Bounded Derivative Feedback Control with Application to Magnetic Levitation ⋆

Muhammad Hamad Zaheer * **Khalid M. Arthur** ** **Se Young Yoon** ***

* *Department of Electrical and Computer Engineering, University of New Hampshire, Durham, NH 03824, USA (e-mail: mz1038@wildcats.unh.edu).*
** *Department of Electrical and Computer Engineering, University of New Hampshire, Durham, NH 03824, USA (e-mail: ka1070@wildcats.unh.edu)*
*** *Department of Electrical and Computer Engineering, University of New Hampshire, Durham, NH 03824, USA (e-mail: seyoung.yoon@unh.edu)*

Abstract: In this paper, we study the stabilization of dynamic systems with uncertain equilibrium states and in the presence of bounded control. We propose state and output derivative feedback control schemes to stabilize the dynamic system, and to drive the system states to its true equilibrium state even when the location of such equilibrium is uncertain. Control bounds in the feedback control are also considered in this paper, and stability conditions are derived for the cases when the control energy is bounded, and when the maximum control is bounded. Stability conditions are derived in the form of matrix inequalities for both cases of control bounds, and numerical methods are discussed to synthesize feasible control solutions. The effectiveness of the proposed method is illustrated by an experimental implementation.

Keywords: Dynamic output feedback, nonlinear control, bounded control, continuous control, convex optimization

1. INTRODUCTION

Knowledge of the equilibrium states plays a vital role in the analysis and control of dynamic systems, as it provides a resting point around which the stability of the system is evaluated. However, finding the exact value of the equilibrium points of dynamic systems is non-trivial as it involves solving systems of complex nonlinear equations with uncertain parameters. The equilibrium points may also not be fixed, but change with the time-varying dynamics of the system. Therefore, conventional control methodologies, which require exact knowledge of the equilibrium states, are ineffective in controlling systems with uncertain equilibrium states. Uncertainty in the equilibrium points may require a non-zero steady state control signal to stabilize the system at a non-equilibrium point, resulting in greater energy consumption by the actuator (Arthur et al. (2018)).

Time-delayed feedback control has been explored in Pyragas (1992) to control nonlinear systems with uncertain equilibrium states. A time-delay feedback control scheme for stabilizing chaotic systems under unstable periodic orbits is presented in Pyragas (1992), Pyragas (1995) and Pyragas (2001). In Kokame et al. (2001b,a), the authors extended these results to the control of nonlinear systems with uncertain steady states. An analytical method for selecting the appropriate controller gains and time-delay constant is presented in Hövel and Schöll (2005), and an optimization based technique is used in Chen and Yu (1999) to determine the time-delay constant of the time-delay feedback controller for chaotic systems.

Adaptive control techniques can also be used to control nonlinear systems with uncertain equilibrium points by tracking the unknown equilibrium states. In Arthur et al. (2018) and Bazanella et al. (2000), the authors developed an adaptation mechanism to determine the true equilibrium point, and then control the system using conventional methods of state transformation and feedback control. On the other hand, the effects of bounded control on the stability of the closed-loop system are not taken into account, although actuator constraints were cited as a primary motivation for developing a solution for compensating the uncertain equilibrium.

State-derivative feedback control has been considered for the control of nonlinear systems without explicit knowledge of the equilibrium states (Shigekuni and Takimoto (2013)). State-derivative has been considered as a special case of the state difference control in Kokame et al. (2001a) and Ulsoy (2015). In some practical applications, the state-derivative signals can even be more accessible than the state measurement signals, as accelerometers provide access to acceleration measurements in applications such as vibration control, vehicle suspension systems (Abdelaziz and Valášek (2004)). In Abdelaziz and Valášek (2005), a

⋆ This work was supported in part by the National Science Foundation, under the grant number ECCS 1607023.

state-derivative feedback controller is proposed for stabilizing linear systems, and these results are extended to include robustness to dynamic uncertainties in Assunção et al. (2007); Faria et al. (2009); Abdelaziz (2010). However, to the best of our knowledge, the problem of bounded state-derivative control has not been considered in the context of uncertain equilibrium states.

