Presented as late breaking results contribution
21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Static optimal output feedback design for linear quadratic regulator using nonlinear optimal controller design

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Abstract: The paper proposes static optimal output feedback design that achieves linear quadratic regulator for a single-input, single-output, continuous-time, time-invariant system. The proposed approach constructs the closed-loop system via static feedback control, then optimizes the initial state value representing a constant static output feedback gain. The Euler-Lagrange equation is derived for characterizing the optimal initial state value for the finite time quadratic cost criterion. The paper also gives some analysis regarding the optimal condition for the static output feedback gain using the solution of the Ricatti differential equation. Finally, the paper shows a numerical example that supports the proposed derivation of the optimal static output feedback gain.

Keywords: linear quadratic regulator, static output feedback design, Euler-Lagrange equation

1. INTRODUCTION

The linear quadratic regulator is a basic optimal control strategy based on linear state space representations. The standard design realizes state feedback form where the state feedback gain is derived from the solution of the algebraic Ricatti equation. Anderson et al. (1971). The combined state observer and feedback formulation allows us to realize the output feedback formulation. Gessing (2001). Meanwhile, in practical points of view, static output feedback is applicable to many cases due to the conciseness of the implementation rather than either state feedback control or the combined state observer and feedback control. Hence, several studies on static output feedback design have been made. Syrmos et al. (1997); Vesely (2001); Yuan (1996). In these studies, the necessary conditions for the optimal static output feedback gain were addressed. However, they are nonlinear algebraic equations, which leads to some complexity to solve the equations.

The present work also concerns a static output feedback controller that achieves linear quadratic regulator. In contrast to the existing works, it focuses on a single-input, single-output, continuous-time, time-invariant system, and constructs the closed-loop system via static output feedback control, then optimizes the initial state value representing a constant static output feedback gain. The optimal static output feedback gain is derived using the Euler-Lagrange equation that characterizes the optimal initial state value for the finite time quadratic cost criterion. The Euler-Lagrange equation enables us to handle with nonlinear systems. In addition, the paper also provides analysis regarding the static output optimal feedback gain using the Ricatti differential equation, and the static optimal gain can be represented using the solution. Finally, the paper shows a numerical example that supports the proposed derivation of the optimal static output feedback gain.

2. PROBLEM STATEMENT

Consider a single-input, single-output, linear time invariant system described by a state space representation.

$$\frac{d}{dt}\boldsymbol{x}(t) = A\boldsymbol{x}(t) + \boldsymbol{b}u(t),\ \boldsymbol{x}(0) = \boldsymbol{x}_0, \tag{1}$$

$$y(t) = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}(t), \tag{2}$$

where $y(t), u(t) \in \mathbb{R}^1$ is the controlled output and input signals of time $t \geq 0$, respectively. $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the n dimensional state vector of time $t \geq 0$. $(A, \boldsymbol{b})$ and $(A, \boldsymbol{c}^{\mathrm{T}})$ are stabilizable and detectable, respectively. The present work considers the output feedback controller

$$u(t) = -ky(t), \tag{3}$$

so that the following finite time, quadratic cost criterion could be minimized:

$$J = \int_0^T \left[qy(t)^2 + ru(t)^2\right] dt + \varphi[\boldsymbol{x}(T)], \tag{4}$$

where $k \in \mathbb{R}^1$ is a scalar feedback gain, and $q > 0, r > 0$ are weighting parameters for the magnitude of output and input signals, respectively. $\varphi(\boldsymbol{x}(T))$ is the cost criterion at the terminal time T. In the present work, the terminal cost criterion is set as

$$\varphi[\boldsymbol{x}(T)] = q_f y(T)^2, \tag{5}$$

where $q_f > 0$ is the terminal weighting parameter.

