Stochastic Control for DC–DC Power Converters: A Generalized Minimum Variance Control Approach

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Abstract: This paper proposes a Generalized Minimum Variance (GMV) controller to regulate the output of the DC-DC power converters in order to decrease the impact of the noise in the system performance. The Dual Active Bridge and the Buck converters are used to illustrate the proposed methodology. The proposed controller is designed using the stochastic augmentation methodology that provides a Proportional-Integral-Derivative (PID) controller, in the Reference Signal Tracking (RST) structure, the ability to adequately treat noise of a stochastic nature. The GMV control reduces the variance of the control signal and in turn leads to a better output characteristic. The simulation results show that the GMV controller achieve better performance in the sense of minimum variance and energy consumption in comparison with a robust PID controller.

Keywords: minimum variance control, stochastic control, stochastic Augmentation, dc-dc converter, parametric uncertain, robust control.

1. INTRODUCTION

Since the integration of power electronics systems are rising every time into industrial applications, making more efficient control systems has become strong interested trend in power electronic applications (Lucas et al., 2019c; Hossain et al., 2018). Thereby, the aim is to enhance the efficiency of the power electronic converter.

DC-DC power converters have been extensively utilized in various kinds of dc voltage regulation to adapt the voltages between source and load (Hossain et al., 2018; Lucas et al., 2019b).

DC-DC power converters are generally controlled by Pulse Width Modulation (PWM) strategy (Lucas et al., 2019c). This pulse timing processing scheme is based on comparing the control signal to the periodic signal in every carrier period. As a result, large switching noise peaks come to appear (Sugahara and Matsunaga, 2016; Sangswang and Nwankpa, 2004). This switching process leads practical difficulties in clearing noise regulations (Sangswang and Nwankpa, 2004). Thus, an appropriate control is necessary to improve the quality and reliability of such control loops.

Sangswang and Nwankpa (2004) show the impact of noise in a switching PWM converter. However, in most cases where random fluctuations are always present, deterministic concepts simply ignore existing disturbances in converter modeling, assuming that the system is free of noise.

In real applications, such an assumption is never justified since all systems generate one or more types of noise internally and are subjected to external interferences (Sangswang and Nwankpa, 2004). In fact, the noise processes are due to disturbances from various components in power electronic system including parasitic effects, measurement and sensor inaccuracies, EMI, ambient temperature effect, switching signal, and ripples (Sangswang and Nwankpa, 2004).

Therefore, the noise may degrade the performance of the power converters when they are regulated by deterministic approaches because, in many cases, they lack the addressal of physical realizations of uncertainties in system parameters (Sangswang and Nwankpa, 2004).

In other related areas, stochastic approaches have been addressed to quantitatively express the reliability of the dynamical systems (Silveira et al., 2016; Trentini et al., 2016; Silveira and Coelho, 2011; Silva and Silveira, 2018; Pinheiro et al., 2016).

This paper proposes a Generalized Minimum Variance (GMV) controller to regulate the output voltage of a DC-DC Buck Converter and a Dual Active Bridge (DAB) DC-

⋆ This work was partially funding by the Personnel Improvement Coordination of Superior Level (CAPES/Brazil) and ESPOL Polytechnic University under Doctoral grant Finance Code 001 and the R&D Project GI-GISE-FIEC-01-2018, respectively.

DC Converter, in order to decrease the impact of noise on the systems. The GMV controller is compared to a robust controller based on robust parametric control approach (Bhattacharyya et al., 1995; Lucas et al., 2019c).

The GMV controller is a stochastic controller (Aström, 1970) that considers a full description of system's deterministic and stochastic parts (Aström and Wittenmark, 1973; Clarke and Gawthrop, 1975).

The stochastic control methodology is based on Stochastic Augmentation (SA) by minimum variance (Silveira et al., 2016; Trentini et al., 2016; Clarke and Gawthrop, 1975).

To the best of the authors' knowledge, the proposed GMV control approach, applied to the DC-DC power converters, is presented in this paper for the first time.

Therefore, the main contribution of this work is the adaptation of the GMV control for DC-DC power converters to overcome the negative effect of noise in these systems, resulting in more efficient systems.

The remainder of this paper is organized as follows. Section 2 presents a brief review of the GMV control based on linear RST controllers. Section 3 shows the evaluation of the proposed controller into DC-DC power converters. Finally, Section 4 presents the main conclusions.

2. REVIEW OF GMV CONTROL-BASED INCREMENTAL RST CONTROLLER

In order to design the GMV controller, first, a Reference Signal Tracking (RST) structure for linear digital controllers is introduced that counts on three arbitrary polynomial filters, which are responsible for plant's output closed-loop behavior, given by

$$\begin{cases} R(z^{-1}) = r_0 + r_1 z^{-1} + \cdots + r_{n_a} z^{-n_r} \\ S(z^{-1}) = s_0 + s_1 z^{-1} + \cdots + s_{n_b} z^{-n_s} \\ T_{RST}(z^{-1}) = t_0 + t_1 z^{-1} + \cdots + t_{n_c} z^{-n_t} \end{cases} \quad (1)$$

Hence, Incremental RST control law is given by,

$$\Delta u(k) = \frac{T_{RST}(z^{-1}) y_r(k) - S(z^{-1}) y(k)}{R(z^{-1})} \quad (2)$$

where $y(k)$, $y_r(k)$ and $u(k)$ are sampled output, controller reference and control signal, respectively, and $\Delta = 1 - z^{-1}$.