In feedback control applications to practical systems, it is often the case that we require a bounded control input for stabilization. The requirement for bounded control arises due to energy conservation and actuator saturation. The literature on bounded control is extensive. Control laws for stabilization of linear systems with bounds on input energy and peak are derived in Boyd et al. (1994). Global stability of stable linear systems with actuator saturation is discussed in Klai et al. (1993). For unstable and controllable systems, a control law for local stabilization is derived in Gomes da Silva et al. (2003). To increase the region of stability, antiwindup control is used in Grimm et al. (2003); da Silva and Tarbouriech (2005). Robust control in the presence of sensor actuation is investigated in Turner and Tarbouriech (2006). In this paper, the feedback control under control input constraints will be considered for the state and output derivative feedback control framework.

In this paper, we investigate the design of state and output derivative feedback controllers to stabilize dynamic systems with uncertain equilibrium states, and subject to control constraints. Stability conditions are derived for the cases when the euclidean norm and the infinity norm of the control input is bounded. The technical results of this paper are verified through experimental implementation on a magnetic levitation test-bed.

The remainder of this paper is organized as follows. Section 2 introduces the control problem considered in this paper. In Section 3 the state-derivative controller is introduced, and design procedures for the controller gains are presented for the control cases under bounded control energy and bounded infinity norm. In Section 4 design procedure for output-derivative feedback controller is presented for the case of bounded input infinity norm. In Section 5 an experimental implementation illustrates the effectiveness of the proposed method. Conclusions and discussions on future works are presented in Section 6.

Standard notations are used throughout the paper. The transpose of a matrix A is represented as A^T. For a matrix X, X_{ij} represents its $(i,j)^{th}$ element. The Euclidean norm of a vector u is represented as $\|u\|$, and the corresponding induced norm for a matrix X is $\|X\|$. The infinity norm of a vector u is represented as $\|u\|_\infty$, and the corresponding induced norm for a matrix X is $\|X\|_\infty$. The positive-definiteness (or semi-definiteness) of a matrix P is specified as $P > 0$ ($P \geq 0$).

2. PROBLEM STATEMENT

Let us consider a nonlinear system given by

$$\begin{aligned} \dot{x} &= f(x,u,p), \\ y &= g(x), \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^{n_u}$ and $y \in \mathbb{R}^{n_y}$ are the state, input and output vectors respectively. Let p be a vector of unknown model parameters, and define $x_0 \in \mathbb{R}^n$ as the nominal equilibrium point of the nonlinear system (1) for a nominal value of p, $p = p_0$,

$$f(x_0, 0, p_0) = 0.$$

The actual equilibrium point x_p depends on the actual value of p, $f(x_p, 0, p) = 0$, and it is therefore uncertain.

When the nominal equilibrium x_0 equals the true equilibrium of (1), a stabilizing control can be obtained under a mild controllability condition in a state feedback form

$$u(t) = -K(x(t) - x_0). \tag{2}$$

The feedback gain K stabilizes the linear approximation of the nonlinear system (1) about the equilibrium point x_0,

$$\begin{aligned} \delta\dot{x}(t) &= A\delta x(t) + Bu(t), \\ \delta y &= C\delta x(t), \end{aligned} \tag{3}$$

where $\delta x(t) = x(t) - x_p$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n_u}$ and $C \in \mathbb{R}^{n\times n_y}$ are the Jacobian matrices $\partial f(x,u)/\partial x$, $\partial f(x,u)/\partial u$ and $\partial g(x)/\partial y$ evaluated at the equilibrium point x_p, respectively.

On the other hand, when the true value of the parameter p and the corresponding equilibrium point x_p are uncertain, the control law (2) is no longer implementable. For small enough perturbation p from p_0, using the nominal equilibrium information in (2) may stabilize the system at a non-equilibrium point due to continuity, but it would also require a non-zero steady state control input to the plant (Shigekuni and Takimoto (2013)). A continuous non-zero steady state control action will result in greater continuous control effort, which can significantly degrade performance and stability for systems with strict actuator constraints (Arthur et al. (2018)). Therefore, conventional state feedback controllers are not effective for stabilizing system with uncertain equilibrium points.

Actuators in practical applications are commonly subject to operating constraints such as limited power and peak output. Ignoring these constraints during the controller design process results in degraded performance and even instability of the controlled system. The challenges of actuator constraints are even more urgent for systems with uncertain equilibrium states, in which operating around a non-equilibrium point of the system may demand a continuous non-zero control effort by the actuators.

