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Nonlinear Attitude Tracking Maneuver of Spacecraft with Reaction Control System and Reaction Wheel ⋆

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Abstract: Missions involving rapid and large-angle attitude maneuvers have been conceived in astronomical and earth observation satellites in recent years. Since the rotational motion of a spacecraft in such missions is nonlinear, it will be required to design an attitude control system that takes into account nonlinear motion. As an actuator capable of generating a large torque is also going to be required, it will also be necessary to consider characteristics of an actuator in designing a control system. Actuators capable of generating a large torque include reaction control system (RCS). RCS gives an on/off input as it uses the reaction force from fuel injection by thrusters, it can generate a large moment. In addition, the control system of current application satellites normally uses both RCS and reaction wheel (RW) conventionally used for attitude control. This paper considers large angle attitude maneuver of spacecraft by a combination of RCS and RW. To this end, characteristics of RCS and RW are defined, and a model for control system design is derived. Then, we design a nonlinear controller so that the closed-loop system becomes input-to-state stable (ISS) using the concept of backstepping approach. Finally, the effectiveness of proposed control method is verified by numerical simulations.

Keywords: Spacecraft, Reaction Control System, Reaction Wheel, Large Attitude Maneuver.

1. INTRODUCTION

Missions involving rapid and large angle attitude maneuvers have been conceived in astronomical and earth observation satellites in recent years (Kamiya et al. (2006); Somov et al. (2008)). Since the rotational motion of a spacecraft in such missions is nonlinear, it will be required to design an attitude control system that takes into account nonlinear motion. As an actuator capable of generating a large torque is also going to be required, it will also be necessary to consider characteristics of an actuator in designing a control system. Actuators capable of generating a large torque include control moment gyroscope (CMG) and reaction control system (RCS). While a CMG can generate a larger torque in comparison with an reaction wheel (RW) conventionally used for attitude control, there is a singularity where it cannot generate a torque to a certain direction (Wie et al. (2001)). It is thus necessary to consider avoiding such a singularity in designing a control system. On the other hand, while an RCS gives an on/off input as it uses the reaction force from fuel injection by thrusters, it can generate a large moment just as a CMG does and has a simple structure compared with a CMG and no problem of avoiding a singularity. In addition, the control system of current application satellites normally uses both RCS and RW.

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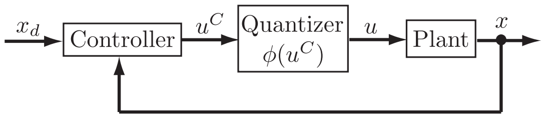

Fig. 1. Block diagram of control system including quantizer.

Authors thus proposed an attitude control method using NMPC based on the conception of control using both RCS and RW which (1) rapidly makes an attitude reach the neighborhood of the target attitude by an RCS and (2) performs high-precision control in the neighborhood of the target attitude by an RW (Sakamoto et al. (2016)). While this method is a very effective one as it also gives the design conditions for the weighting matrix in the terminal cost that ensures the stability of a closed loop system, computational complexity becomes an issue as it requires solving a nonlinear optimization problem on-line. On the other hand, for problems of controlling a linear system with an on/off or quantized input, a method has been proposed for designing a continuous controller ensuring the stability of a closed loop system composed of the continuous controller and a quantizer as shown in Fig. 1, which has no on-line computational complexity issue as a fixed controller is used (Ninomiya et al. (2004); Koike and Chida (2012)). It is also conceivable to use this controller as a backup controller in case a nonlinear optimization problem with NMPC fails to be solved within a sampling time. This study constructs a nonlinear control method based on the idea in Koike and Chida (2012) as a solution

