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Exploiting Models of Different Granularity in Robust Predictive Control

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Abstract— **The use of detailed models over long horizons in predictive control can be computationally challenging. Furthermore, always-present uncertainty renders the use of such sophisticated detailed models over long time horizons questionable due to the resulting variability of the trajectories. We propose a multi-stage scheme that combines the use of models of different granularity – using detailed models for short-term predictions, while performing long-term predictions with less detailed models. Using projection and invariance properties for the different model complexities and the transitions between them, we show that this scheme is recursively feasible. In a simulation study, we show how two models of different complexity can be combined for steering a mobile robot through a landscape with obstacles.**

I. Introduction

Predictive control techniques compute control actions based on the predictions of the system's behavior that are obtained using a mathematical model. In particular, Model Predictive Control (MPC) solves an optimization problem in a receding-horizon fashion to compute optimal control actions subject to constraints, minimizing a given objective, see e.g. [1]–[4]. The accuracy of the model used for the predictions has a crucial impact on the closed-loop performance, as well as on the satisfaction of constraints and even on stability properties. Often, researchers and practitioners spend large efforts on the development of detailed mathematical models for optimal control. This trend can be observed in different fields, most notably in chemical engineering ([5], [6]).

However, while detailed models allow one to overcome model uncertainty, it is often still unavoidable due to the difficulty to accurately identify model parameters and because of the presence of unpredictable disturbances and variations. Such uncertainties put into question whether it is reasonable to use very detailed (but uncertain) models for long-term predictions as the unavoidable disturbance will deteriorate the prediction quality. One might wonder, if less detailed models could be sufficient for long horizons, as long as detailed models are used for short-horizon predictions.

Following this idea, we propose to use models of different granularity along the prediction horizon in this work. That is, detailed models are used for the near future, so that strict requirements can be met, but coarser models are used for distant and uncertain predictions, since the use of a detailed model might not increase the amount of information obtained from the predictions. The use of coarser models for distant predictions might not only reduce the computational effort by reducing the number of decision variables. It may also better suit the process of decision-making under uncertainty. That is, in many situations, long-term planning needs different types of decisions or different descriptions of the reality. For example, an automatically-driven car (see e.g. [7]) should decide on steering and acceleration on the short term, but it may decide on predefined tasks (such as overtaking, turning, stopping, etc.) on a larger horizon. To some degree, this idea of combining short-horizon control with long-horizon planning can even be based on behaviors observed in nature and biology, like the walking motion of a human or animal.

In practice, using different models for different prediction lengths is widely used ([8]). Optimization on the company level is typically performed statically, while on the factory and machine level, some kind of dynamic model is used. The optimizations are typically performed separately, however. Similar ideas have been described in the field of optimization, e.g. in [9], where the numerical approximations of the nonlinear program are updated at higher frequencies for the first part of the predictions, compared to the predictions of the distant future. However, to the best of our knowledge, no systematic theory for guaranteeing recursive feasibility and stability has been derived, yet.

Successfully connecting models of different granularity for the proposed control scheme relies on several assumptions. Most notably, a projection that guarantees consistency has to be found. Such projections as well as the difference in granularity potentially introduce uncertainties into the models, which need to be taken into account. Different strategies have been proposed to counteract the effect of uncertainty in MPC, usually named as robust MPC approaches. Examples are min-max MPC ([10], [11]), relaxation-based MPC ([12]), scenario-tree-based MPC ([13], [14]), and tube-based MPC ([15], [2]), among others ([16]).

While any robust MPC scheme could be exploited for the presented idea, we focus on tube-based methods to counteract the uncertainty that is introduced into the predictions by the use of a coarse model. For the outlined approach, we derive the conditions for the recursive feasibility of a predictive controller that uses different models along the prediction horizon based on invariance notions. We illustrate the approach with simulation results of a mobile robot.

The remainder of the paper is structured as follows. The problem setup with the necessary assumptions and the construction of an optimal control problem is presented in Section II. This leads to the main contribution of the paper – the proof of recursive feasibility – in Section III. The approach is illustrated by simulation results presented in Section IV before the paper is concluded with a summary and ideas on possibly future extensions in Section V.

