

**Please note the following:**

- The exam consists of 6 problems for solution; the points you can get for each problem are given in parentheses next to the number of the problem. Partial solutions are also accepted. For problems 2–6 it is not enough, however, to state the result only, but you should clearly display your approach and your way to the solution.
  - You can reach 43 points. For passing the exam you have to achieve a total of (at least) **18 points** out of all problems.
  - You are allowed to use: Text books, mathematical and/or statistical tables, manuscripts and notes from lectures and/or exercises, pocket calculators subject to the restrictions of the FWW.
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**Problem 1 (5 points)**

Each of following statements is either TRUE or FALSE. So just answer TRUE or FALSE, an explanation is not necessary.

- If univariate data  $x_i$  ( $1 \leq i \leq n$ ) are transformed to  $y_i = 10x_i$  ( $1 \leq i \leq n$ ), then the ratio of mean and standard deviation,  $\bar{x}/s_x$  and  $\bar{y}/s_y$ , remains the same.
- If  $A$  and  $B$  are two events of a probability space and  $A \subset B$ , then  $P(A \cap B) = P(B)$ .
- If  $X \sim B(1, p)$ , then  $1 - X \sim B(1, 1 - p)$ .
- If  $X$  is a univariate random variable with  $E(X) = \mu$  and  $\text{Var}(X) = \sigma^2$ , then  $E(X^2) = \mu^2 + \sigma^2$ .
- If  $X \sim \mathcal{N}(\mu, \sigma^2)$ , then  $X/2 \sim \mathcal{N}(\mu/2, \sigma^2/2)$ .

**Problem 2 (8 points)**

Consider the following sample:

13.9 12.0 13.6 6.4 9.9 9.5 5.2 10.8 6.8 14.2

- Compute the empirical distribution function at  $x = 8.0$ ,  $x = 10.0$ , and  $x = 12.0$ .
- The sum of the squared sample values is 1134.7. Calculate sample mean and standard deviation.
- Determine sample median and interquartile range.

**Problem 3 (6 points)**

In a production process wires are produced. The length of a produced wire (in meter) is known to follow a normal distribution with expectation  $\mu = 30$  and variance  $\sigma^2 = 9$ . A wire is said to be within tolerance limits if its length is between 27.5 and 35.0.

- Calculate the probability that a produced wire is within tolerance limits.
- Is it true that, in the long run, 90% or more of produced wires have length greater than 25 meter?

**Problem 4 (6 points)**

The duration (in minutes) of a job is viewed as a random variable with expectation  $\mu = 60$  and standard deviation  $\sigma = 5$ .

- Calculate the lower Chebychev bound for the probability that a job will take between 45 and 75 minutes.
- Under the additional assumption that the duration is normally distributed, calculate the probability that a job will take between 45 and 75 minutes.

**Problem 5 (10 points)**

A (univariate) continuous random variable  $X$  has the density

$$f_X(x) = \begin{cases} 3/x^4 & , \text{ if } x \geq 1 \\ 0 & , \text{ if } x < 1 \end{cases}$$

- Calculate the distribution function  $F_X(x)$ .
- Calculate expectation and standard deviation of  $X$ .
- Calculate the probability of the event  $0 < X < 2$ .

**Problem 6 (8 points)**

Consider two random variables  $X_1$  and  $X_2$  whose possible values are 0, 3, and 4, and whose joint distribution is given by

		$x_2$		
		0	3	4
$P(X_1 = x_1, X_2 = x_2) :$	0	$\frac{1}{12}$	$\frac{1}{3}$	0
	$x_1$ 3	$\frac{1}{3}$	0	0
	4	0	0	$\frac{1}{4}$

- Calculate the marginal distributions  $P_{X_1}$  and  $P_{X_2}$ .
- Are  $X_1$  and  $X_2$  uncorrelated ?