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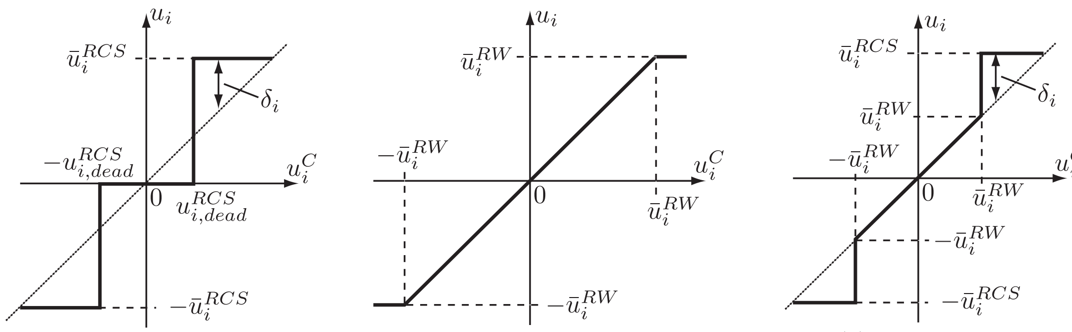

(a) RCS (b) RW (c) RCS and RW

Fig. 2. Actuator nonlinearity.

to the problem of spacecraft attitude tracking control using both RCS and RW.

The following notations are used throughout the paper. Let $\mathbb{R}$ denotes the real numbers. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ are the sets of real vectors and matrices, respectively. For real vector $a \in \mathbb{R}^n$, a^T is the vector transpose, $\|a\|$ denotes the Euclidean norm, and $a^\times \in \mathbb{R}^{3\times 3}$ is a skew symmetric matrix

$$a^\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

derived from the vector $a = [a_1\ a_2\ a_3]^T \in \mathbb{R}^3$. For a real symmetric matrix A, $A > 0$ implies the positive definite matrix. The identity matrix of size 3×3 is denoted by I_3. $\lambda_A^{\max} \in \mathbb{R}$ and $\lambda_A^{\min} \in \mathbb{R}$ are the maximal and the minimal eigenvalues of the matrix A, respectively.

2. RELATIVE EQUATION OF MOTION OF SPACECRAFT

The rotational motion equations of the spacecraft fixed body frame $\{b\}$ are given by the equations (1) and (2) if the modified Rodrigues parameters (MRPs) are used as the attitude parameters (Tsiotras (1998)):

$$\dot{\sigma} = \frac{1}{2}\left\{\frac{1-\|\sigma\|^2}{2}I_3 + \sigma\sigma^T + \sigma^\times\right\}\omega = G(\sigma)\omega, \tag{1}$$

$$\dot{\omega} = J^{-1}\left\{-\omega^\times J\omega + u\right\}, \tag{2}$$

where (1) is the kinematics equation that represents the attitude of $\{b\}$ with respect to the inertial frame $\{i\}$, (2) is the rotational dynamics equation, $\sigma \in \mathbb{R}^3$ [-] is the MRPs, $\omega \in \mathbb{R}^3$ [rad/s] is the angular velocity, $u \in \mathbb{R}^3$ [Nm] is the control torque (input), and $J \in \mathbb{R}^{3\times 3}$ [kgm^2] is the moment of inertia.

We considered a control problem in which a spacecraft tracks the desired attitude (MRPs) $\sigma_d \in \mathbb{R}^3$ and angular velocity $\omega_d \in \mathbb{R}^3$ in fixed frame $\{d\}$. The MRPs of relative attitude $\sigma_e \in \mathbb{R}^3$ and the relative angular velocity $\omega_e \in \mathbb{R}^3$ in the frame $\{b\}$ are given by

$$\sigma_e = \frac{(1-\|\sigma_d\|^2)\sigma - (1-\|\sigma\|^2)\sigma_d + 2\sigma^\times\sigma_d}{1+\|\sigma\|^2\|\sigma_d\|^2 + 2(\sigma_d)^T\sigma}, \tag{3}$$

$$\omega_e = \omega - C\omega_d, \tag{4}$$

where $C \in \mathbb{R}^{3\times 3}$ is the direction cosine matrix from $\{b\}$ to $\{d\}$ that is expressed as follows (Meng et al. (2009)):

$$C = I_3 + \frac{8({\sigma_e}^\times)^2 - 4(1-\|\sigma_e\|^2){\sigma_e}^\times}{(1+\|\sigma_e\|^2)^2}. \tag{5}$$