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II. Problem Setup

We consider two models of a nonlinear discrete time system,

$$\begin{aligned} x^+ &= f(x,u) \\ \text{s.t. } x &\in \mathbb{X} \\ u &\in \mathbb{U} \end{aligned} \qquad (1) \qquad\qquad \begin{aligned} z^+ &= g(z,v) \\ \text{s.t. } z &\in \mathbb{Z} \\ v &\in \mathbb{V}, \end{aligned} \qquad (2)$$

where $x \in \mathbb{R}^{n_x}$ and $z \in \mathbb{R}^{n_z}$ denote the states, x^+ and z^+ the states at the next time instant, and $u \in \mathbb{R}^{n_u}$ and $v \in \mathbb{R}^{n_v}$ the inputs. The inputs and states are constrained by the compact and convex sets $\mathbb{U}$/$\mathbb{V}$ and $\mathbb{X}$/$\mathbb{Z}$, respectively.

Following the main idea of the paper, the models f and g are of different granularity, that is, (1) is a "detailed" model of a plant, while (2) is a "coarse" representation of that plant. Usually, $n_z \leq n_x$ and $n_v \leq n_u$. Note that the coarse model can have different inputs – such as set points – in comparison to the detailed model. We connect these different models over the prediction by the following projection.

Assumption 2.1 (Projection): We assume that there exists a surjective projection function $\mathrm{Proj} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_z} \times \mathbb{R}^{n_v}$ that maps the states x and inputs u of the detailed system (1) to the states z and inputs v of the coarse system (2).

Remark 2.2: One way to fulfill Assumption 2.1 is to use a reduced model of f for g, e.g. one that uses only a subset of the system states of f, or to exploit model reduction techniques ([17]).

Assumption 2.3 (Projection of Constraint Sets): We further assume that the constraint sets $\mathbb{Z}$ and $\mathbb{V}$ for the coarse model (2) are calculated by projecting the detailed model (1)'s constraint sets $\mathbb{X}$ and $\mathbb{U}$ using the projection function from Assumption 2.1, i.e, $(\mathbb{Z}, \mathbb{V}) = \mathrm{Proj}(\mathbb{X}, \mathbb{U})$. Proj being surjective implies that there exists at least one pair $(x, u) \in \mathbb{X} \times \mathbb{U}$ for each $(z, v) \in \mathbb{Z} \times \mathbb{V}$ so that $(z, v) = \mathrm{Proj}(x, u)$.

A. MPC with Models of Different Granularity

We use the detailed model f for the first part of the prediction in an optimal control problem, from the first time step at $k = 0$ up to a transition time step k_s (*short horizon*), and the coarse model g afterwards, from $k = k_\mathrm{s}$ up to a final time step k_f (*long horizon*).

During the long-horizon stage of the prediction, we capture the possible lack of some state and input information caused by using a coarse instead of a detailed model by an appended additive uncertainty d as well as uncertain initial conditions for the prediction with that coarse model. Extending (2), z_d and v_d denote the states and inputs of such an uncertain coarse system

$$\begin{aligned} z_\mathrm{d}^+ &= g(z_\mathrm{d}, v_\mathrm{d}) + d \\ \text{s.t. } z_\mathrm{d} &\in \mathbb{Z} \\ v_\mathrm{d} &\in \mathbb{V} \\ d &\in \mathbb{D}. \end{aligned} \qquad (3)$$

System (3), affected by the bounded uncertainty $d \in \mathbb{D}$, still has to satisfy the state and input constraints $\mathbb{Z}$ and $\mathbb{V}$. Note that bounding model uncertainty is in general difficult.

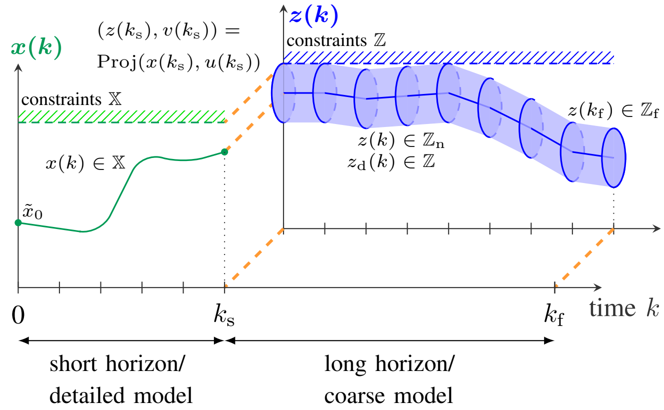


Fig. 1. Sketch of the control scheme. A detailed model is used for the short-horizon prediction from 0 to k_s, a coarse model is used for the long horizon up to k_f. Tube-based MPC controls the uncertain model, so that it satisfies constraints. A projection between the models guarantees consistency.

However, recent advances in uncertain model predictions ([18], [19]) raise hope that this will be less challenging in the near future. Furthermore, for certain systems, like the robot example considered in Section IV, this is possible based on physical insights.