3. STATE-DERIVATIVE FEEDBACK CONTROL

State-derivative feedback controllers have been considered for stabilizing dynamic systems (1) when the true equilibrium state information is known or uncertain during implementation. The state-derivative feedback controller is given by

$$u(t) = -K\delta\dot{x}(t) = -K\dot{x}(t), \tag{4}$$

where $K \in \mathbb{R}^{n_u\times n}$ is the controller gain. Note that (4) does not require information of the equilibrium point for implementation. Therefore, even if the true equilibrium point shifts from its nominal value x_0, the state-derivative controller can stabilize the system at the true equilibrium by driving δx to zero.

Using state-derivative feedback controller for the system in (3), we get

$$\begin{aligned} \delta\dot{x} &= A\delta x - BK\delta\dot{x}, \\ (I + BK)\delta\dot{x} &= A\delta x. \end{aligned}$$

For full rank $I + BK$, the closed loop system is given by,

$$\delta\dot{x} = (I + BK)^{-1} A\delta x. \quad (5)$$

The objective is then to design a feedback controller (4) such that the closed-loop system (5) is stable and the input is constrained to a specified level. The assumptions and lemmas that will aid the derivation of our main results are discussed below,

Assumption 1. The equilibrium state x_p of (1) is an isolated equilibrium, and the state matrix A is full rank.

Remark 2. We have considered $I + BK$ to have full rank. If the system (A, B) is controllable then a K can always be found to make $(I + BK)$ full rank (Abdelaziz and Valášek (2004)).

Lemma 3. If the closed loop system (5) is stable with Lyapunov function $V = \delta x^T P \delta x$ for a matrix $P > 0$, then the condition $\delta x^T(0) P \delta x(0) \leq 1$ implies $\delta x(t) \in \mathcal{E}$ for all $t > 0$, where $\mathcal{E} = \left\{\xi | \xi^T P \xi \leq 1\right\}$ (Boyd et al. (1994)).

Lemma 4. Consider two matrices X and Y with appropriate dimensions and a symmetric invertible matrix Γ. The following property holds

$$X^T Y + Y^T X \leq X^T \Gamma X + Y^T \Gamma^{-1} Y. \quad (6)$$

3.1 State-Derivative Feedback Control under Bounded Control Energy

In this subsection, we consider the case where the Euclidean norm of the control effort is bounded,

$$\|u\| \leq \mu,$$

for some $\mu \in \mathbb{R}$, and derive necessary conditions for the stability of the derivative feedback system. The stability conditions are introduced in the form of matrix inequalities in the following theorem, which is presented without a proof because of space constraints.

Theorem 5. If there exists a matrix $Q > 0 \in \mathbb{R}^{n\times n}$ and a constant $\sigma \in \mathbb{R}$ such that

$$AQ + QA^T - \sigma BB^T < 0, \quad (7)$$

$$\begin{bmatrix} Q & -(\sigma/2)A^{-1}B \\ * & \mu^2 I \end{bmatrix} \geq 0, \quad (8)$$

$$\begin{bmatrix} 1 & \delta x^T(0) A^T \\ * & \Phi \end{bmatrix} \geq 0, \quad (9)$$

where $\Phi = Q - (\sigma/2)BB^T A^{-T} - (\sigma/2)A^{-1}BB^T + (\sigma^2/4)BB^T A^{-T} Q^{-1} A^{-1} BB^T$, then the control law $u = (\sigma/2)B^T A^{-T} Q^{-1} \delta\dot{x}(t)$ subject to the bounding condition $\|u\| \leq \mu$ stabilizes the system (3) for the initial condition $\delta x(0)$.

3.2 State-Derivative Feedback Control under Bounded Input Infinity Norm

Next, we consider the case where the control is subject to the bounding condition,

$$-\mu \leq u_i \leq \mu,$$

for elements of the control vector $u = [u_i], i = 1, ..., n_u$, and some $\mu > 0 \in \mathbb{R}$. The above bounding condition is equivalent to a bound on the infinity norm of the control

$$\|u\|_\infty \leq \mu.$$

We proceed to extend the results of Theorem 5 to bound the infinity norm of the input vector. The proof is omitted because of space constraints.