Substitution of the equations (3) and (4) into the equations (1) and (2) using the identity $\dot{C} = -\omega_e^\times C$ yields the following relative motion equations:

$$\dot{\sigma}_e = G(\sigma_e)\omega_e, \tag{6}$$

$$\dot{\omega}_e = J^{-1}\Big[-\{\omega_e + C\omega_d\}^\times J\{\omega_e + C\omega_d\} \\ -J\left\{C\dot{\omega}_d - \omega_e^\times C\omega^d\right\} + u\Big]. \tag{7}$$

We assumed that σ and ω are directly measurable, and J is known. In addition, we assumed the following about σ_d, ω_d, and $\dot{\omega}_d$.

Assumption 1. The desired states σ_d, ω_d and $\dot{\omega}_d$ are uniformly continuous and bounded for all $t \in [0,\ \infty)$.

As both RCS and RW are used as actuators in this paper, an RCS output $u_i = u_i^{RCS}$ $(i = 1, 2, 3)$ for the command u_i^C takes discrete values, with a dead zone (Fig. 2(a)), and an RW output $u_i = u_i^{RW}$ takes continuous values, with a saturation (Fig. 2(b)). Now, for the magnitude $u_{i,dead}^{RCS}$ of the RCS dead zone and the RW saturation value $\bar{u}_i^{RW}$, let $|u_{i,dead}^{RCS}| \le |\bar{u}_i^{RW}|$. Then, if both RCS and RW are used, an output from the input converter (quantizer) ϕ can be considered to be applied to a control input u_i (Fig. 2(c)).

$$u_i = \phi(u_i^C) = \begin{cases} \bar{u}_i^{RCS}, & \text{if } u_i^C > \bar{u}_i^{RCS} \\ u_i^C, & \text{if } \left|u_i^C\right| \le \left|\bar{u}_i^{RW}\right| \\ -\bar{u}_i^{RCS}, & \text{if } u_i^C < -\bar{u}_i^{RCS} \end{cases}. \tag{8}$$

The block diagram of the control system is shown in Fig. 3. Now, define a quantization error

$$\delta = \phi(u^C) - u^C \quad \left(\phi(u^C) = \left[\phi(u_1^C)\ \phi(u_2^C)\ \phi(u_3^C)\right]^T\right) \tag{9}$$

that occurs when RCS is used and assume that the norm of quantization error satisfy

$$\|\delta\| \le \bar{\delta},\ \forall t \ge 0, \tag{10}$$

where $\bar{\delta}$ is a positive constant. By equation (9) and $u = \phi(u^C)$, equation (7) is transformed into

$$\dot{\omega}_e = J^{-1}\Big[-\{\omega_e + C\omega_d\}^\times J\{\omega_e + C\omega_d\} \\ -J\left\{C\dot{\omega}_d - \omega_e^\times C\omega^d\right\} + u^C + \delta\Big]. \tag{11}$$

and can be regarded as a system to which bounded disturbance δ is applied. This control system is shown in Fig. 4. In the next section, equation (11) taken to represent the dynamics in the design model.

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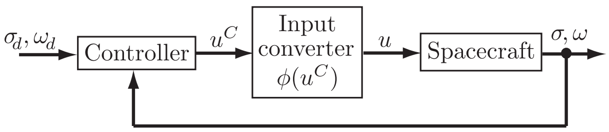

Fig. 3. Block diagram of control system.

3. NONLINEAR ATTITUDE TRACKING CONTROL USING RCS AND RW

Firstly, the command value (control law) u^C for the closed-loop system of (6) and (11) to be input-state stable (ISS) is derived under assumption of equation (10) when only RCS as actuator is used. Secondly, we indicate that the closed-loop system using both RCS and RW can be asymptotically stabilized, i.e.,

$$(\sigma_e, \omega_e) \to (0, 0)\ (t \to \infty).$$

The control law is derived based on the concept of back-stepping. In addition, Lemmas when using the derivation of the control law are shown below.

