the energy balance equations for the producer, storage tank, and consumer, for each $i \in \mathcal{V}$, which respectively read as

$$V_i^p \dot{T}_i^p = q_i^p \left(T_i^{S_c} - T_i^p\right) + P_i^p, \tag{2a}$$

$$V_i^{S_h} \dot{T}_i^{S_h} = q_i^p \left(T_i^p - T_i^{S_h}\right) + \sum_{k\in\mathcal{N}_i^h} q_k^{\tau,h}(T_j^{S_h} - T_i^{S_h}), \tag{2b}$$

$$V_i^{S_c} \dot{T}_i^{S_c} = q_i^c \left(T_i^c - T_i^{S_c}\right) + \sum_{k\in\mathcal{N}_i^c} q_k^{\tau,c}(T_j^{S_c} - T_i^{S_c}), \tag{2c}$$

$$V_i^c \dot{T}_i^c = q_i^c \left(T_i^{S_h} - T_i^c\right) - P_i^c, \tag{2d}$$

where $\mathcal{N}_i^h$, $\mathcal{N}_i^c$ is the set of hot, cold water distribution pipes targeting node $i \in \mathcal{V}$, respectively.

On the other hand, the volume (mass) balance at the hot and cold layer of the i-th storage tank read, respectively as

$$\dot{V}_i^{S_h} = q_i^p - q_i^c + \sum_{k\in\mathcal{N}_i} q_k^{\tau,h}, \tag{3a}$$

$$\dot{V}_i^{S_c} = q_i^c - q_i^p - \sum_{k\in\mathcal{N}_i} q_k^{\tau,c}, \tag{3b}$$

where $\mathcal{N}_i$ is the set of edges incident to node i. [3]

Remark 2. In a clear abuse of notation, we refer to the quantities P_i^p and P_i^c as (thermal) powers, however, both quantities should appear with the coefficient $\frac{1}{C_p\rho}$, where C_p and ρ are the water's specific heat and density, respectively. Nonetheless, such omission does not affect our results.

Remark 3. In virtue of Assumption 2, from (3) we get that

$$\dot{V}_i^{S_h} + \dot{V}_i^{S_c} = 0 \;\Rightarrow\; V_i^{S_h}(t) + V_i^{S_c}(t) = V_i^{\max},\ \forall t \geq 0$$

for all $i \in \mathcal{V}$, provided $V_i^{S_h}(0) + V_i^{S_c}(0) = V_i^{\max}$, which holds by Assumption 1. Since V^{S_c} becomes a variable dependent on V^{S_h}, in the sequel we discard the second equation in (3). In addition, the variable $V_i^{S_c}$ appearing in (2) is substituted by

$$V_i^{S_c} = V_i^{\max} - V_i^{S_h},\ \forall i \in \mathcal{V}.$$

2.3 System Model in Matrix Form

Let $T^p = \mathrm{col}(T_i^p)$, $T^{S_h} = \mathrm{col}(T_i^{S_h})$, $T^{S_c} = \mathrm{col}(T_i^{S_c})$ and $T^c = \mathrm{col}(T_i^c)$. In addition, let $V^{S_h} = \mathrm{col}\left(V_i^{S_h}\right)$ and $V^{S_c} = \mathrm{col}\left(V_i^{S_c}\right)$. We define the *state vector* as $X = \mathrm{col}(x, z)$, where

$$x = \begin{bmatrix} T^p \\ T^{S_h} \\ T^{S_c} \\ T^c \end{bmatrix} \in \mathbb{R}^{4n}, \quad z = \mathrm{col}\left(V_i^{S_h}\right) \in \mathbb{R}^n. \tag{4}$$

On the other hand, the vector of *control inputs* is written as $U = \mathrm{col}(u, v^p, v^c, w)$, where

$$u = \mathrm{col}(P_i^p) \in \mathbb{R}^n, \quad v^p = \mathrm{col}\left(q_i^p\right) \in \mathbb{R}^n, \tag{5}$$

$$v^c = \mathrm{col}\left(q_i^c\right) \in \mathbb{R}^n, \quad w = \mathrm{col}\left(q_k^\tau\right) \in \mathbb{R}^m, \tag{6}$$

Furthermore, the effect of the consumers' power demand is codified by the *constant* vector

$$d = \left[0_n^\top\ 0_n^\top\ 0_n^\top\ -\mathrm{col}\left(P_i^c\right)^\top\right]^\top \in \mathbb{R}^{4n}. \tag{7}$$