Theorem 6. If there exist matrices $Q > 0 \in \mathbb{R}^{n\times n}$, $X \in \mathbb{R}^{n_u \times n_u}$ and a constant $\sigma \in \mathbb{R}$ such that

$$AQ + QA^T - \sigma BB^T < 0, \quad (10)$$

$$\begin{bmatrix} X & -(\sigma/2)A^{-1}B \\ * & Q \end{bmatrix} \geq 0, \quad (11)$$

$$X_{ii} - \mu^2 \leq 0, \quad (12)$$

$$\begin{bmatrix} 1 & \delta x^T(0) A^T \\ * & \Phi \end{bmatrix} \geq 0, \quad (13)$$

where $\Phi = Q - (\sigma/2)BB^T A^{-T} - (\sigma/2)A^{-1}BB^T + (\sigma^2/4)BB^T A^{-T} Q^{-1} A^{-1} BB^T$ and $i = 1, ..., n_u$, then the control law $u = (\sigma/2)B^T A^{-T} Q^{-1} \delta\dot{x}(t)$ stabilizes the system (3) for the initial condition $\delta x(0)$ and subject to the bounding condition $\|u\|_\infty \leq \mu$.

4. OUTPUT-DERIVATIVE FEEDBACK CONTROL

A dynamic output-derivative feedback controller of order n is given by,

$$\begin{aligned} \dot{z} &= (A + LCA + BF + LCBF)z - L\dot{y}, \\ u &= Fz, \end{aligned} \quad (14)$$

where $L \in \mathbb{R}^{n\times n_y}$ and $F \in \mathbb{R}^{n_u \times n}$ are design parameters. Note that $\delta\dot{y} = C\delta\dot{x} = C\dot{x} = \dot{y}$ around the uncertain equilibrium, and the information of such equilibrium is not needed for implementation of the dynamic controller.

Define error state as $e \triangleq \delta x - z$. The dynamics of the closed loop system in terms of state x and error e are given by,

$$\begin{bmatrix} \delta\dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A + BF & -BF \\ 0 & A + LCA \end{bmatrix} \begin{bmatrix} \delta x \\ e \end{bmatrix} \quad (15)$$

Theorem 7. If there exist matrices $R, Q > 0 \in \mathbb{R}^{n\times n}$, $\Gamma, X > 0 \in \mathbb{R}^{n_u \times n_u}$, $Y \in \mathbb{R}^{n_u \times n}$ and $Z \in \mathbb{R}^{n\times n_y}$ such that

$$RA + A^T R + ZCA + A^T C^T Z^T < 0, \quad (16)$$

$$AQ + QA^T + BY + Y^T B^T + B\Gamma B^T < 0, \quad (17)$$

$$1 - \begin{bmatrix} \delta x(0) \\ e(0) \end{bmatrix}^T \begin{bmatrix} Q^{-1} & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \delta x(0) \\ e(0) \end{bmatrix} \geq 0, \quad (18)$$

$$\begin{bmatrix} X & Y & Y \\ * & Q & 0 \\ * & * & QRQ \end{bmatrix} \geq 0, \quad (19)$$

$$X_{ii} - \mu^2 \leq 0, \quad (20)$$

where $i = 1, ..., n_u$, then the output-derivative feedback controller given by (14) with $L = R^{-1}Z$ and $F = YQ^{-1}$, subject to the bounding condition $\|u\|_\infty \leq \mu$ stabilizes the system (3) for the initial conditions $\delta x(0)$ and $z(0) = \delta x(0) - e(0)$.

Proof. We choose a quadratic Lyapunov function $V_1(e(t)) = e(t)^T Re(t)$, where $R > 0 \in \mathbb{R}^{n\times n}$. The time derivative of V_1 along the trajectories of (15) is given by,

$$\dot{V}_1(e(t)) = e^T(RA + A^T R + RLCA + A^T C^T L^T R)e.$$

From the Lyapunov stability theorem, we get the stability condition as,

$$RA + A^T R + ZCA + A^T C^T Z^T < 0, \quad (21)$$

where $Z = RL$.

