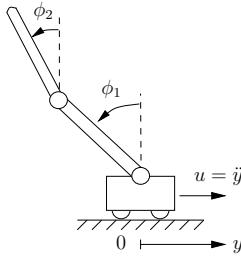


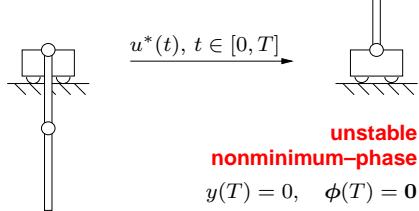
## Swing-Up of the Double Pendulum on a Cart



- Equations of motion**
- |   |
|---|
| $\Sigma_{IO} : \ddot{y} = u$                            |
| $\Sigma_{IN} : \dot{\phi} = \beta(\phi, \dot{\phi}, u)$ |
- Input-output dynamics  $\Sigma_{IO}$  for cart position  $y(t)$
  - Internal dynamics  $\Sigma_{IN}$  for angles  $\phi(t) = [\phi_1(t), \phi_2(t)]^\top$

- Control task:** finite-time transition between equilibrium points

stable  
minimum-phase  
 $y(0) = 0, \phi(0) = -\pi$



⇒ **Nonlinear boundary value problem (BVP)** for  $y(t)$  and  $\phi(t)$ ,  $t \in [0, T]$  depending on input  $u(t)$  and swing-up time  $T$

### Feedforward control design

- Feedforward control with **inverse input-output dynamics**  $\Sigma_{FF} = \Sigma_{IO}^{-1}$

$u^*(t) = \ddot{y}^*(t), t \in [0, T]$  with output trajectory  $y^*(t)$

- BVP of internal dynamics**  $\Sigma_{IN}^* \Rightarrow$  **overdetermined**

$$\ddot{\phi}^* = \beta(\phi, \dot{\phi}, \ddot{y}^*), \phi^*(0) = -\pi, \phi^*(T) = 0, \dot{\phi}^*|_{0,T} = 0$$

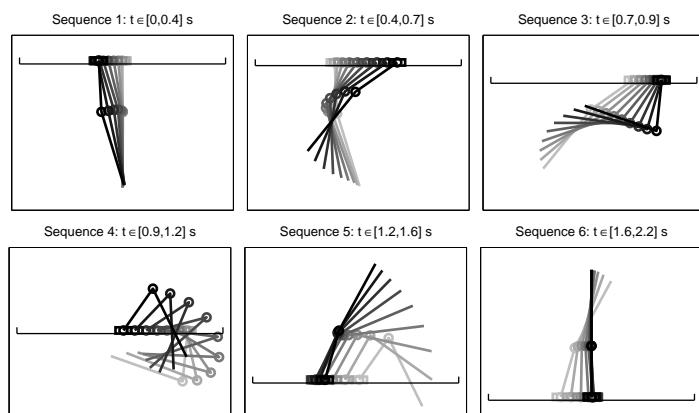
- Output trajectory  $y^*(t) = \Upsilon(t, p)$  with **free parameters**  $p = (p_1, \dots, p_4)$

$$\Upsilon(t, p) = a_0 + a_1 \cos\left(\pi \frac{t}{T}\right) + \sum_{i=2}^5 p_{i-1} \cos\left(i\pi \frac{t}{T}\right)$$

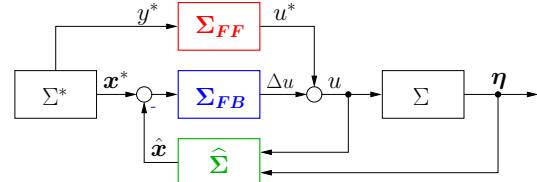
BCs for  $y^*(t)$  satisfied due to  $a_0 = -p_1 - p_3$  and  $a_1 = -p_2 - p_4$

- Numerical solution with **Matlab function bvp4c**

- Snapshots** of swing-up maneuver for  $t \in [0, 2.2] \text{ s}$

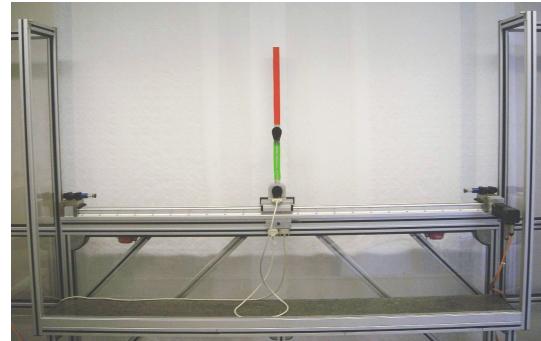


### Two-degree-of-freedom (2dof) control scheme

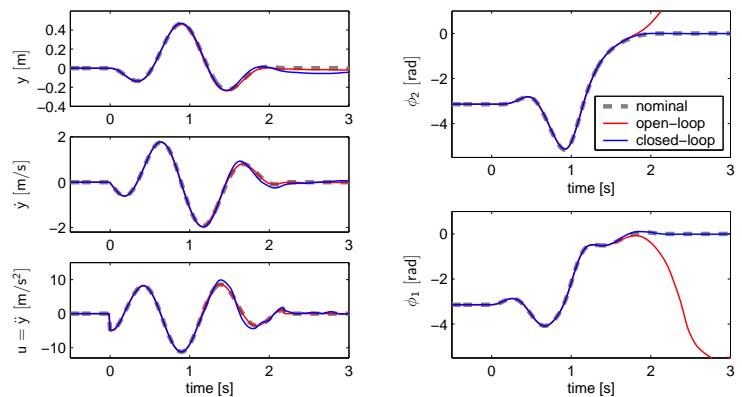


- Feedforward**  $\Sigma_{FF}$  for tracking performance with optimization-based adjustment of model parameters to enhance accuracy
- Linear feedback**  $\Sigma_{FB}$  for stabilization along nominal trajectories  $x^*(t)$  (reverse-time integration of Riccati-ODE ⇒ time-varying feedback gains)
- Tracking observer**  $\hat{\Sigma}$  of Luenberger type for state estimation  $\hat{x}$

### Experimental results



- Construction of pendulum by **MPG Magdeburg** and **Hasomed GmbH**
- Implementation under **RT-Linux** with sampling time 1 ms



⇒ **Experimental trajectories** show high tracking performance and difference between **open-loop / closed-loop** control

⇒ **Video:** <http://www.isys.uni-stuttgart.de/~graichen/2xpendulum.mpg>

### References

- K. Graichen, V. Hagenmeyer, M. Zeitz. A new approach to inversion-based feedforward control for nonlinear systems. *Automatica*, vol. 41, pp. 2033–2041, 2005.
- K. Graichen, M. Treuer, M. Zeitz. Swing-up of the double pendulum on a cart by feed-forward and feedback control with experimental validation. *Automatica* (acc.), 2006.
- K. Graichen. *Feedforward Control Design for Finite-Time Transition Problems of Nonlinear Systems with Input and Output Constraints*. PhD Thesis (to be published), 2006.