3. OPTIMAL OUTPUT STATIC GAIN

From Eq. (1) and Eq. (2), the closed loop system can be described as

$$\frac{d}{dt}\boldsymbol{x}(t) = A\boldsymbol{x}(t) - k\boldsymbol{b}\boldsymbol{c}^T\boldsymbol{x}(t),\ \boldsymbol{x}(0) = \boldsymbol{x}_0, \tag{6}$$

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Now let the static output feedback gain k be regarded as a state variable $v(t)$. Since $v(t)$ is a constant value, the differential of $v(t)$ turns out to equal zero. Hence, the state space representation of the closed-loop system is expressed as

$$\frac{d}{dt}\begin{bmatrix}\boldsymbol{x}(t)\\ v(t)\end{bmatrix}=\begin{bmatrix}A\boldsymbol{x}(t)-v(t)\boldsymbol{b}\boldsymbol{c}^T\boldsymbol{x}(t)\\ 0\end{bmatrix}, \tag{7}$$

$$\begin{bmatrix}\boldsymbol{x}(0)\\ v(0)\end{bmatrix}=\begin{bmatrix}\boldsymbol{x}_0\\ k\end{bmatrix}. \tag{8}$$

Note that the closed-loop system Eq. (7) and Eq. (8) are a nonlinear autonomous system, which implies there is no external control input. In addition, the final element of the initial state variable is unsettled, and it is handled as the optimizing variable.

Subsequently, the section derives the optimality condition for the the final element of the initial state variable using a nonlinear optimal controller design approach. To start with, let the closed-loop system Eq. (7) and Eq. (8) be represented as

$$\dot{\boldsymbol{x}}_v(t)=\begin{bmatrix}\boldsymbol{f}_v(\boldsymbol{x}_v(t))\\ 0\end{bmatrix}, \tag{9}$$

$$\boldsymbol{f}_v(\boldsymbol{x}_v(t))=A\boldsymbol{x}(t)-v(t)\boldsymbol{b}\boldsymbol{c}^T\boldsymbol{x}(t), \tag{10}$$

$$\boldsymbol{x}_v(t)=\begin{bmatrix}\boldsymbol{x}(t)\\ v(t)\end{bmatrix},\ \boldsymbol{x}_v(0)=\begin{bmatrix}\boldsymbol{x}_0\\ k\end{bmatrix}, \tag{11}$$

where $\dot{\boldsymbol{x}}_v(t)$ stands for $\frac{d}{dt}\boldsymbol{x}_v(t)$. Then, the cost criterion in Eq. (4) is described as

$$J=\int_0^T L(\boldsymbol{x}_v)dt+\varphi[\boldsymbol{x}(T)], \tag{12}$$

where $L(\boldsymbol{x}_v)$ is expressed as

$$L(\boldsymbol{x}_v)=q\boldsymbol{x}^{\mathrm{T}}\boldsymbol{c}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}+rv(t)^2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{c}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}. \tag{13}$$

Now, we will derive the optimal condition of the optimizing variable k using a nonlinear optimal control approach. From the objctive function Eq. (12) and the diffential equation in terms of the state variable $\boldsymbol{x}_v(t)$ in Eq. (11), the Lagrange function becomes

$$J^* = J+\int_0^T \boldsymbol{\lambda}_v^{\mathrm{T}}\left(\begin{bmatrix}\boldsymbol{f}_v(\boldsymbol{x}_v)\\ 0\end{bmatrix}-\dot{\boldsymbol{x}}_v\right)dt, \tag{14}$$

where $\boldsymbol{\lambda}_v$ is the co-state variable. Let the co-state variable be composed of

$$\boldsymbol{\lambda}_v^{\mathrm{T}}=\begin{bmatrix}\boldsymbol{\lambda}^{\mathrm{T}} & \nu\end{bmatrix},\ \ \boldsymbol{\lambda}\in\mathbb{R}^n,\ \nu\in\mathbb{R}^1 \tag{15}$$

Hence, from Eq. (7) and Eq. (8), it follows that Eq. (14) becomes

$$J^* = J+\int_0^T\left(\boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{f}(\boldsymbol{x}_v)-\boldsymbol{\lambda}_v^{\mathrm{T}}\dot{\boldsymbol{x}}_v\right)dt, \tag{16}$$

Defining the Hamiltonian H as

$$H(\boldsymbol{x}_v,\boldsymbol{\lambda})=L(\boldsymbol{x}_v)+\boldsymbol{\lambda}^T\boldsymbol{f}_v(\boldsymbol{x}_v). \tag{17}$$

The Lagrange function J^* in Eq. (16) results in

$$J^*=\int_0^T H(\boldsymbol{x}_v,\boldsymbol{\lambda})dt+\varphi[\boldsymbol{x}(T)]-\int_0^T\boldsymbol{\lambda}_v^{\mathrm{T}}\dot{\boldsymbol{x}}_v dt. \tag{18}$$