We choose a second quadratic Lyapunov function $V_2(\delta x(t)) = \delta x(t)^T P \delta x(t)$, where $P > 0 \in \mathbb{R}^{n\times n}$. The time derivative of V_2 along the trajectories of (15) is given by,

$$\dot{V}_2(\delta x(t)) = \delta x^T[AQ + QA^T + BY + Y^T B^T]\delta x - \delta x^T BYe - e^T Y^T B^T \delta x, \tag{22}$$

where $Q = P^{-1}$ and $Y = FQ$. Using Lemma 4 we have,

$$-\delta x^T BYe - e^T Y^T B^T \delta x < \delta x^T B\Gamma B^T \delta x + e^T Y^T \Gamma^{-1} Ye, \tag{23}$$

for $\Gamma > 0 \in \mathbb{R}^{n_u \times n_u}$, and thus, the derivative of Lyapunov function (22) is bounded as,

$$\dot{V}_2(\delta x) \leq \delta x^T[AQ + QA^T + BY + Y^T B^T + B\Gamma B^T]\delta x + e^T Y^T \Gamma^{-1} Ye.$$

Let us define,

$$\mathcal{P} = AQ + QA^T + BY + Y^T B^T + B\Gamma B^T, \tag{24}$$

$$\varepsilon = \lambda_{min}(-\mathcal{P}), \tag{25}$$

Therefore,

$$\dot{V}_2 < -\varepsilon\|\delta x\|^2 + \|\Gamma^{-1/2} Ye\|. \tag{26}$$

From (21) we get that $e(t)$ is asymtotically stable. Therefore, it follows that $\dot{V}_2 < 0$ for

$$\mathcal{P} < 0. \tag{27}$$

We consider that the initial conditions for state and error satisfy the LMI,

$$\begin{bmatrix} \delta x(0) \\ e(0) \end{bmatrix}^T \begin{bmatrix} Q^{-1} & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \delta x(0) \\ e(0) \end{bmatrix} \leq 1. \tag{28}$$

From the condition $\|u\|_\infty \leq \mu$ we get,

$$\begin{aligned} \max_{t\geq 0} \|u(t)\|_\infty &= \max_{t\geq 0} \|Fz(t)\|_\infty , \\ &= \max_{t\geq 0} \left\| [F \ -F] \begin{bmatrix} \delta x(t) \\ e(t) \end{bmatrix} \right\|_\infty . \end{aligned} \tag{29}$$

Using Lemma 3 we get,

$$\begin{aligned} \max_{t\geq 0} \|u(t)\|_\infty &\leq \max_{[\delta x^T e^T]^T \in \mathcal{E}} \left\| [F \ -F] \begin{bmatrix} \delta x(t) \\ e(t) \end{bmatrix} \right\|_\infty , \\ &\leq \max_{[\delta x^T e^T]^T \in \mathcal{E}} \left\| Y \begin{bmatrix} Q^{-1} & -Q^{-1} \end{bmatrix} \begin{bmatrix} \delta x(t) \\ e(t) \end{bmatrix} \right\|_\infty , \\ &\leq \sqrt{\max(Y(Q^{-1} + Q^{-1}R^{-1}Q^{-1})Y^T)_{ii}}, \\ &\leq \mu. \end{aligned} \tag{30}$$

The above condition can be written as,

$$(\mu^2 I - Y(Q^{-1} + Q^{-1}R^{-1}Q^{-1})Y^T)_{ii} \geq 0, \tag{31}$$

and,

$$X_{ii} \leq \mu^2. \tag{32}$$

where $i = 1, ..., n_u$. Using Schur's complement on (31) we get,

$$\begin{bmatrix} X & Y & Y \\ Y^T & Q & 0 \\ Y^T & 0 & QRQ \end{bmatrix} \geq 0, \tag{33}$$

which concludes the proof.