Considering a type-0 AutoRegressive-Moving-Average with eXogenous inputs (ARMAX) plant model (3),

$$A(z^{-1}) y(k) = B(z^{-1}) z^{-d} u(k) + C(z^{-1}) \xi(k) \quad (3)$$

with,

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + \cdots + a_{n_a} z^{-n_a} \\ B(z^{-1}) &= b_0 + b_1 z^{-1} + \cdots + b_{n_b} z^{-n_b} \\ C(z^{-1}) &= 1 + c_1 z^{-1} + \cdots + c_{n_c} z^{-n_c} \end{aligned}$$

being d the sample delay, z^{-1} is the backward shift operator and $\xi(k)$ is a Gaussian white noise sequence, the the following closed-loop expression can be obtained

$$y(k) = \underbrace{\frac{z^{-d} B T_{RST}}{R\Delta A + z^{-d} BS}}_{\text{Deterministic}} y_r(k) + \underbrace{\frac{C}{R\Delta A + z^{-d} BS}}_{\text{Sensitivity}} \xi(k) \quad (4)$$

where $\Delta A(z^{-1})$ is the augmented model of $A(z^{-1})$ due to the incremental structure of the RST controller, resulting in the Autoregressive Integrated Moving Average model with eXogenous inputs (ARIMAX) model (5).

$$\Delta A(z^{-1}) y(k) = B(z^{-1}) z^{-d} \Delta u(k) + C(z^{-1}) \xi(k) \quad (5)$$

For the deterministic transfer function (4), the sample delay d appears explicitly in the numerator, which means that the closed-loop system does not compensate it.

The GMV control is one of the simplest Model-based Predictive Control (MPC) techniques (Clarke and Gawthrop, 1975). It counts on Minimum Variance Predictor (MVP) (Aström and Wittenmark, 1973) in a generalized form similar to an RST controller, such that, a predictive generalized output (Clarke and Gawthrop, 1975),

$$\begin{aligned} \phi(k+d) = &P(z^{-1}) y(k+d) - T(z^{-1}) y_r(k+d) \\ &+ Q(z^{-1}) u(k) \end{aligned} \quad (6)$$

with $P(z^{-1})$, $T(z^{-1})$ and $Q(z^{-1})$ being arbitrary weighting filters for system's output, reference and control signal respectively. The generalized output $\phi(k+d)$ is posed into a stochastic optimization problem of minimizing the GMV cost function,

$$\text{J} = \mathbb{E}[\phi^2(k+d)] \quad (7)$$

while $\mathbb{E}[.]$ denotes the mathematical expectation operator.

Due to the incremental structure of the RST Controller, the generalized output is redefined as

$$\begin{aligned} \phi(k+d) = &P(z^{-1}) y(k+d) - T(z^{-1}) y_r(k+d) \\ &+ Q(z^{-1}) \Delta u(k) \end{aligned} \quad (8)$$

Clarke and Gawthrop (1975) formulation of the GMV control problem is based on stochastic plant models, i.e. ARMAX models (3) where $C(z^{-1})/A(z^{-1})$ introduces the stochastic model of a Gaussian disturbance sequence $\xi(k)$.

In (7), future reference data, $y_r(k+d)$, is supposed to be known a priori, but $y(k+d)$ is not. Then arises the problem of predicting the output d-steps ahead to compensate the time delay. Shifting (3) d-steps ahead gives

$$\Delta A(z^{-1}) y(k+d) = B(z^{-1}) \Delta u(k) + C(z^{-1}) \xi(k+d) \quad (9)$$

note that this model (9) come from the augmented model (5).

Due to $\xi(k+d)$, the best achievable solution to determine $y(k+d)$ and consequently $\phi(k+d)$ is through optimal estimation theory and the Kalman filter results that leads to the minimum variance prediction of $\hat{y}(k+d|k)$, i.e. the d-steps ahead estimated output given by Clarke and Gawthrop (1975) as being

$$\hat{y}(k+d|k) = \frac{B(z^{-1}) E(z^{-1})}{P(z^{-1}) C(z^{-1})} u(k) + \frac{F(z^{-1})}{P(z^{-1}) C(z^{-1})} y(k) \quad (10)$$

where $E(z^{-1})$ and $F(z^{-1})$ are the MVP polynomials determined by the solution of the Diophantine equation for the incremental case,

$$P(z^{-1})C(z^{-1}) = \Delta A(z^{-1})E(z^{-1}) + z^{-d}F(z^{-1}) \quad (11)$$

where

$$E(z^{-1}) = 1 + e_1 z^{-1} + \cdots + e_{(d-1)}z^{-(d-1)}$$
$$F(z^{-1}) = f_0 + f_1 z^{-1} + \cdots + f_{(n_f-1)}z^{-(n_f-1)}$$

The discrete time delay d is what defines the $E(z^{-1})$ and $F(z^{-1})$ polynomials complexity.