Consider the first variation of J^*, it follows that

$$\begin{aligned}\delta J^* =& \int_0^T\left(\frac{\partial}{\partial\boldsymbol{x}}H\delta\boldsymbol{x}+\frac{\partial}{\partial v}H\delta v+\frac{\partial}{\partial\boldsymbol{\lambda}}H\delta\boldsymbol{\lambda}\right)dt\\ &+\frac{\partial}{\partial\boldsymbol{x}(T)}\varphi\left(\boldsymbol{x}(T)\right)\delta\boldsymbol{x}(T)\\ &-\int_0^T\boldsymbol{\lambda}_v^{\mathrm{T}}\delta\dot{\boldsymbol{x}}_v dt-\int_0^T\dot{\boldsymbol{x}}_v^{\mathrm{T}}\delta\boldsymbol{\lambda}_v dt.\end{aligned} \tag{19}$$

Using the partial integration, the third and forth terms in the right-hand side of Eq. (19) becomes

$$-\int_0^T\boldsymbol{\lambda}_v^T\delta\dot{\boldsymbol{x}}_v dt-\int_0^T\dot{\boldsymbol{x}}_v^{\mathrm{T}}\delta\boldsymbol{\lambda}_v dt \tag{20}$$

$$=-\left[\boldsymbol{\lambda}_v^{\mathrm{T}}\delta\boldsymbol{x}_v\right]_0^T+\int_0^T\dot{\boldsymbol{\lambda}}_v^{\mathrm{T}}\delta\boldsymbol{x}_v dt-\int_0^T\dot{\boldsymbol{x}}_v^{\mathrm{T}}\delta\boldsymbol{\lambda}_v dt \tag{21}$$

$$\begin{aligned}&=-\boldsymbol{\lambda}_v^{\mathrm{T}}(T)\delta\boldsymbol{x}_v(T)+\boldsymbol{\lambda}_v(0)^{\mathrm{T}}\delta\boldsymbol{x}_v(0)\\ &\quad+\int_0^T\dot{\boldsymbol{\lambda}}_v^{\mathrm{T}}\delta\boldsymbol{x}_v dt-\int_0^T\dot{\boldsymbol{x}}_v^{\mathrm{T}}\delta\boldsymbol{\lambda}_v dt\\ &=-\boldsymbol{\lambda}^{\mathrm{T}}(T)\delta\boldsymbol{x}(T)-\nu(T)\delta v(T)\\ &\quad+\boldsymbol{\lambda}(0)^{\mathrm{T}}\delta\boldsymbol{x}(0)+\nu(0)\delta v(0)\\ &\quad+\int_0^T\dot{\boldsymbol{\lambda}}_v^{\mathrm{T}}\delta\boldsymbol{x}_v dt-\int_0^T\dot{\boldsymbol{x}}_v^{\mathrm{T}}\delta\boldsymbol{\lambda}_v dt\end{aligned} \tag{22}$$

Noting that $\boldsymbol{x}(0)$ is a fixed value, which leads to $\delta\boldsymbol{x}(0)=0$, $\dot{\boldsymbol{x}}_v^{\mathrm{T}}\delta\boldsymbol{\lambda}_v=\dot{\boldsymbol{x}}^{\mathrm{T}}\delta\boldsymbol{\lambda}$, and $\dot{\boldsymbol{\lambda}}_v^{\mathrm{T}}\delta\boldsymbol{x}_v=\dot{\boldsymbol{\lambda}}^{\mathrm{T}}\delta\boldsymbol{x}+\dot{\nu}\delta v$. Thus, Eq. (19) is summarized as

$$\begin{aligned}\delta J^* =& \int_0^T\left(\frac{\partial}{\partial\boldsymbol{x}}H+\dot{\boldsymbol{\lambda}}^{\mathrm{T}}\right)\delta\boldsymbol{x}dt\\ &+\int_0^T\left(\frac{\partial}{\partial\boldsymbol{\lambda}}H-\dot{\boldsymbol{x}}^{\mathrm{T}}\right)\delta\boldsymbol{\lambda}dt\\ &+\left(\frac{\partial}{\partial\boldsymbol{x}}\varphi[\boldsymbol{x}(T)]-\boldsymbol{\lambda}^{\mathrm{T}}(T)\right)\delta\boldsymbol{x}(T)\\ &+\nu(0)\delta v(0)-\nu(T)\delta v(T)\\ &+\int_0^T\left(\frac{\partial}{\partial v}H+\dot{\nu}\right)\delta v dt.\end{aligned} \tag{23}$$