In order to guarantee that using an uncertain model like (3) for the long-horizon stage of the prediction does not result in a violation of constraints, a robust control scheme needs to be applied, like those mentioned in Section I. We concentrate on ideas from tube-based MPC in this paper, as formulated e.g. in [2], [20], and [21]. Other robust MPC approaches would also work. In Section III, we build upon such ideas to prove recursive feasibility for the presented control scheme.

Commonly, in tube-based MPC, a standard MPC approach is used to find an optimal solution trajectory for a nominal system like (2), restricted by tighter constraint sets $\mathbb{Z}_\mathrm{n}$ and $\mathbb{V}_\mathrm{n}$. Then, local control laws v_d at each time step guarantee that an extended uncertain system like (3) is robustly stable around that nominal trajectory. This effectively spans a tube around the nominal trajectory, so that satisfaction of the tighter constraint sets $\mathbb{Z}_\mathrm{n}$ and $\mathbb{V}_\mathrm{n}$ by the nominal system states z with the controller v is equivalent to the uncertain system states z_d with controller v_d not violating the original constraint sets $\mathbb{Z}$ and $\mathbb{V}$, cf. Fig. 1.

Assumption 2.4 (Feasible Tube-based MPC): We assume that, given a constraint set $\mathbb{D}$ for the additive uncertainty d of an uncertain model (3), there exist tightened constraint sets $\mathbb{V}_\mathrm{n}$ and $\mathbb{Z}_\mathrm{n}$, such that, if $z \in \mathbb{Z}_\mathrm{n}$ and $v \in \mathbb{V}_\mathrm{n}$ for the nominal system (2), there exists $z_\mathrm{d} \in \mathbb{Z}$ and $v_\mathrm{d} \in \mathbb{V}$ for the uncertain dynamics (3).

In general, the control law v_d and the tightened constraint sets $\mathbb{Z}_\mathrm{n}$ and $\mathbb{V}_\mathrm{n}$ depend on the uncertainty constraint set $\mathbb{D}$. Details about the determination of a such a control law v_d can for example be found in [2].

The set $\mathbb{D}$ is typically derived from known bounds on the uncertainty in tube-based MPC approaches. Here, however, it needs to be calculated based on the "difference" in granularity of the two used system models. It serves to maintain consistency between the models, so that the uncertain coarse model (3) can be interpreted as an over-approximation of the detailed model (1) – while living in different state and input spaces.

Assumption 2.5 (Consistency of Models): We assume that we can calculate a constraint set $\mathbb{D}$, such that, if a control law v_{d}, robustly stabilizing a pair (z, v), can be found for that $\mathbb{D}$, there exists a pair (x, u) for the same time instance, for which $(z, v) = \mathrm{Proj}(x, u)$ and for which $\exists x^+ = f(x, u) \in \mathbb{X}$. It is further assumed that $\mathbb{D}$ is compact and contains the origin in its interior.

Then, the constraint sets $\mathbb{Z}_{\mathrm{n}}$ and $\mathbb{V}_{\mathrm{n}}$ for the nominal coarse system can be computed depending on $\mathbb{D}$.

B. Optimal Control Problem

We now formulate an optimization problem for the described scheme that finds optimal control inputs based on cost functions l_x, $V_{\mathrm{f},x}$, l_z, and $V_{\mathrm{f},z}$, starting from the current state $\tilde{x}_0$:

$$\min_{\substack{\{u(0),\ldots,u(k_{\mathrm{s}})\},\\ \{v(k_{\mathrm{s}}),\ldots,v(k_{\mathrm{f}}-1)\}}} \sum_{k=0}^{k_{\mathrm{s}}-1} l_x(x(k), u(k)) + V_{\mathrm{f},x}(x(k_{\mathrm{s}})) + \sum_{k=k_{\mathrm{s}}}^{k_{\mathrm{f}}-1} l_z(z(k), v(k)) + V_{\mathrm{f},z}(z(k_{\mathrm{f}})) \tag{4a}$$

$$\text{s.t.} \quad x(0) = \tilde{x}_0, \tag{4b}$$

$$x(k+1) = f(x(k), u(k)), \quad x(k) \in \mathbb{X}, \quad u(k) \in \mathbb{U}, \quad \forall k = 0, \ldots, k_{\mathrm{s}} - 1, \tag{4c}$$