According to Clarke and Gawthrop (1975), a solution to (9) is

$$\begin{cases} n_e = d-1 \\ n_f = \max\{n_p + n_c - 1\ ,\ n_a - 1\} \end{cases} \quad (12)$$

then, the incremental GMV control law that will ensure in steady-state, which the variance of the generalized output (σ_ϕ^2) is a minimum, is given by

$$\Delta u(k) = \frac{C(z^{-1})T(z^{-1})y_r(k+d) - F(z^{-1})y(k)}{B(z^{-1})E(z^{-1}) + \lambda C(z^{-1})Q(z^{-1})} \quad (13)$$

where $\lambda \in \mathbb{R}^+$ is a scalar control energy weighting factor and is set arbitrarily by the designer in order to achieve the design requirements.

The closed-loop polynomial (12) for the incremental GMV control is given by (Silveira et al., 2016)

$$y(k) = \underbrace{\frac{BT}{\lambda Q\Delta A + BP}}_{\text{Deterministic}} y_r(k+d) + \underbrace{\frac{C\lambda Q + BE}{\lambda Q\Delta A + BP}}_{\text{Sensitivity}} \xi(k) \quad (14)$$

Comparing (4) and (14), it is noticed that, for the GMV controller, the sample delay d is entirely compensated if the reference is known d-steps in advance. Nevertheless, both deterministic transfer functions are equivalent if

$$\begin{cases} Q(z^{-1}) = R(z^{-1}) \\ P(z^{-1}) = S(z^{-1}) \\ T(z^{-1}) = T_{RST}(z^{-1}) \end{cases} \quad (15)$$

On the other hand, due to the term $E(z^{-1})$, which is gained from the solution of the Diophantine (11), the sensitivity function of the GMV control differs from the sensitivity function of the RST control. Hence, $E(z^{-1})$ is directly responsible for the stochasticity effect of the controller (Trentini et al., 2016). Therefore, the GMV controller will exhibit the same I/O behavior as the RST controller but with predictive characteristics, since the reference and the output signals are used d-steps ahead.

In order to have a fairest comparison setup, one may test the GMV case without the d-steps ahead reference sequence, i.e., with $y_r(k)$. Notice, however, that if the same is done to $y(k+d)$, the controller will no longer produce the minimization of σ_ϕ^2 in the minimum variance sense (Silveira et al., 2016).

3. GMV CONTROLLER DESIGN FOR DC-DC POWER CONVERTERS

This section presents the application of the above theory into DC-DC power converters. Two converter topologies are established, empirically modeled by offline Least Squares identification, to illustrate the GMV controller design.

3.1 Dual Active Bridge Converter Example

The DAB DC-DC converter can be represented as a first-order plant (Lucas et al., 2019a). Hence, the following ARMAX model represented the dynamic of the DAB converter.

$$A_1(z^{-1})y_1(k) = B_1(z^{-1})z^{-1}u_1(k) + C_1(z^{-1})\xi_1(k) \quad (16)$$

with,

$$A_1(z^{-1}) = 1 + a_1^{(1)}z^{-1}$$
$$B_1(z^{-1}) = b_0^{(1)}$$
$$C_1(z^{-1}) = 1 + c_1^{(1)}z^{-1} + c_2^{(1)}z^{-2}$$

Lucas et al. (2018) proposed a robust PI controller to regulate the output of a DAB converter with a phase-shift control scheme. This controller is represented in the Laplace complex domain as,

$$C_1(s) = \frac{U_1(s)}{E_1(s)} = \frac{K_p^{(1)}s + K_i^{(1)}}{s} \quad (17)$$

Defining,

$$s := \tfrac{1}{T_{s_1}}(1 - z^{-1}) \qquad \text{(implicit Euler method)}$$

for converting (17) to the digital domain with a sample time T_{s_1} which depends on the switching frequency of the DAB converter, the RST structure is obtained after some algebraic manipulations,

$$\Delta u_1(k) = \left(h_0^{(1)} + h_1^{(1)}z^{-1}\right)e(k) \quad (18)$$

which results in,

$$\begin{cases} R_1(z^{-1}) = 1 \\ S_1(z^{-1}) = h_0^{(1)} + h_1^{(1)}z^{-1} \\ T_{RST_1}(z^{-1}) = h_0^{(1)} + h_1^{(1)}z^{-1} \end{cases} \quad (19)$$

Then, $P_1(z^{-1})$, $T_1(z^{-1})$ and $Q_1(z^{-1})$ are found using RST and GMV analogy (15).

$$\begin{cases} P_1(z^{-1}) = h_0^{(1)} + h_1^{(1)}z^{-1} \\ Q_1(z^{-1}) = 1 \\ T_1(z^{-1}) = h_0^{(1)} + h_1^{(1)}z^{-1} \end{cases} \quad (20)$$

To design the GMV controller, it is necessary to find the polynomials $F_1(z^{-1})$ and $E_1(z^{-1})$ through the Diophantine equation (21).

$$P_1(z^{-1})C_1(z^{-1}) = \Delta A_1(z^{-1})E_1(z^{-1}) + z^{-1}F_1(z^{-1}) \quad (21)$$