The variation $\delta\boldsymbol{x}$, $\delta\lambda$, $\delta\boldsymbol{x}(T)$, $\delta v(0)$, $\delta v(T)$, δv independently vary, so the following differential equation could be obtained in order that the first variation $\delta J^*=0$.

$$\frac{\partial}{\partial\boldsymbol{x}}H+\dot{\boldsymbol{\lambda}}^{\mathrm{T}}=0, \tag{24}$$

$$\frac{\partial}{\partial\boldsymbol{x}}\varphi\left[\boldsymbol{x}(T)\right]-\boldsymbol{\lambda}^{\mathrm{T}}(T)=0, \tag{25}$$

$$\frac{\partial}{\partial\boldsymbol{\lambda}}H-\dot{\boldsymbol{x}}^{\mathrm{T}}=0, \tag{26}$$

$$\nu(0)=0,\quad \nu(T)=0, \tag{27}$$

$$\frac{\partial}{\partial v}H+\dot{\nu}^{\mathrm{T}}=0. \tag{28}$$

The obtained differential equations are a kind of the Euler-Lagrange equations that characterize the nonlinear optimal control laws. They are derived for the specific purpose of characterizing the static output feedback gain for linear quadratic regulator.

Substituting Eq. (5), Eq. (10), and Eq. (17) into the optimal conditions Eq. (24) $\sim$ Eq. (28), and using $v(t)\equiv k,\quad t\geq 0$, the Euler-Lagrange equations result in the following differential equations.

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} - k\boldsymbol{b}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}, \quad \boldsymbol{x}(0) = \boldsymbol{x}_0, \tag{29}$$
$$\dot{\boldsymbol{\lambda}} = -q\boldsymbol{c}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} - rk^2\boldsymbol{c}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} - A^{\mathrm{T}}\boldsymbol{\lambda} + k\boldsymbol{c}\boldsymbol{b}^{\mathrm{T}}\boldsymbol{\lambda}, \tag{30}$$
$$\boldsymbol{\lambda}(T) = q_f\boldsymbol{c}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}(T), \tag{31}$$
$$\dot{\nu} = -rk\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} + \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}\boldsymbol{b}^{\mathrm{T}}\boldsymbol{\lambda}, \tag{32}$$
$$\nu(0) = 0, \quad \nu(T) = 0. \tag{33}$$

In these differential equations, the initial value of $\boldsymbol{x}(0)$ and $\nu(0)$ are given, and the terminal value of $\nu(T)$ and the terminal condition of $\boldsymbol{x}(T)$ and $\boldsymbol{\lambda}$ are given in Eq. (31) and Eq. (33). They are a so-called TPBV (Two-point boundary valued) problem. In general, they can be solved by a shooting method.

Meanwhile, using the properties of linear differential equations, the solution of the problem can be characterized by using a Ricatti differential equation. In the next section, we consider a solving way to obtain the optimal output feedback gain k satisfying the above differentilal equations.

4. OPTIMAL GAIN CALCULATION

The section provides a calculation method of the static output feedback gain k that satisfies the differential equation Eq. (29) $\sim$ Eq. (33).

To begin with, let the co-state $\boldsymbol{\lambda}$ be expressed as

$$\boldsymbol{\lambda}(t) = P(t)\boldsymbol{x}(t), \tag{34}$$

where $P(t) \in \mathbb{R}^{n\times n}$ is assumed to be the positive semi definite matrix. Substituting $P(t)$ into the differential equation Eq. (30), and using Eq. (29), we get

$$\begin{aligned}\dot{P}\boldsymbol{x} = &-PA\boldsymbol{x} - A^{\mathrm{T}}P\boldsymbol{x} + kP\boldsymbol{b}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} + k\boldsymbol{c}\boldsymbol{b}^{\mathrm{T}}P\boldsymbol{x}\\ &- q\boldsymbol{c}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} - r^{-1}k^2\boldsymbol{c}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}.\end{aligned} \tag{35}$$