Using the above definitions, the systems (2) and (3) can be compactly represented as follows.

$$M(z)\dot{x} = -A(v^p, v^c, w)x + Bu + d, \tag{8a}$$

$$\dot{z} = v^p - v^c + \mathcal{B}w, \tag{8b}$$

with matrices

$$M(z) = \mathrm{block.diag}\left(\Lambda(V^p),\ \Lambda(z),\ \Lambda(V^{\max} - z),\ \Lambda(V^c)\right), \tag{9}$$

where $V^{\max} = \mathrm{col}\left(V_i^{\max}\right)$. Also,

$$A = \begin{bmatrix} \Lambda(v^p) & 0_{n\times n} & -\Lambda(v^p) & 0_{n\times n} \\ -\Lambda(v^p) & \Lambda(v^p) + \mathcal{B}_+\Lambda(w)\mathcal{B}^\top & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} & \Lambda(v^c) + \mathcal{B}_-\Lambda(w)\mathcal{B}^\top & -\Lambda(v^c) \\ 0_{n\times n} & -\Lambda(v^c) & 0_{n\times n} & \Lambda(v^c) \end{bmatrix} \tag{10}$$

[3] In the sequel we simply write $q_k^\tau := q_k^{\tau,h} = q_k^{\tau,c}$ (see Assumption 2).

where $\mathcal{B}_+$ and $\mathcal{B}_-$ are matrices with the positive and negative entries of $\mathcal{B}$, respectively. In addition,

$$B = \begin{bmatrix} I_n \\ 0_{3n\times n} \end{bmatrix} \tag{11}$$

The state and input spaces of system (8) are taken, respectively, as

$$\mathcal{X} = \{(x, z) \in \mathbb{R}^{5n}_{>0}\} \tag{12}$$

and

$$\mathcal{U} = \{(u, v^p, v^c, w) \in \mathbb{R}^{3n+m}_{>0}\}. \tag{13}$$

Remark 4. The matrix $A(v^p, v^c, w)$ is linear with respect to its arguments. Then, there exist matrices $A_{p,i}$, $A_{c,i}$ and $A_{\tau,i}$ such that

$$A(v, w) = \sum_{i=1}^{n} v_i^p A_{p,i} + \sum_{i=1}^{n} v_i^c A_{c,i} + \sum_{i=1}^{m} w_i A_{\tau,i},$$

which, due to space constraints, are not shown here.

3. PASSIVITY PROPERTIES

We present in this section our main result, which establishes conditions for the shifted passivity of the model (8). Consider first the following standing assumption.

Assumption 3. There exists an equilibrium tuple

$$(\bar{X}, \bar{U}) \in \mathcal{X} \times \mathcal{U}$$

for the system (8).

We are in position now to present our main result.

Proposition 1. Consider the system (8) and let $(\bar{X}, \bar{U})$ denote an equilibrium tuple. Define

$$\mathcal{S}(x, z) = \frac{1}{2}(x - \bar{x})^\top M(z)(x - \bar{x}) + \frac{1}{2}(z - \bar{z})^\top (z - \bar{z})$$

and the mappings

$$\mathcal{F}_z(x, \bar{x}) = \tfrac{1}{2}\left[(x - \bar{x})^\top \mathcal{D}_1 (x - \bar{x}) \ \cdots \ (x - \bar{x})^\top \mathcal{D}_n (x - \bar{x})\right]^\top,$$

$$\mathcal{F}_p(x, \bar{x}) = \left[(x - \bar{x})^\top A_{p,1} x \ \cdots \ (x - \bar{x})^\top A_{p,n} x\right]^\top,$$

$$\mathcal{F}_c(x, \bar{x}) = \left[(x - \bar{x})^\top A_{c,1} x \ \cdots \ (x - \bar{x})^\top A_{c,n} x\right]^\top,$$

$$\mathcal{F}_\tau(x, \bar{x}) = \left[(x - \bar{x})^\top A_{\tau,1} x \ \cdots \ (x - \bar{x})^\top A_{\tau,m} x\right]^\top,$$

where

$$\mathcal{D}_i = \text{block.diag}\left(0_{n\times n}, e_i e_i^\top, -e_i e_i^\top, 0_{n\times n}\right).$$

Assume that $(\bar{v}^p, \bar{v}^c, \bar{w})$ is such that

$$\text{symm}\{A(\bar{v}^p, \bar{v}^c, \bar{w})\} \geq 0.$$

Then, along trajectories of (8), the following holds.