$$(z(k_{\mathrm{s}}), v(k_{\mathrm{s}})) = \mathrm{Proj}(x(k_{\mathrm{s}}), u(k_{\mathrm{s}})) \subseteq \mathbb{Z}_{\mathrm{n}} \times \mathbb{V}_{\mathrm{n}}, \tag{4d}$$

$$z(k+1) = g(z(k), v(k)), \quad z(k) \in \mathbb{Z}_{\mathrm{n}}, \quad v(k) \in \mathbb{V}_{\mathrm{n}}, \quad \forall k = k_{\mathrm{s}}, \ldots, k_{\mathrm{f}} - 1, \tag{4e}$$

$$z(k_{\mathrm{f}}) \in \mathbb{Z}_{\mathrm{f}} \subseteq \mathbb{Z}_{\mathrm{n}} \ . \tag{4f}$$

The cost function (4a) shows the two-stage nature of the prediction. Following the constraint (4c), the detailed model f is used for the short-horizon prediction, while the coarse model g is used for the long-horizon prediction, via (4e). The constraint (4d) maps between the two models at the transition time k_{s}. The control invariant set $\mathbb{Z}_{\mathrm{f}}$ in constraint (4f) is used as a terminal region.

Note that $u(k_{\mathrm{s}})$ is an optimization variable (degree of freedom) for (4a), even though it is not used in the cost functions or in the prediction of the detailed model in constraint (4c). Due to the construction of the projection function Proj in Assumption 2.1, $u(k_{\mathrm{s}})$ is however necessary for finding a corresponding $v(k_{\mathrm{s}})$ in constraint (4d). At the same time, this can be interpreted as the long-horizon prediction stage being able to cope with uncertain initial conditions, similar to how tube-based MPC approaches can deal with those, see e.g. [2].

In the next section, we show that the presented optimization problem is recursively feasible. Considering a simple model for a mobile robot, we demonstrate the control approach in Section IV.

III. Recursive Feasibility

Establishing recursive feasibility for the presented approach, as a necessary requirement for use in a receding-horizon control scheme, can rely on standard MPC theory. However, special attention is required at the transitions where the prediction starts using the coarse model instead of the detailed one.

Assumption 3.1 (Initial Feasibility): We assume that the optimization problem (4), constructed based on the mentioned assumptions, is feasible for $k = 0$. That is, an optimizer can find a solution sequence $\{u(0|0), \ldots, u(k_{\mathrm{s}}|0); v(k_{\mathrm{s}}|0), \ldots, v(k_{\mathrm{f}} - 1|0)\}$.

Here, $u(i|j)$ denotes the control input at time i, calculated at time j. This notation can be used equivalently with x, z, and v. With Assumption 3.1 holding, Assumption 2.4 is also satisfied, since the tube-based MPC design is embedded into the optimization problem.

Proposition 3.2 (Recursive Feasibility): If Assumptions 2.1, 2.3, 2.5, and 3.1 hold, the optimization problem (4) remains feasible for all $k \geq 1$.

Proof: Since the optimization problem is feasible according to Assumption 3.1, we can shift the prediction horizon by one time step in the next iteration of the receding-horizon scheme. Then, the solution sequence for u from $k = 1$ to $k_{\mathrm{s}} - 1$ can be reused as it is still feasible after the shift. That is, $\{u(1|1), \ldots, u(k_{\mathrm{s}} - 1|1)\}$ can be set to $\{u(1|0), \ldots, u(k_{\mathrm{s}} - 1|0)\}$ as there is no model-plant mismatch.

The same is possible for $u(k_{\mathrm{s}}|1) = u(k_{\mathrm{s}}|0)$, even though $u(k_{\mathrm{s}}|0)$ has not been used for the prediction of a state $x(k_{\mathrm{s}} + 1|0) \in \mathbb{X}$ (compare constraint (4c)). That holds, as $x(k_{\mathrm{s}} + 1|1) \in \mathbb{X}$ will hold due to the consistency of the models assumed in Assumption 2.5, as a consequence of the feasibility of $(z(k_{\mathrm{s}}|0), v(k_{\mathrm{s}}|0)) \in \mathbb{Z}_{\mathrm{n}} \times \mathbb{V}_{\mathrm{n}}$. A new solution piece has to be found only for $u(k_{\mathrm{s}}+1|1)$. This will be possible as there was a feasible pair $(z(k_{\mathrm{s}} + 1|0), v(k_{\mathrm{s}} + 1|0)) \in \mathbb{Z}_{\mathrm{n}} \times \mathbb{V}_{\mathrm{n}}$, which could be reused as $(z(k_{\mathrm{s}} + 1|1), v(k_{\mathrm{s}} + 1|1))$, so that, by construction of the projection function in Assumption 2.1, there must be a pair $(x(k_{\mathrm{s}} + 1|1), u(k_{\mathrm{s}} + 1|1))$.