The order of the $F_1(z^{-1})$ and $E_1(z^{-1})$ polynomials are given by (12)

$$\begin{cases} E_1(z^{-1}) = e_0^{(1)} \\ F_1(z^{-1}) = f_0^{(1)} + f_1^{(1)}z^{-1} + f_2^{(1)}z^{-2} \end{cases} \quad (22)$$

Let's define

- $M_1(z^{-1}) = P_1(z^{-1})C_1(z^{-1})$
- $\Delta A_1(z^{-1}) = (1 - z^{-1})A_1(z^{-1})$

where

$$M_1(z^{-1}) = m_0^{(1)} + m_1^{(1)} z^{-1} + m_2^{(1)} z^{-2} + m_3^{(1)} z^{-3},$$
$$\Delta A_1(z^{-1}) = 1 + \hat{a}_1 z^{-1} + \hat{a}_2 z^{-2}$$

thereby, one solution to the incremental Diophantine equation (21) is given by:

$$\begin{cases} e_0^{(1)} = m_0^{(1)} \\ f_0^{(1)} = m_1^{(1)} - e_0^{(1)} \hat{a}_1 \\ f_1^{(1)} = m_2^{(1)} - e_0^{(1)} \hat{a}_2 \\ f_2^{(1)} = m_3^{(1)} \end{cases} \quad (23)$$

Then, the control law (24) must be implemented.

$$\Delta u_1(k) = \frac{C_1(z^{-1})T_1(z^{-1})y_r(k+d) - F_1(z^{-1})y(k)}{B_1(z^{-1})E_1(z^{-1}) + C_1(z^{-1})\lambda_1 Q_1(z^{-1})} \quad (24)$$

where the signal control u represents the phase-shift φ control for the single phase-shift modulator in a DAB converter.

To complete the design procedure, it is necessary to test empirically λ_1 to ensure the design specifications (Silveira et al., 2016). $\lambda_1 = 1$ may be considered to start the design procedure.

3.2 Buck Converter Converter Example

It is well-known that the DC-DC buck converter is a second-order plant. Hence, the following ARMAX model represented the dynamic of the buck converter.

$$A_2(z^{-1})y_2(k) = B_2(z^{-1})z^{-1}u_2(k) + C_2(z^{-1})\xi_2(k) \quad (25)$$

with,

$$A_2(z^{-1}) = 1 + a_1^{(2)} z^{-1} + a_2^{(2)} z^{-2}$$
$$B_2(z^{-1}) = b_0^{(2)} + b_1^{(2)} z^{-1}$$
$$C_2(z^{-1}) = 1 + c_1^{(2)} z^{-1} + c_2^{(2)} z^{-2}$$

Lucas et al. (2018) proposed a robust PID controller for a buck converter which is represented in the Laplace complex domain as,

$$C_2(s) = \frac{U_2(s)}{E_2(s)} = \frac{K_d^{(2)} s^2 + K_p^{(2)} s + K_i^{(2)}}{s(s+\alpha)} \quad (26)$$

Defining,

$$s := \tfrac{1}{T_{s_2}}(1 - z^{-1}) \qquad \text{(implicit Euler method)}$$

for converting (26) to the digital domain with a sample time T_{s_2} which depends on the switching frequency of the Buck converter, the RST structure is obtained after some algebraic manipulations,

$$\Delta u_2(k) = \frac{h_0^{(2)} + h_1^{(2)} z^{-1} + h_2^{(2)} z^{-2}}{1 + \alpha_d z^{-1}} e(k) \quad (27)$$

which results in,

$$\begin{cases} R_2(z^{-1}) = 1 + \alpha_d z^{-1} \\ S_2(z^{-1}) = h_0^{(2)} + h_1^{(2)} z^{-1} + h_2^{(2)} z^{-2} \\ T_{RST_2}(z^{-1}) = h_0^{(2)} + h_1^{(2)} z^{-1} + h_2^{(2)} z^{-2} \end{cases} \quad (28)$$

Then, $P_2(z^{-1})$, $T_2(z^{-1})$ and $Q_2(z^{-1})$ are found using RST and GMV analogy (15).

$$\begin{cases} P_2(z^{-1}) = h_0^{(2)} + h_1^{(2)} z^{-1} + h_2^{(2)} z^{-2} \\ Q_2(z^{-1}) = 1 + \alpha_d z^{-1} \\ T_2(z^{-1}) = h_0^{(2)} + h_1^{(2)} z^{-1} + h_2^{(2)} z^{-2} \end{cases} \quad (29)$$

To design the GMV controller, it is necessary to find the polynomials $F_2(z^{-1})$ and $E_2(z^{-1})$ through the Diophantine equation (30).

$$P_2(z^{-1})C_2(z^{-1}) = \Delta A_2(z^{-1})E_2(z^{-1}) + z^{-1}F_2(z^{-1}) \quad (30)$$

The order of the $F_2(z^{-1})$ and $E_2(z^{-1})$ polynomials are given by (12)

$$\begin{cases} E_2(z^{-1}) = e_0^{(2)} \\ F_2(z^{-1}) = f_0^{(2)} + f_1^{(2)} z^{-1} + f_2^{(2)} z^{-2} + f_3^{(2)} z^{-3} \end{cases} \quad (31)$$