$$\dot{\mathcal{S}} \leq \begin{bmatrix} u - \bar{u} \\ v^p - \bar{v}^p \\ v^c - \bar{v}^c \\ w - \bar{w} \end{bmatrix}^\top \begin{bmatrix} B^\top (x - \bar{x}) \\ \mathcal{F}_z(x, \bar{x}) + (z - \bar{z}) - \mathcal{F}_p(x, \bar{x}) \\ -\mathcal{F}_z(x, \bar{x}) - (z - \bar{z}) - \mathcal{F}_c(x, \bar{x}) \\ \mathcal{B}^\top \left(\mathcal{F}_z(x, \bar{x}) + (z - \bar{z})\right) - \mathcal{F}_\tau(x, \bar{x}) \end{bmatrix}.$$

Consequently, the system is shifted passive with storage function $\mathcal{S}$.

In the following corollary, which follows directly from Proposition 1, we consider the case in which the flows in the piping system and the volumes in both layers of every storage tank are fixed at their respective stationary states, *i.e.*, the system (8b) is not taken into account. [4]

[4] This setting is meaningful when there is a distinct time scale separation between the dynamics of the hydraulic (fast) and thermal (slow) parts of the system.

Corollary 1. Consider that V^{S_h}, V^{S_c} are constant and in satisfaction of Assumption 1. Moreover, assume the tuple $(\bar{v}^p, \bar{v}^c, \bar{w})$ is such that $\text{symm}\{A(\bar{v}^p, \bar{v}^p, \bar{w})\} \geq 0$. Then, the system (8a) is shifted passive with storage function

$$x \mapsto \frac{1}{2}(x - \bar{x})^\top M(z)(x - \bar{x})$$

and shifted output

$$y_1 - \bar{y}_1 = B^\top (x - \bar{x}).$$

4. DISCUSSION

We have shown in this brief manuscript that a cluster of interconnected district heating subsystems is shifted passive under certain conditions. The analysis has been carried out for a nonlinear model of the system, which simultaneously treats both temperature and volume dynamics of each storage tank. Our current research is focused on finding distributed conditions to verify $\text{symm}(A(v^p, v^c, w)) \geq 0$, which has been instrumental to establish our main results; moreover, we are exploring the design of decentralized and distributed passivity-based controllers aimed at achieving simultaneous temperature and volume regulation in the system.

REFERENCES

Cucuzzella, M., Kosaraju, K.C., and Scherpen, J.M. (2019). Distributed passivity-based control of DC microgrids. *Proceedings of the American Control Conference*, 2019-July, 652–657. doi:10.23919/acc.2019.8814756.

De Persis, C., Jensen, T.N., Ortega, R., and Wisniewski, R. (2014). Output regulation of large-scale hydraulic networks. *IEEE Transactions on Control Systems Technology*, 22(1), 238–245. doi: 10.1109/TCST.2012.2233477.

De Persis, C. and Kallesoe, C.S. (2011). Pressure regulation in nonlinear hydraulic networks by positive and quantized controls. *IEEE Transactions on Control Systems Technology*, 19(6), 1371–1383. doi: 10.1109/TCST.2010.2094619.

De Persis, C., Weitenberg, E.R.A., and Dörfler, F. (2018). A power consensus algorithm for DC microgrids. *Automatica*, 89, 364–375. doi: 10.1016/j.automatica.2017.12.026.

Scholten, T.W., De Persis, C., and Tesi, P. (2015). Modeling and control of heat networks with storage: The single-producer multiple-consumer case. *IEEE Transactions on Control Systems Technology*, 25(2), 2242–2247. doi:10.1109/ECC.2015.7330872.

Scholten, T. (2017). *Control of Flow Networks with Constraints and Optimality Conditions.* Ph.D. thesis, University of Groningen.

Scholten, T., Trip, S., and De Persis, C. (2017). Pressure Regulation in Large Scale Hydraulic Networks with Input Constraints. In *Proc. of the 2017 IFAC World Congress*, volume 50, 5367–5372. Elsevier. doi:10.1016/j.ifacol.2017.08.1068. URL `https://doi.org/10.1016/j.ifacol.2017.08.1068`.

Trip, S., Scholten, T., and Persis, C.D. (2019). Optimal regulation of flow networks with transient constraints. *Automatica*, 104, 141–153. doi: 10.1016/j.automatica.2019.02.046.