Now, let the positive semi-definite matrix function $P(t)$ be determined by the following Ricatti differential equations.

$$\dot{P} = -PA - A^{\mathrm{T}}P - q\boldsymbol{c}\boldsymbol{c}^{\mathrm{T}} + r^{-1}P\boldsymbol{b}\boldsymbol{b}^{\mathrm{T}}P. \tag{36}$$

In order to satisfy the Eq. (31), the terminal value of $P(T)$ should have the following value.

$$P(T) = q_f\boldsymbol{c}\boldsymbol{c}^{\mathrm{T}}. \tag{37}$$

Hence, the solution of the Ricatti differential equation Eq. (36) can be solved backwards from $t = T$ to $t = 0$. In addition, since $P(T)$ is a positive semi-definite matrix, the solution of the differential equation $P(t)$, $0 \le t \le T$ is also proved to be a positive semi-definite matrix.

The next, define the positive definite function $V(t)$ as

$$V(t) = \boldsymbol{x}^{\mathrm{T}}P(t)\boldsymbol{x}. \tag{38}$$

By calculatin the derivative of $V(t)$ and using the Ricatti differential equation Eq. (36), we get

$$\begin{aligned}\dot{V}(t) = &-\boldsymbol{x}^{\mathrm{T}}q\boldsymbol{c}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} - \boldsymbol{x}^{\mathrm{T}}r^{-1}k^2\boldsymbol{c}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}\\ &+ \boldsymbol{x}^{\mathrm{T}}(rk\boldsymbol{c} - P\boldsymbol{b})r^{-1}(rk\boldsymbol{c}^{\mathrm{T}} - \boldsymbol{b}^{\mathrm{T}}P)\boldsymbol{x}.\end{aligned} \tag{39}$$

Integrating Eq. (39) from $t = 0$ to $t = T$, it follows

$$\begin{aligned}V(T) - V(0) = &-\int_0^T \boldsymbol{x}^{\mathrm{T}}q\boldsymbol{c}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} + \boldsymbol{x}^{\mathrm{T}}r^{-1}k^2\boldsymbol{c}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}dt\\ &+ \int_0^T \boldsymbol{x}^{\mathrm{T}}(rk\boldsymbol{c} - P\boldsymbol{b})r^{-1}(rk\boldsymbol{c}^{\mathrm{T}} - \boldsymbol{b}^{\mathrm{T}}P)\boldsymbol{x}dt\end{aligned} \tag{40}$$

Hence, we finally get

$$\begin{aligned}&\int_0^T \boldsymbol{x}^{\mathrm{T}}q\boldsymbol{c}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} + \boldsymbol{x}^{\mathrm{T}}r^{-1}k^2\boldsymbol{c}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}dt + V(T)\\ &= V(0) + \int_0^T \boldsymbol{x}^{\mathrm{T}}(rk\boldsymbol{c} - P\boldsymbol{b})r^{-1}(rk\boldsymbol{c}^{\mathrm{T}} - \boldsymbol{b}^{\mathrm{T}}P)\boldsymbol{x}dt.\end{aligned} \tag{41}$$

The left-hand side of Eq. (41) is equivallent to the quadratic cost criterion J defined by Eq. (4). Therefore, the minimization of Eq. (4) is equivalent to the second term of the right-hand side of Eq. (41). Namely, the optimal static gain is expressed as

$$k_{\mathrm{opt}} = \arg\min_k \int_0^T \boldsymbol{x}^{\mathrm{T}}(rk\boldsymbol{c} - P\boldsymbol{b})r^{-1}(rk\boldsymbol{c}^{\mathrm{T}} - \boldsymbol{b}^{\mathrm{T}}P)\boldsymbol{x}dt, \tag{42}$$

where $\boldsymbol{x}$ is the solution of the differential equation Eq. (29), and P is the solution of the Ricatti differential equation Eq. (36). From Eq. (42), it follows that the static optimal output feedback gain is calculated using the solution of the Ricatti differential equation as well as the state feedback case.