Similar to how $u(0|0)$ is applied to the controlled plant in a receding-horizon manner and then no longer taken into account in the further prediction, $v(k_{\mathrm{s}}|0)$ can be neglected in the next time step. Then, in a similar fashion as before, the solution sequence for v from $k = k_{\mathrm{s}} + 1$ to $k_{\mathrm{f}} - 1$ can be reused. Thus, $\{v(k_{\mathrm{s}} + 1|1), \ldots, v(k_{\mathrm{f}} - 1|1)\}$ can be set to $\{v(k_{\mathrm{s}} + 1|0), \ldots, v(k_{\mathrm{f}} - 1|0)\}$. A new $v(k_{\mathrm{f}}|1)$ can then always be found, as the tube at k_{f} is a control invariant set due to the terminal constraint (4f).

In summary, parts of the control input sequence from the previous iteration can be reused for both the detailed and the coarse system and an initial solution sequence exists from Assumption 3.1. Thus, the optimization problem (4) is recursively feasible. □

Note that we focus solely on recursive feasibility in this work. Showing stability in a tailored sense is also possible.

IV. Example

In this section, we use the presented scheme to control the motion of a simple robot through a known landscape with obstacles. The goal for the robot is to move from a starting point $\tilde{x}_0$ to a destination point $\tilde{x}_\mathrm{f}$ without touching the obstacles, which are modeled as time-varying state constraints, cf. Fig. 2.

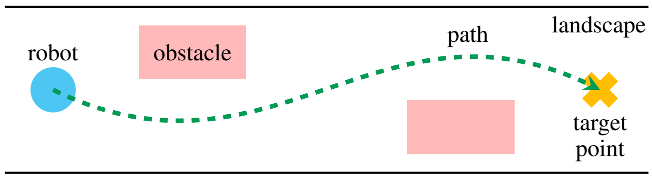


Fig. 2. Visualization of the considered control problem, a mobile robot moving to a target point in a landscape with obstacles.

A. Modeling a Simple Mobile Robot

We consider two models for the 2D motion of a mobile robot with omnidirectional wheels. A detailed model is given by a double integrator with sampling time Δt:

$$\underbrace{\begin{pmatrix} p_x \\ v_x \\ p_y \\ v_y \end{pmatrix}^{+}}_{x^+} = \underbrace{\begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} p_x \\ v_x \\ p_y \\ v_y \end{pmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 & 0 \\ \Delta t \frac{1}{m} & 0 \\ 0 & 0 \\ 0 & \Delta t \frac{1}{m} \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{pmatrix} F_x \\ F_y \end{pmatrix}}_{u}. \tag{5}$$

p_x and p_y resemble the robot's position in the 2D space and v_x and v_y its velocity in the two directions. The acceleration forces F_x and F_y serve as control inputs. The model parameters are $m = 0.5\,\mathrm{kg}$ and $\Delta t = 0.5\,\mathrm{s}$. Both the states and the control inputs are box-constrained: The states p_x and p_y must be in $[0, 30]\,\mathrm{m}$, p_y must be in $[-1, 1]\,\mathrm{m}$, the velocities v_x and v_y must be in $[-3, 3]\,\mathrm{m/s}$ and the acceleration forces F_x and F_y must be in $[-3, 3]\,\mathrm{N}$. Additionally, there are constraints on the positional state p_y resembling obstacles in the landscape of motion: $0.4\,\mathrm{m} \leq p_y \leq 1\,\mathrm{m}$ for $3\,\mathrm{s} \leq t \leq 3.5\,\mathrm{s}$ and $-1\,\mathrm{m} \leq p_y \leq -0.3\,\mathrm{m}$ for $7\,\mathrm{s} \leq t \leq 7.5\,\mathrm{s}$.

