

## Differentialgleichungen

$$\dot{x}(t) = a x(t) \rightarrow x(t) = e^{at} \rightarrow a < 0 \rightarrow \text{asympt. stabil}$$

$$a=0 \rightarrow \text{stabil}$$

$$a>0 \rightarrow \text{instabil}$$

$$\dot{x}_0(t) = 0 = a x_0(t)$$

## Linearisierung

$\rightarrow$  Jacobimatrix 6ilde  $\xrightarrow{\text{alle } \lambda < 0} \rightarrow$  asympt. stabil  
 $J^M$  mind. ein  $\lambda = 0 \rightarrow$  stabil  
 $\downarrow$  mind. ein  $\lambda > 0 \rightarrow$  instabil

$$( \dot{y}(t) = J M \cdot \underline{y}(t) )$$



## Transformation

$$\ddot{y}(t) + a \dot{y}(t) + b y(t) = 0$$

$$y(t) = x_1(t)$$

$$\dot{y}(t) = \dot{x}_1(t) = x_2(t)$$

$$\ddot{x}_2(t) = \ddot{y}(t) = -a \dot{y}(t) - b y(t) \rightarrow \text{wieder } \lambda \text{ betrachten}$$

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix} \underline{x}$$

## Koordinatentransformation

zunächst wie Differenzengl. now  $y^{(k+1)} = \dot{x}$   
 $y^{(k)} = x$

$$\rightarrow \dot{\underline{z}}(t) = \underbrace{T^{-1} \cdot A \cdot T}_{\left[ \begin{smallmatrix} 0 & 1 \\ -b & -a \end{smallmatrix} \right]} \underline{z}(t)$$

$$\rightarrow z_1(t) = \lambda_1 z_1(t) \rightarrow z_1(t) = e^{\lambda_1 t} \quad z_0 = T^{-1} \underline{x}_0$$

$$\rightarrow z_2(t) = \lambda_2 z_2(t) \rightarrow z_2(t) = e^{\lambda_2 t}$$

$$\rightarrow x(t) = T \underline{z}(t) = z_{10} \cdot e^{\lambda_1 t} \left( \begin{array}{c} 1 \\ 0 \end{array} \right) + z_{20} e^{\lambda_2 t} \left( \begin{array}{c} 0 \\ 1 \end{array} \right)$$

$\xrightarrow{\text{EW zu } \lambda_1}$

(erste Spalte von T)

$\xrightarrow{\text{EW zu } \lambda_2}$

# Differenzengleichungen

$$y(k+1) = ay(k)$$

$$Y(k) = a^k y_0 \rightarrow |a| < 1 \text{ asympt. stabil}$$

RL:  $y_0 = y(k+1) = y(k)$

- $|a| < 1$  stabil
- $|a| = 1$  stabil
- $|a| > 1$  instabil

## Linearisierung:

$$x(k+1) = f(x(k)) \approx f(x_s) + \frac{df}{dx(k)}|_{x(k)=x_s} (x(k) - x_s) \quad \underbrace{(x(k) - x_s)}_{\Psi(k)}$$

$$x(k+1) - f(x_s) = \Psi(k+1)$$

$$\Psi(k+1) = \frac{df}{dx(k)}|_{x(k)=x_s} \Psi(k)$$

wenn das

$  \cdot   < 1$	asympt. stabil
$  \cdot   = 1$	stabil
$  \cdot   > 1$	instabil

## Transformation:

$$y(k+2) + ay(k+1) + by(k) = 0$$

$$\rightarrow y(k) = x_1(k)$$

$$\rightarrow y(k+1) = x_1(k+1) = x_2(k)$$

$$\begin{aligned} \rightarrow x_2(k+1) &= y(k+2) \\ &= -by(k) - ay(k+1) \end{aligned}$$

$$\left. \begin{aligned} x(k+1) &= \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix} x(k) \\ &\quad \downarrow A \end{aligned} \right\}$$

→ Eigenwerte von A:

wenn alle $ \lambda  < 1$	→ asympt. stabil
wenn einer $ \lambda  = 1$	→ stabil
wenn einer $ \lambda  > 1$	→ instabil

## Koordinatentransformation:

$$\begin{aligned} \underline{x}(k) &= T \underline{z}(k) \\ \underline{x}(k+1) &= A \underline{z}(k) \end{aligned} \quad \left. \begin{aligned} \Rightarrow T \underline{z}(k+1) &= AT \underline{z}(k) \rightarrow \underline{z}(k+1) = \underbrace{T^{-1}AT}_{\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}} \underline{z}(k) \end{aligned} \right\}$$

$$\rightarrow z_1(k+1) = \lambda_1 z_1(k) \rightarrow z_1(k) = \lambda_1^k z_{10}$$

$$\rightarrow z_2(k+1) = \lambda_2 z_2(k) \rightarrow z_2(k) = \lambda_2^k z_{20}$$

$$\underline{z}_0 = T^{-1} \underline{x}_0$$