Lemma 2. For all $\sigma_e \in \mathbb{R}^3$, the following equations hold (Tsiotras (1998)):

$$\sigma_e^T G(\sigma_e) = b\sigma_e^T,\ G(\sigma_e)^T G(\sigma_e) = b^2 I_3$$
$$\left(b = \frac{1 + \|\sigma_e\|^2}{4} > 0\right).$$

3.1 Control law design when only RCS as actuator is used

Derivation of virtual input α Assume that ω_k^e is the virtual input to subsystem (6), and define the stabilizing function such that

$$\omega_e = \alpha = -f_1\sigma_e \tag{12}$$

where $f_1 > 0 \in \mathbb{R}$ is the feedback gain. The candidate Lyapunov function for (6) is defined as

$$V_1 = \frac{1}{2}\|\sigma_e\|^2. \tag{13}$$

From Lemma 2, the differential of (13) along the trajectories of the closed-loop system becomes

$$\begin{aligned}\dot{V}_1 &= \sigma_e^T G(\sigma_e)\alpha \\ &= b\sigma_e^T \alpha \\ &= -f_1 b\|\sigma_e\|^2 < 0, {}^\forall\sigma_e \neq 0\end{aligned} \tag{14}$$

since $b > 0$ and $f_1 > 0$. Therefore, if $\omega_e \to \alpha\ (t \to \infty)$, then $\sigma_e \to 0$.

Derivation of control torque u^C The error variable between the state ω_e and α is defined as

$$z := \omega_e - \alpha = \omega_e + f_1\sigma_e. \tag{15}$$

The control input u^C that makes the closed loop system with state variables σ_e and z to be ISS is derived.

From (15), subsystem (6) becomes

$$\dot{\sigma}_e = G(\sigma_e)z - f_1 b\sigma_e, \tag{16}$$

and the dynamics with respect to z is

$$\dot{z} = \dot{\omega}_e + f_1\dot{\sigma}_e$$

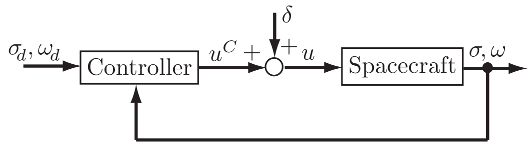

Fig. 4. Equivalent expression of block diagram of control system.

$$\begin{aligned}&= J^{-1}\Big[-\{z - f_1\sigma_e + C\omega_d\}^\times J\{z - f_1\sigma_e + C\omega_d\} \\ &\quad -J\left\{C\dot{\omega}_d - (z - f_1\sigma_e)^\times C\omega^d\right\} + u^C + \delta\Big] \\ &\quad + f_1\{G(\sigma_e)z - f_1 b\sigma_e\}.\end{aligned} \tag{17}$$

Now, by setting u^C to

$$\begin{aligned}u^C &= \{z - f_1\sigma_e + C\omega_d\}^\times J\{z - f_1\sigma_e + C\omega_d\} \\ &\quad + J\left\{C\dot{\omega}_d - (z - f_1\sigma_e)^\times C\omega^d\right\} \\ &\quad - f_1 J\{G(\sigma_e)z - f_1 b\sigma_e\} - b\sigma_e - f_2 z,\end{aligned} \tag{18}$$

(17) becomes

$$\dot{z} = J^{-1}(-b\sigma_e - f_2 z + \delta), \tag{19}$$

where $f_2 > 1 \in \mathbb{R}$ is the feedback gain. The candidate Lyapunov function for (16) and (19) is defined as

$$V_2 = V_1 + \frac{1}{2}z^T Jz. \tag{20}$$

Since (14) becomes

$$\dot{V}_1 = b\sigma_e^T z - f_1 b\|\sigma_e\|^2$$

from (15), the differential of (20) along the trajectories of the closed-loop system becomes

$$\dot{V}_2 = -f_1 b\|\sigma_e\|^2 - f_2\|z\|^2 + z\delta. \tag{21}$$

By using completing square, (21) is given by

$$\begin{aligned}\dot{V}_2 &\le -f_1 b\|\sigma_e\|^2 - (f_2 - 1)\|z\|^2 + \frac{1}{4}\|\delta\|^2 \\ &\le -\chi^T Q\chi + \frac{1}{4}\|\delta\|^2 \\ &\le -\lambda_Q^{\min}\|\chi\|^2 + \frac{1}{4}\bar{\delta}^2\end{aligned} \tag{22}$$
$$\left(Q = \mathrm{diag}\{f_1 bI_3, (f_2 - 1)I_3\},\ \chi^T = \begin{bmatrix}\sigma_e^T & z^T\end{bmatrix}\right).$$

Therefor, the condition of ISS (Khalil (2001)) holds since $Q > 0$.