Further, we consider a simpler description of the robot's motion as the coarse model, given by

$$\underbrace{\begin{pmatrix} p_x \\ p_y \end{pmatrix}^{+}}_{z^+} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{A}_\text{coarse}} \underbrace{\begin{pmatrix} p_x \\ p_y \end{pmatrix}}_{z} + \underbrace{\begin{bmatrix} \Delta t & 0 \\ 0 & \Delta t \end{bmatrix}}_{\mathbf{B}_\text{coarse}} \underbrace{\begin{pmatrix} v_x \\ v_y \end{pmatrix}}_{v}, \tag{6}$$

which uses the detailed model's position states p_x and p_y, but does not know about the velocity dynamics v_x and v_y and instead treats those as control inputs. The given model parameters and constraint sets are reused correspondingly. Using such simplified models is standard in robotics and path planning, however, such approaches are typically applied heuristically. For this example, it is possible to find a simple projection function (compare Assumption 2.1)

$$\begin{pmatrix} z \\ v \end{pmatrix} = \mathrm{Proj}\left(\begin{pmatrix} x \\ u \end{pmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{pmatrix} x \\ u \end{pmatrix} \tag{7}$$

that maps both models at the switching time (see constraint (4d)).

B. Controller Design

Following the reasoning in Section II, we can derive an uncertain coarse model

$$\underbrace{\begin{pmatrix} p_{x,\mathrm{d}} \\ p_{y,\mathrm{d}} \end{pmatrix}^{+}}_{z_\mathrm{d}^+} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{A}_\text{coarse}} \underbrace{\begin{pmatrix} p_{x,\mathrm{d}} \\ p_{y,\mathrm{d}} \end{pmatrix}}_{z_\mathrm{d}} + \underbrace{\begin{bmatrix} \Delta t & 0 \\ 0 & \Delta t \end{bmatrix}}_{\mathbf{B}_\text{coarse}} \underbrace{\begin{pmatrix} v_{x,\mathrm{d}} \\ v_{y,\mathrm{d}} \end{pmatrix}}_{v_\mathrm{d}} + \underbrace{\begin{pmatrix} d_x \\ d_y \end{pmatrix}}_{d}, \tag{8}$$

whose uncertain states and inputs are still constrained, i.e., $z_\mathrm{d}^+ \in \mathbb{Z}$ and $v_\mathrm{d}^+ \in \mathbb{V}$. In this example, there is no uncertain initial condition for the states of the coarse model, as those are also states of the detailed model. In general, however, an initial uncertainty can be caused by the projection at the transition time (cf. Fig. 1).

In order to design the proposed controller, it is necessary to compute the set $\mathbb{D}$ so that Assumption 2.5 is satisfied. While, in general, this computation can be challenging, in many cases, physical knowledge about the system can be used to derive a suitable $\mathbb{D}$. For the given example, the coarse model neglects the robot's velocity dynamics and instead assumes that it can be changed instantaneously as a control input. Due to those dynamics, this assumption does not hold for the detailed model, because of the constrained acceleration forces. Since the maximum acceleration force of the detailed model is $\pm 3\,\mathrm{N}$, resulting in a maximum acceleration of $\pm 6\,\mathrm{m/s^2}$ for a $0.5\,\mathrm{kg}$ robot, we can tighten the velocity constraints for the coarse model to $\pm 1.5\,\mathrm{m/s}$, with the sampling time $\Delta t = 0.5\,\mathrm{s}$. In this example, this fulfills Assumption 2.5 for any $\mathbb{D}$. To illustrate that tightening the constraint sets does not necessarily lead to conservatism, we also enlarge the obstacles in the coarse prediction by $0.1\,\mathrm{m}$.

We use standard quadratic stage cost functions $l_x(x, u) = (x - x_\text{ref})^\intercal \mathbf{Q}_x (x - x_\text{ref}) + (u - u_\text{ref})^\intercal \mathbf{R}_x (u - u_\text{ref})$ and $l_z(z, v) = (z - z_\text{ref})^\intercal \mathbf{Q}_z (z - z_\text{ref}) + (v - v_\text{ref})^\intercal \mathbf{R}_z (v - v_\text{ref})$. The terminal costs are chosen to be equal to the stage costs and the weighting matrices are $\mathbf{Q}_x = \mathrm{diag}(1, 0, 10, 0)$, $\mathbf{R}_x = \mathrm{diag}(0.1, 0.1)$, $\mathbf{Q}_z = \mathrm{diag}(1, 1)$, and $\mathbf{R}_z = \mathrm{diag}(1, 1)$. The transition time step is set to $k_\mathrm{s} = 5$ and the overall prediction horizon is $k_\mathrm{f} = 20$, equaling $10\,\mathrm{s}$. We use $x_\text{ref} = \begin{pmatrix} 30 & 0 & 0 & 0 \end{pmatrix}^\intercal$ and $u_\text{ref} = \begin{pmatrix} 0 & 0 \end{pmatrix}^\intercal$ and their projections $z_\text{ref} = \begin{pmatrix} 30 & 0 \end{pmatrix}^\intercal$ and $v_\text{ref} = \begin{pmatrix} 0 & 0 \end{pmatrix}^\intercal$ as the references in the cost functions.