Let's define

- $M_2(z^{-1}) = P_2(z^{-1})C_2(z^{-1})$
- $\Delta A_2(z^{-1}) = (1 - z^{-1})A_2(z^{-1})$

where

$$M_2(z^{-1}) = m_0^{(2)} + m_1^{(2)} z^{-1} + m_2^{(2)} z^{-2} + m_3^{(2)} z^{-3} + m_4^{(2)} z^{-4},$$
$$\Delta A_2(z^{-1}) = 1 + \bar{a}_1 z^{-1} + \bar{a}_2 z^{-2} + \bar{a}_3 z^{-3}$$

thereby, one solution to the incremental Diophantine equation (21) is given by:

$$\begin{cases} e_0^{(2)} = m_0^{(2)} \\ f_0^{(2)} = m_1^{(2)} - e_0^{(2)} \bar{a}_1 \\ f_1^{(2)} = m_2^{(2)} - e_0^{(2)} \bar{a}_2 \\ f_2^{(2)} = m_3^{(2)} - e_0^{(2)} \bar{a}_3 \\ f_3^{(2)} = m_4^{(2)} \end{cases} \quad (32)$$

Then, the control law (33) must be implemented

$$\Delta u_2(k) = \frac{C_2(z^{-1})T_2(z^{-1})y_r(k+d) - F_2(z^{-1})y(k)}{B_2(z^{-1})E_2(z^{-1}) + C_2(z^{-1})\lambda_2 Q_2(z^{-1})} \quad (33)$$

where the signal control u represents the duty cycle d_1 in a buck converter.

To complete the design procedure, it is necessary to test empirically λ_2 to ensure the design specifications (Silveira et al., 2016). $\lambda_2 = 1$ may be considered to start the design procedure.

4. GMV CONTROL EVALUATION INTO DC-DC POWER CONVERTERS

The simulation is perfomed using the software MATLAB ® with the toolbox SimPowerSystems ™.

The power converter switching models have been implemented together with the discrete-version of the control laws. Since the switching models are used, power losses are considered in the passive components and the switching components to perform the simulation tests. The simulation data is shown in the Appendix A and B based on the works of Lucas et al. (2019a, 2018), respectively.

Since the GMV control is a model-based controller, the power converters are modeled through an identified stochastic model considering the ARMAX model. Therefore, the noise ξ, which may be considered as a measurement noise, is added into the output of the power converters before performing the stochastic model identification. Sangswang and Nwankpa (2004) claimed that the noise into power electronics converter is approximately Gaussian distributed. Hence, a random noise is introduced into the power converters modeled as a zero-mean Gaussian distributed white noise. Then, the control law (13) is implemented.

The variance of the control signal, and the performance indices (ISE, ITAE, and ISU) are computed in order to evaluate the control performance of the GMV and robust controllers.

Fig. 1 shows the proposed controller applied to the power converter examples addressed in this work.

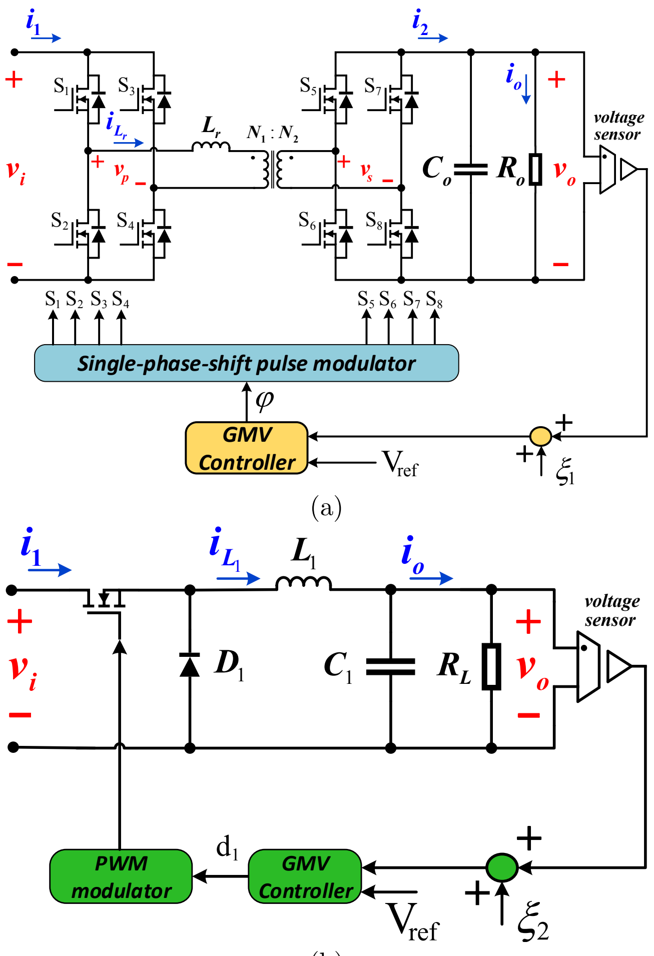


Fig. 1. GMV control scheme. (a) DAB converter. (b) Buck converter.

The white noise signal ξ_1 and ξ_2 (cf. Fig. 1) are added to the output of converters with a relatively variance of $\sigma^2_{\xi_1} = 1e^{-4}$ and $\sigma^2_{\xi_2} = 1e^{-7}$, respectively. Note that the sample time depends on the switching frequency of each power converter.