Remark 8. The inequalities (18) and (19) are not LMIs because of the terms Q^{-1} and QRQ respectively. We can instead convert the non-convex feasibility problem into a nonlinear optimization problem, which can then be solved using linearization methods. Replace (18) and (19) with,

$$1 - \begin{bmatrix} \delta x(0) \\ e(0) \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \delta x(0) \\ e(0) \end{bmatrix} \geq 0, \tag{34}$$

and

$$\begin{bmatrix} X & Y & Y \\ * & Q & 0 \\ * & * & S \end{bmatrix} \geq 0, \tag{35}$$

respectively, where $P = Q^{-1}$ and $S = QRQ$. Using the procedure presented in Moon et al. (2001) and Zaheer et al. (2014), a feasible solution to the matrix conditions of Theorem 7 can be found by solving the nonlinear optimization problem,

$$\left.\begin{array}{ll} \text{Minimize} & \text{Trace}(SU + QP) \\ \text{subject to} & (16), (17), (20), (34), (35) \text{ and} \\ & \begin{bmatrix} U & P \\ * & R \end{bmatrix} \geq 0, \quad \begin{bmatrix} S & I \\ * & U \end{bmatrix} \geq 0, \\ & \begin{bmatrix} Q & I \\ * & P \end{bmatrix} \geq 0. \end{array}\right\} \tag{36}$$

A solution to the above cone complementarity optimization problem can be reached using the linearization method presented in Ghaoui et al. (1997).

5. EXPERIMENTAL RESULTS

To test the effectiveness of our proposed controller, we consider a magnetic levitation test-bed (MagLev Model 730), (Parks (1999)) as shown in Fig. 1. Magnetic levitation is an application of electromagnetism and is the principle behind high speed bullet trains. The MagLev system consists of two magnetic disks (Disk #1 and Disk #2 at bottom and top respectively) controlled by two coils (Coil #1 and Coil #2 at bottom and top respectively). Schematic diagram of the MagLev system indicating the polarities of the coils and magnetic disks is shown in Fig. 1. Details on the setup and operation of the MagLev system can be found in Parks (1999). The knowledge of the magnetic equilibrium is required for the stabilization of the disks. However, its determination is difficult leading to uncertainties in its true location. We will use our proposed controller, which does not require the exact knowledge of equilibrium states to stabilize the system to the true magnetic equilibrium in the presence of these uncertainties.

The equations of motion for magnetic disks are

$$\begin{aligned} m\ddot{y}_1 + c_1\dot{y}_1 + F_{m12} &= F_{u11} - F_{u12} - mg, \\ m\ddot{y}_2 + c_2\dot{y}_2 - F_{m12} &= F_{u22} - F_{u21} - mg, \end{aligned} \tag{37}$$

where y_1 and y_2 are the position of Disk #1 and #2 respectively, m is the mass of magnets, c_i is the damping coefficient of magnet i, F_{m12} is the force between Disks 1 and 2, F_{uij} is the force between from Coil i and Disk j, and g is the acceleration due to gravity. The damping coefficients c_1 and c_2 are assumed to be negligible. The states of our system are chosen as $x = [y_1 \ \dot{y}_1 \ y_2 \ \dot{y}_2]^T$ and input $u = [u_1 \ u_2]^T$. The disk positions y_1 and y_2 are the available output measurements.

For the desired resting point $x_0 = [0.02 \ 0 \ -0.02 \ 0]^T$, a bias current input $u_0 = [u_{10} \ u_{20}]^T$ is determined as $u_0 = [0.5877 \ 0.5877]^T$. The linearized dynamics of the system about $\delta x = x - x_0$ and $\delta u = u - u_0$ is found as

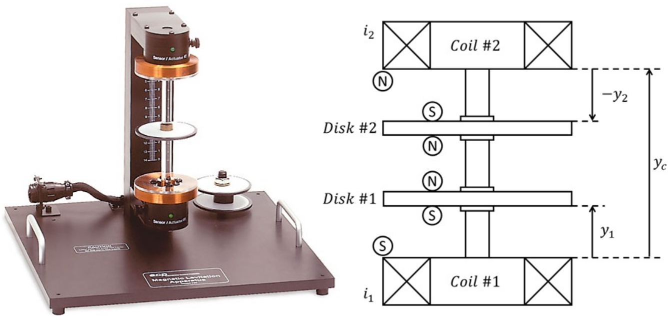

Fig. 1. MagLev Model 730 with schematic diagram.