C. Simulation Results

In the following, we present the results obtained from the solution of optimization problem (4) in a receding horizon manner. Simulations were performed using MATLAB's `fmincon` solver on a standard desktop PC.

Fig. 3 shows the predicted optimal trajectories of the states and control inputs of the detailed model (left) and of the coarse model (right). It can be seen that the constraint for the first obstacle is satisfied during the detailed prediction. During the longer coarse predictions, a tightened constraint as explained above, resembling an enlarged second obstacle, is satisfied.

As can be seen in Fig. 4, the proposed approach successfully drives the system to the target point of the detailed model $x_{\text{ref}} = \begin{pmatrix} 30 & 0 & 0 & 0 \end{pmatrix}^{\mathsf{T}}$. The conservativeness that one might expect from the satisfaction of the tightened constraints in the coarse predictions, see Fig. 3(b), is not visible in the closed-loop results. It vanishes once the obstacles enter the detailed prediction horizon, where their corresponding constraint sets are no longer tightened. As a result, the closed-loop trajectory can exploit the obstacle constraints better than predicted in the coarse predictions (see Fig. 4(a)).

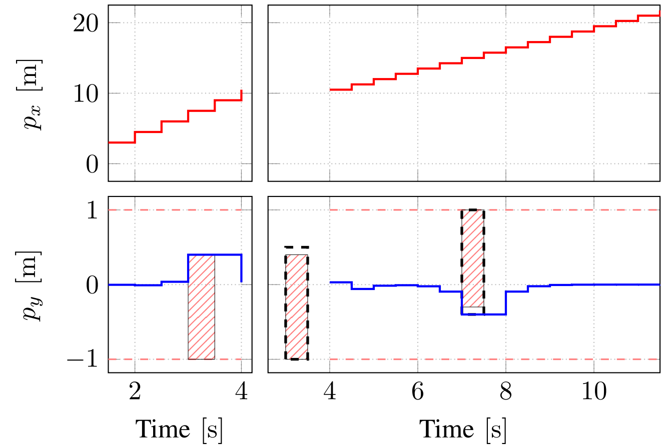
While the same constraint satisfaction can be achieved with a standard MPC controller that uses the detailed model for the full prediction, the presented approach only needs a coarse model for a large part of that prediction. Additionally, even without special optimizations in the code, the approach was roughly 30 % faster per MPC iteration than a standard MPC controller. While exact numbers will depend on the actual control problem, one can in general expect a reduced computational cost.

V. Summary and Conclusions

In this work, we presented an extended Model Predictive Control scheme that exploits the granularity of different models for a plant by using them for a prediction in consecutive stages. While a detailed model is used for the short-term prediction, a coarse or reduced model is used for long-term prediction. This can be interpreted as using the detailed model for the calculation of control inputs for the immediate future, while using a rougher or even uncertain model for path or trajectory planning in the far future, where exact control actions are not yet necessary. By design, one could even use different mathematical system classes or types for the models. Compared to standard MPC approaches, the scheme can also reduce the amount of decisions for an optimizer – and therefore the computational effort – while maintaining equally long predictions horizons.

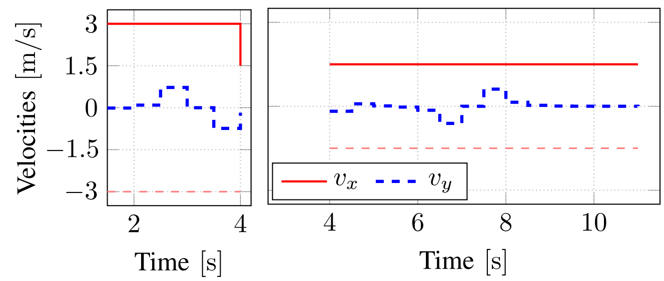
Based on assumptions about the existence of suitable projections between the spaces that the different models live in, we showed that the approach is recursively feasible. Thus, it is possible to use the scheme in a receding-horizon controller, which is then further shown in a simulation example. In that, different models for a simple mobile robot are combined, allowing the robot to predict far into the future in order to find a way through an obstructed landscape, while having to compute a detailed control signal only for a short horizon.

Possible extensions to the presented approach include using several models of different granularity, instead of just two models, as used for brevity in this work. This requires suitable projection functions for those models that share common states and inputs, in their respective state and input spaces. It is also possible to use an uncertain model even in the first stage, which by itself is already controlled by a robust MPC scheme. This idea is visualized in Fig. 5.