Summarizing the above, the following theorem can be obtained.

Theorem 3. If the quantization error δ satisfies (10) and feedback gains $f_1,\ f_2$ are $f_1 > 0,\ f_2 > 1$, then the state variables σ_e and ω_e of the closed-loop system given by equations (6) and (11) with the control law (18) becomes ISS, and state variable χ converge to the following set $\mathcal{S}$.

$$\mathcal{S} := \left\{\chi \in \mathbb{R}^6 \mid \|\chi\| \le \left(\frac{1}{2\sqrt{\lambda_Q^{\min}}}\right)\bar{\delta}\right\}. \tag{23}$$

3.2 Stability of closed loop system when using both RCS and RW

When both RCS and RW are used, the control input u_i^C in the case of $\left|u_i^C\right| \le \left|\bar{u}_i^{RW}\right|$ is a continuous value by RW

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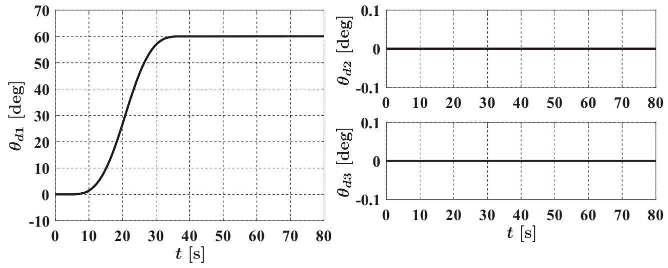


Fig. 5. Switching maneuver

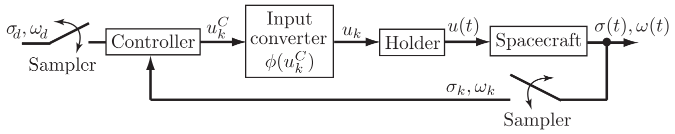
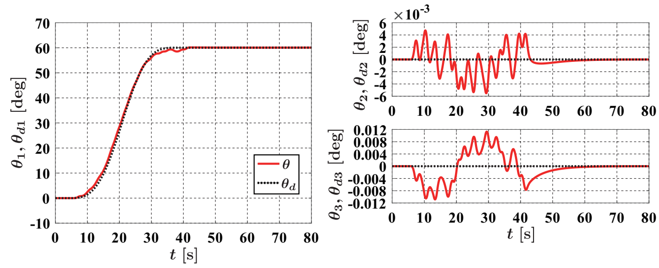
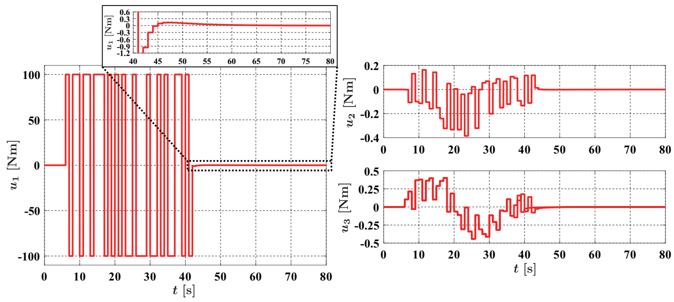


Fig. 6. Block diagram of digital control system.



(a) θ



(b) u

Fig. 7. Simulation results.

and $\delta \equiv 0$ from (8) and Fig. 2(c). From this fact, when the state χ enters the inside of the following set $\bar{\mathcal{S}}$

$$\bar{\mathcal{S}} := \left\{ \chi \in \mathcal{S} \mid \left|u_i^C\right| \leq \left|\bar{u}_i^{RW}\right| \ (i = 1, 2, 3) \right\} \tag{24}$$

by RCS, the state χ can be asymptotically stabilized, i.e., $(\sigma_e, \omega_e) \to (0, 0)$ $(t \to \infty)$ by switching from RCS to RW.