4.1 Dual Active Bridge Example

To evaluate the proposed control approach, the GMV controller is compared with a robust controller proposed by Lucas et al. (2019a) (see Appendix A).

A brief description of the experiments are presented as follow: The system is set to its initial operating condition until the steady state is achieved (V_o = 400 V). Then, the system is subjected to a voltage reference variation (ΔV_{ref_1}) from 400 V to 500 V at t = 0.1s. After that, another variation (t = 0.2s) into the voltage reference (ΔV_{ref_2}) is performed from 500 V to 400. At time t = 0.3s, a load variation (ΔR_{o_1}) is performed from 80Ω to 100Ω. Then, the load returns to its initial condition (80Ω) caused by a load variation (ΔR_{o_2}) at time t = 0.5. Finally, two input voltage variation, ΔV_{i_1} and ΔV_{i_2}, are performed changing the value of 800 V to 900V (t = 0.5s) and 900 V to 800 V (t = 0.6s), respectively.

Fig. 2 shows the simulated results of the closed-loop system performance controlled by GMV and robust control approaches.

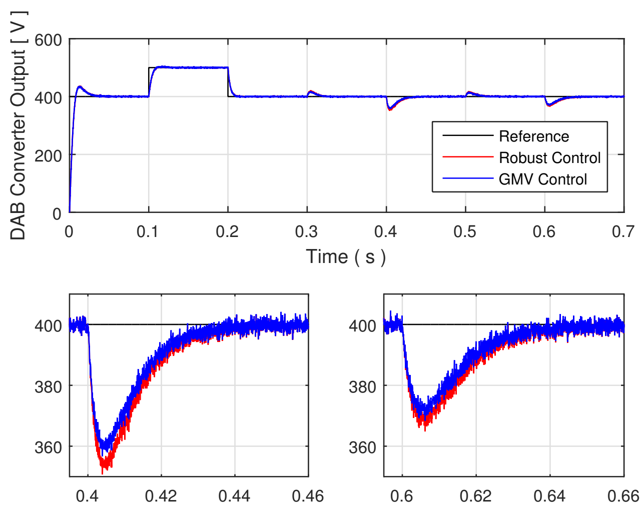


Fig. 2. DAB converter performance with GMV and Robust control approaches.

Fig. 3 shows the phase-shift control for this experiment under the two control approaches.

Note that the zoomed area for ΔR_{o_2} and ΔV_{i_2} are given by Figs. 2 and 3 in order to to better visualize the data of the closed-loop system performance (output and control signals). Moreover, the results in terms of performance indices are shown in Table 1.

Both controllers lead to a stabilizing control. However, for the robust controller, the output voltage has a slightly higher level of oscillation as its GMV controller counterpart (cf. ISE).

In spite of the noise, GMV control and robust control convergence occurs at a similar instant (cf. ITAE). However, the major positive difference comes when we look to the control signal (cf. Fig. 3, ISU and σ^2_u), where the GMV control outperforms the robust control in the energy consumption sense due to its stochastic model based design, which supports the complete model of the noise.

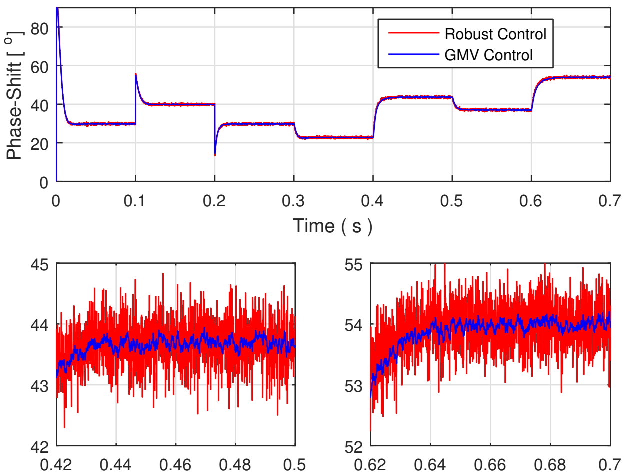


Fig. 3. Phase-Shift control under GMV and Robust control approaches.

Table 1. GMV and Robust controllers comparison by performance indices

Variation	Control	ISE	ITAE	ISU	σ_u^2
ΔV_{ref_1}	GMV	6.463	1.778	58.15	0.00626
	Robust	6.641	1.805	58.23	0.12981
ΔV_{ref_2}	GMV	6.558	3.318	30.61	0.00568
	Robust	6.659	3.365	30.65	0.13543
ΔR_{o_1}	GMV	0.747	3.051	19.01	0.00627
	Robust	1.009	3.383	19.04	0.12679
ΔR_{o_2}	GMV	5.879	10.32	71.11	0.01573
	Robust	7.838	11.85	71.49	0.13443
ΔV_{i_1}	GMV	0.601	4.601	46.59	0.00727
	Robust	0.782	5.096	49.69	0.13755
ΔV_{i_2}	GMV	3.032	11.44	96.25	0.01083
	Robust	3.941	13.06	96.81	0.13206

4.2 Buck Converter Example

To evaluate the proposed control approach, the GMV controller is compared with a robust controller proposed by Lucas et al. (2018) (see Appendix B).