5. NUMERICAL EXAMPLE

Consider the following second order linear state space model.

$$\frac{d}{dt}\begin{bmatrix} x_1(t)\\ x_2(t)\end{bmatrix} = \begin{bmatrix} 0 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix} x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix} 0\\ 1\end{bmatrix}u(t), \tag{43}$$

$$y(t) = [\,1\ \ 0\,]\begin{bmatrix} x_1(t)\\ x_2(t)\end{bmatrix}, \boldsymbol{x}(0) = \begin{bmatrix} 1\\ 0\end{bmatrix}, \tag{44}$$

For the system, the linear quadratic optimal regulator problem was considered. The quadratic cost criterion was set as

$$J = \int_0^T \left[y(t)^2 + u(t)^2\right]dt + y(T)^2, \tag{45}$$

Namely, the weighting parameters are $q = q_f = r = 1$, and both $T = 10$ was applied. At first, a shooting method was applied to solve the TPBV problem in Eq. (29) and Eq. (33). As a result, the optimal static output feedback gain was $k = 1.6404$. Fig. 1 shows the simulation result. from the figure, we can see that the obtained static output feedback gain achieves the control objective. The next, after solving the Ricatti diffential equation Eq. (36), the static output feedback gain was solved using Eq. (42). Fig. 2 shows elements of the solution of the Ricatti diffential equation. The obtained static optimal output feedback gain is $k = 1.6786$, which is almost same as the solution using a shooting method. From the result, it follows that the static optimal feedback gain using the Ricatti equation also achieves the control objective.

6. CONCLUSION

The paper proposed static output feedback design that achieves linear quadratic regulator for a single-input, single-output, continuous-time, time-invariant system. The proposed approach constructs the closed-loop system via static feedback control, and the optimal condition for the initial state value for the finite time quadratic cost criterion was derived using the Euler-Lagrange equation. The

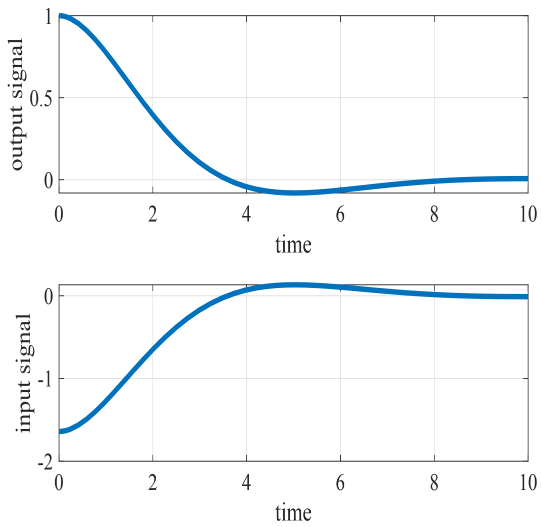

Fig. 1. Simulation resut of static optimal output LQ regulator

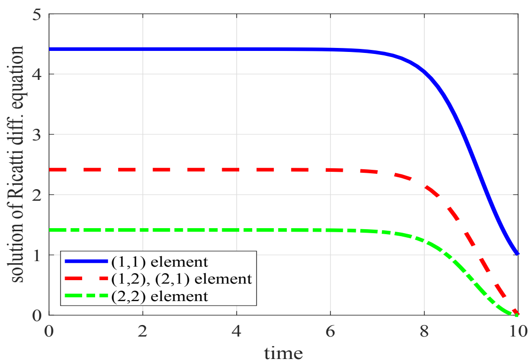

Fig. 2. Solution of Ricatti differential equation

paper also gave analysis regarding the optimal condition for the static output feedback gain using the solution of the Ricatti differential equation. Finally, the paper showed a numerical example that supports the proposed derivation of the optimal static output feedback gain.

ACKNOWLEDGEMENTS

This work was partially supported by JSPS KAKENHI Grant Number 19K04456.

REFERENCES

B. D. O. Anderson and J. B. Moore. Linear optimal control. *Prentice Hall*. 1971.

R. Gessing. Linear quadratic regulator with output feedback and optimal observer. *In Proc. the American Control Conference, VA, June 25-27, 2001*. 3171-3176, 2001.

V. L. Syrmos, T. Abdallah, P. Dorato, and K. Grigoriadis. Static output feedback -a survey. *Automatica*. 33:, 125–137, 1997.

V. Veselý. Static output feedback controller design. *Kybernetika*. 37:205–221, 2001.

L. Yuan, L. E. K. Achenie and W. Jiang. Linear quadratic optimal output feedback control for systems with poles in a specified region. *International Journal of Control*. 64:1151–1164, 1996.