4. NUMERICAL SIMULATION

The properties of the proposed method are discussed in the numerical study. For this purpose, parameters setting of simulation is as below:

$$J = \begin{bmatrix} 7050 & -0.536 & 43.9 \\ -0.536 & 2390 & 1640 \\ 43.9 & 1640 & 6130 \end{bmatrix}, \sigma(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \omega(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\bar{u}_i^{RCS} = 100 \text{ Nm}, \ \bar{u}_i^{RW} = 1 \text{ Nm} \ (i = 1, 2, 3),$$

$$f_1 = 2, \ f_2 = 1000.$$

The initial values $\sigma(0)$ correspond to Euler angles of 1-2-3 system of $\theta(0) = [\theta_1(0) \ \theta_2(0) \ \theta_3(0)]^T = [0 \ 0 \ 0]^T$ [deg]. The desired states σ_d, ω_d and $\dot{\omega}_d$ in this simulation are the switching maneuver as shown in Fig. 5. As shown in Fig. 6, since the control system of the spacecraft is a digital control system, zero-order hold is assumed and the sampling interval T [s] is set to $T = 1$ in this simulation. The results of the numerical simulation are shown in Fig. 7. It can be seen that switching maneuver has been achieved by appropriately switching between RCS and RW.

5. CONCLUSION

In this paper, we considered the spacecraft attitude tracking control problem that requires agile and large angle attitude maneuvers using both RCS and RW and proposed a nonlinear control method. In the proposed method, it is indicated that the closed-loop system when using RCS only becomes ISS under the assumption that the quantization error is bounded, and the closed-loop system can be asymptotically stabilized when both RCS and RW are used. The effectiveness of the proposed control method is verified by numerical simulations. Minimizing fuel consumption by RCS injection, robustness against parameter fluctuations by fuel injection, and optimal switching between RCS and RW will be subject to future work.

REFERENCES

Kamiya, T., Maeda, K., and Ogura, N. (2006). Preshaping profile for flexible spacecraft rest-to-rest maneuver. *AIAA Guidance and Control Conference*, AIAA 2006–6181.

Khalil, H. (2001). *Nonlinear Systems - third edition.* Prentice Hall, New Jersey.

Koike, M. and Chida, Y. (2012). Multivariate control design considering quantization error and input time-delay for pneumatic isolation table. *Proc. of ASME 2012 5th Annual Dynamic Systems and Control Conference joint with the JSME 2012 11th Motion and Vibration Conference*, 1–10.

Meng, Z., Ren, W., and You, Z. (2009). Decentraized cooperative attitude tracking using modified rodriguez parameters. *Proceedings of the 48th IEEE Conference on Decision and Control held jointly with 28th Chinese Control Conference*, 853–858.

Ninomiya, T., Kida, T., Yamaguchi, I., Hamada, Y., and Sasa, S. (2004). On-off state feedback control of linear system by input-state linearization. *Journal of the Japan Society for Aeronautical ans Space Sciences*, 52, 189–195 (in Japanese).

Sakamoto, A., Ikeda, Y., Yamaguchi, I., and Kida, T. (2016). Nonlinear model predictive control for large angle attitude maneuver of spacecraft with rw and rcs. *Proceedings of the 55th IEEE Conference on Decision and Control*, 3202–3209.

Somov, Y., Butyrin, S., and Somov, S. (2008). Guidance and robust gyromoment precise attitude control of agile observation spacecraft. *17th IFAC World Congress*, 3422–3427.

Tsiotras, P. (1998). Further passivity results for the attitude control problem. *IEEE Transactions on Automatic Control*, 43, 1597–1600.

Wie, B., Bailey, D., and Heiberg, C. (2001). Singularity robust steering logic for redundant single-gimbal control moment gyros. *AIAA Journal of Guidance, Control, and Dynamics*, 24, 865–872.