A brief description of the experiments are presented as follow: The system is set to its initial operating condition until the steady state is achieved (V_o = 5 V). Then, the system is subjected to a voltage reference variation (ΔV_{ref_1}) from 5 V to 8 V in t = 0.3s. After that, another variation (t = 0.6s) into the voltage reference (ΔV_{ref_2}) is performed from 8 V to 5. At time t = 0.9s, a load variation (ΔR_{L_1}) is performed from 4Ω to 6Ω. Then, the load returns to its initial condition (4Ω) caused by a load variation (ΔR_{L_1}) at time t = 1.2. Finally, two input voltage variation, ΔV_{i_2} and ΔV_{i_2}, are performed changing the value of 15 V to 17V (t = 0.5s) and 17 V to 15 V (t = 0.6s), respectively.

Fig. 4 shows the simulated results of the closed-loop system performance controlled by GMV and robust control approaches.

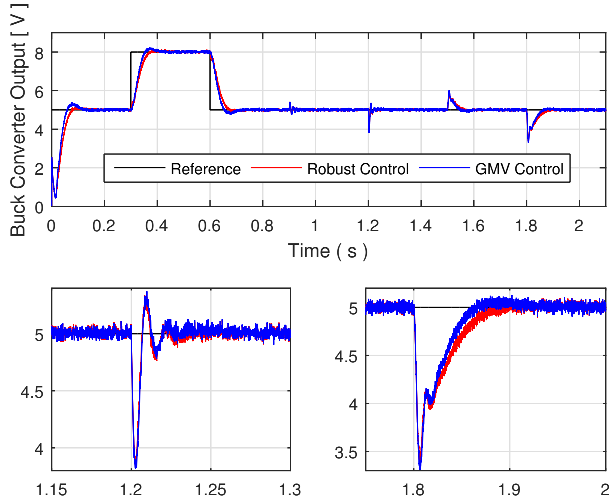


Fig. 4. Buck converter performance with GMV and Robust control approaches.

Fig. 5 shows the duty cycle control for this experiment under the two control approaches.

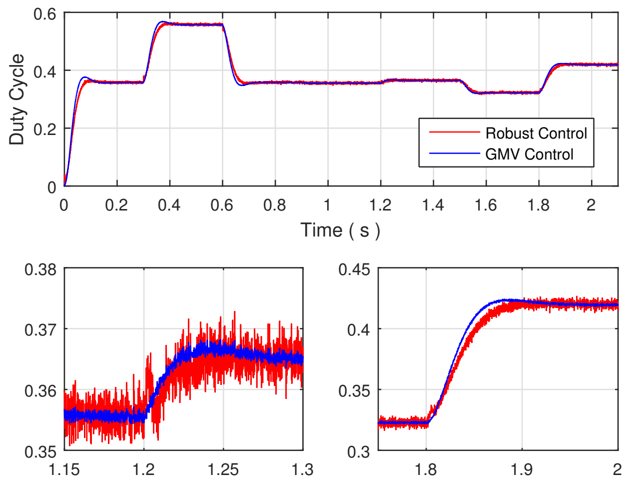


Fig. 5. Duty cycle control under GMV and Robust control approaches.

Note that the zoomed area for ΔR_{L_2} and ΔV_{i_2} are given by Figs. 4 and 5 in order to to better visualize the data of the closed-loop system performance (output and control signals).

Same as the DAB converter test, the output voltage of the buck converter seems to be quite similar for both controllers. However, a further view on the control signal (cf. Fig. 5) shows considerable differences. Thereby, the GMV controller presents smoothed characteristic, causing less stress on the control signal. In contrast, the robust controller presents a highly dynamic control signal, changing its amplitude at any time.

The results in terms of performance indices are shown in Table 2. Notice that the GMV controller is more economic (cf. ISU and σ_u^2) and present approximately the same settling time (cf. ITAE).

Table 2. GMV and Robust controllers comparison by performance indices

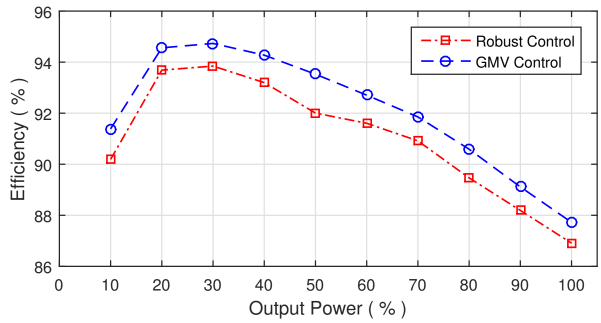
Variation	Control	ISE	ITAE	ISU	σ_u^2
ΔV_{ref_1}	GMV	44.24	7.888	24.84	0.00032
	Robust	48.10	8.683	25.17	0.04883
ΔV_{ref_2}	GMV	44.36	14.83	11.74	0.00032
	Robust	48.09	16.30	11.97	0.04882
ΔR_{o_1}	GMV	0.273	2.978	10.81	0.00132
	Robust	0.279	3.043	10.82	0.05007
ΔR_{o_2}	GMV	0.126	5.150	11.32	0.00166
	Robust	0.128	5.024	11.33	0.04894
ΔV_{i_1}	GMV	2.646	10.45	9.014	0.00032
	Robust	2.764	11.22	9.954	0.04623
ΔV_{i_2}	GMV	9.039	21.55	14.16	0.00119
	Robust	9.814	24.25	14.02	0.05022