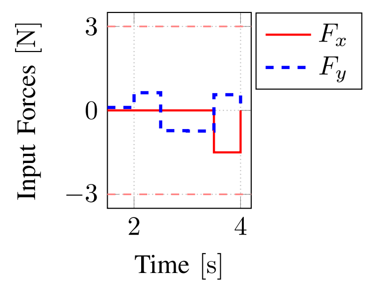
(a) Predicted position states for the detailed model.

(b) Predicted position states for the coarse model. The dashed black line indicates the tightened constraint sets.



(c) Predicted velocity states for the detailed model.

(d) Calculated control inputs for the coarse model.



(e) Calculated control inputs for the detailed model.

Fig. 3. Prediction sequences for the detailed (left) and coarse models (right) at the third time step of the simulation ($t = 1.5\,\text{s}$). The transition between the models occurs at $t = 4\,\text{s}$.

Building upon the presented results for recursive feasibility, future works could focus on other system properties like stability. It would also be possible to investigate how other types of uncertainty, e.g. uncertain parameters, could be exploited. Further research could be put into finding efficient ways for calculating the constraint sets that are used by the approach.

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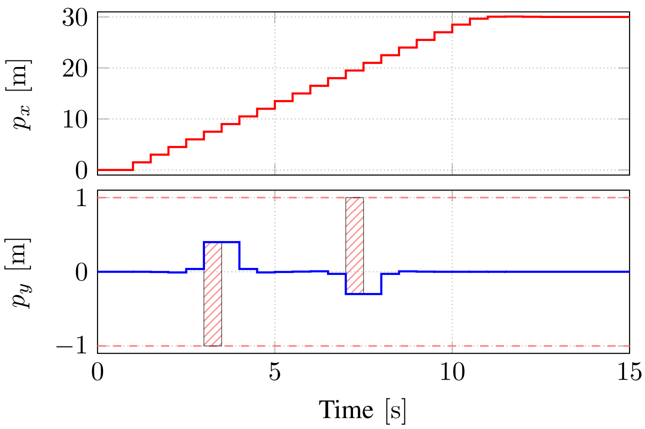
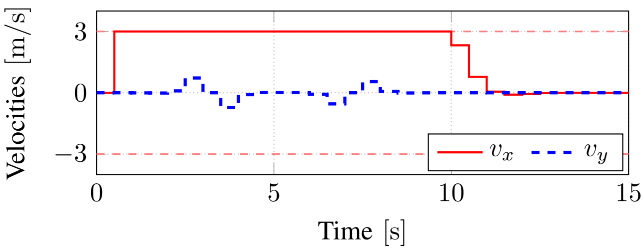
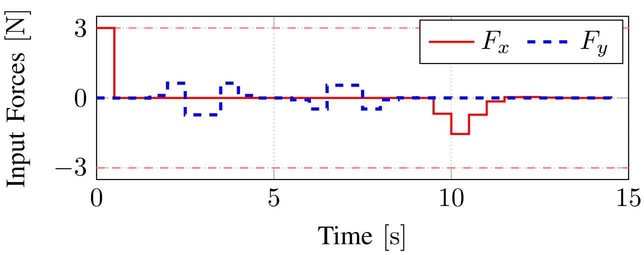


(a) Position states p_x and p_y in the landscape with obstacles.



(b) Velocity states v_x and v_y.



(c) Control inputs F_x and F_y.

Fig. 4. Simulation results for the closed-loop control of the robot.

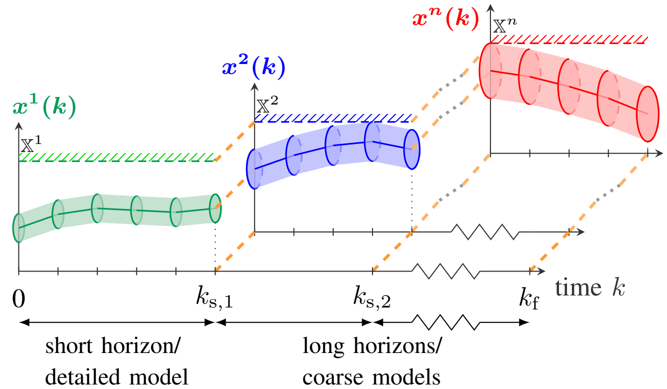


Fig. 5. Sketch of an extended control scheme that uses multiple, possibly uncertain, models that are connected with projections for consistency.

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