4.3 Efficiency of DC-DC Power Converters with GMV and Robust Control Approaches

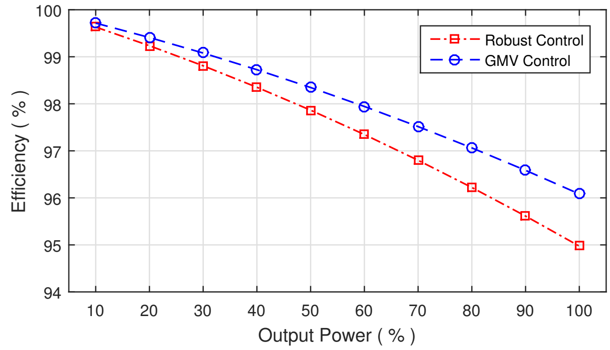
The efficiency of the DAB and buck converter is computed by simulation as a function of the load power, to demonstrate the effectiveness of the proposed controller. Thereby, the efficiency of the DAB and the buck converters is improved when they are regulated under the GMV controller as shown Fig. 6.

4.4 Performance and Efficiency discussion

The study developed by Almawlawe et al. (2014) addressed the advantages to use a GMV control approach to regulate a dc-dc boost converter. By combining of GMV control and discrete-time quasi-sliding mode control, the authors achieve zero steady-state error, resulting in high output voltage accuracy in the presence of parameters perturbations, in addition to filtering the switching control component, which reduces the chattering phenomenon. However, the control signal analysis of the switching components and the efficiency of the dc-dc power converters are not addressed. In addition, no comparison with other control approaches is performed, aiming to justify the implementation of a more complex control scheme. Finally, sliding mode control approaches need high switching frequencies in real time applications in order to reduce delays, this implies the use of high speed digital signal processing and high speed switching power electronic devices. In contrast, this work introduces an outstanding contribution for designing stochastic controllers from SISO linear controllers, which provides the ability to adequately treat noise of a stochastic nature, in addition to analyzing the performance and efficiency of the converter. The proposed control methodology enhance the system performance (cf. ISE and ITAE), reducing the variance of the control signal (cf. ISU and σ_u^2), which causes less stress in the switches devices, thereby, power losses in switching devices are decreased, thus, the efficiency is improved.



(a)



(b)

Fig. 6. Efficiency of converters under GMV and Robust control approaches. (a) DAB converter. (b) Buck Converter

5. CONCLUSION

In this paper, we present the evaluation of a generalized minimum variance controller. Moreover, a framework to compare different SISO linear controllers on a unified base is introduced. The stochastic part is introduced into the controller using stochastic augmentation of deterministic controllers based on RST-Structure.

A dual active bridge and a buck converter were chosen to compare the effectiveness of the GMV controller with a robust controller. The simulated results shows the GMV controller outperforms the robust controller. For both power converters, the control signals (phase-shift φ, and duty cycle d_1) show better characteristics, significantly reducing their variances, i.e. the GMV controller uses less energy causing less switching losses.

The proposed control approach is an effective method to minimize the control variance improving the efficiency of the power electronic converters addressed in this work. Therefore, it has the potential to improve the efficiency of power converter systems where efficiency is a major concern.

ACKNOWLEDGEMENTS

Kevin E. Lucas and Daniel J. Pagano acknowledge CAPES/BRAZIL and CNPq/BRAZIL for partially funding its work under Doctoral grant (Finance Code 001), and Project 302229/2018-3, respectively. Douglas A. Plaza, David A. Vaca-Benavides and Sara J. Ríos acknowledge ESPOL Polytechnic University for the financial support given to this research.

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Appendix A. DAB CONVERTER DATA

Table A.1. Parameters of the DAB converter system

Par.	Unit	Value	Description
DAB Converter Parameters			
V_i	V	800	Source input voltage
V_o	V	400	Output voltage
L_r	mH	1.10	Auxiliary inductor
C_o	μF	104.17	Output Capacitor
D_φ	rad.	$\pi/6$	Nominal phase-shift
f_{sw_1}	kHz	20	Switching frequency
R_o	Ω	80	Resistive Load
N_2/N_1		0.50	Transformer turn ratio
Robust Controller Parameters $C_1(s)$			
$K_p^{(1)}$		0.004513	Proportional coefficient
$K_i^{(1)}$		0.6372	Integral coefficient

Appendix B. BUCK CONVERTER DATA

Table B.1. Parameters of the Buck converter system

Par.	Unit	Value	Description
Buck Converter Parameters			
V_i	V	15	Source input voltage
V_o	V	5	Output voltage
L_1	mH	2	Filter inductor
C_1	μF	2000	Output Capacitor
f_{sw_2}	kHz	10	Switching frequency
R_L	Ω	4	Resistive Load
Robust Controller Parameters $C_2(s)$			
$K_p^{(2)}$		0.03504	Proportional coefficient
$K_i^{(2)}$		225.6	Integral coefficient
$K_d^{(2)}$		0.0007019	Derivative coefficient
α		99.5	Filter coefficient