$$\begin{bmatrix}\delta\dot{y}_1\\ \delta\ddot{y}_1\\ \delta\dot{y}_2\\ \delta\ddot{y}_2\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ a_{21} & 0 & a_{23} & 0\\ 0 & 0 & 0 & 1\\ a_{41} & 0 & a_{43} & 0\end{bmatrix}\begin{bmatrix}\delta y_1\\ \delta\dot{y}_1\\ \delta y_2\\ \delta\dot{y}_2\end{bmatrix} + \begin{bmatrix}0 & 0\\ 16.06 & 0\\ 0 & 0\\ 0 & 16.06\end{bmatrix}\begin{bmatrix}\delta u_1\\ \delta u_2\end{bmatrix},$$
$$\delta y = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\left[\delta y_1 \;\; \delta\dot{y}_1 \;\; \delta y_2 \;\; \delta\dot{y}_2\right]^T, \tag{38}$$

where $a_{21} = -477.4247$, $a_{23} = 0.0137$, $a_{41} = 0.0137$ and $a_{43} = 477.3973$.

5.1 Proportional-Integral Control

First, we stabilize the system using Proportional-Integral (PI) controller and the results will be used as a reference for comparison. The disks are stabilized at the nominal location of $[0.02 \;\; -0.02]$ as shown in Fig. 2. However, the steady-state control signals δu_1 and δu_2 are not equal to 0 as shown in Fig. 3. Therefore, these disk positions do not correspond to the true equilibrium associated with u_{10} and u_{20} and a non-zero steady-state control effort is required to maintain the system away from its true equilibrium which is undesirable. Moreover, we can't control the peak of input and bounding conditions on control can cause instability of the system.

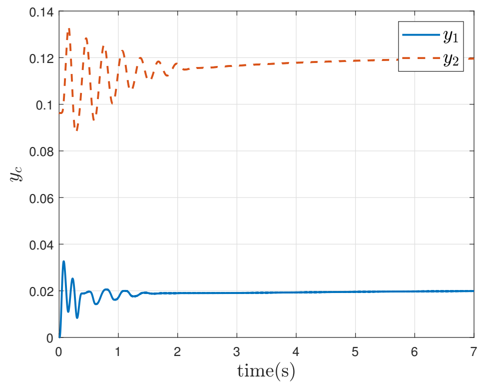

Fig. 2. Output of MagLev system stabilized with PI controller.

5.2 Dynamic Output-Derivative Control

To stabilize the magnetic disks at the true equilibrium, we use Theorem 7 to design an output-derivative feedback controller using the linearized equations of motion (38). We find the value of output-derivative feedback controller gains L and K for the bounding condition as $\|\delta u\|_\infty \leq 0.5$.

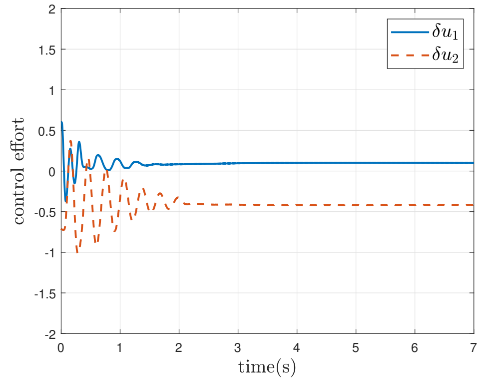

Fig. 3. Input to MagLev system stabilized with PI controller.

We observe in Fig. 4 that the system is not stabilized at the nominal equilibrium position of $[0.02 \;\; -0.02]$ but rather at $[0.015 \;\; -0.041]$. The steady-state control signals δu_1 and δu_2 are 0 as shown in Fig. 5, which indicates that the final resting positions of the disks are the true equilibrium points associated with u_{10} and u_{20}. The peak input level is within the bounding condition level of the control input.

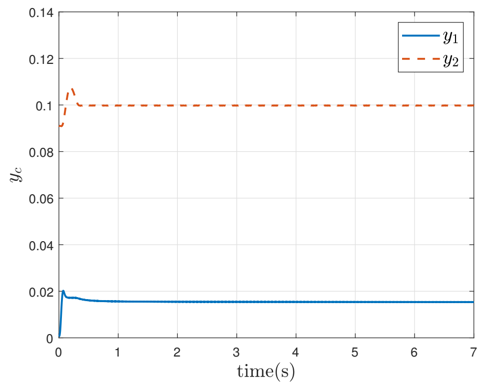

Fig. 4. Output of MagLev system stabilized at true equilibrium position.

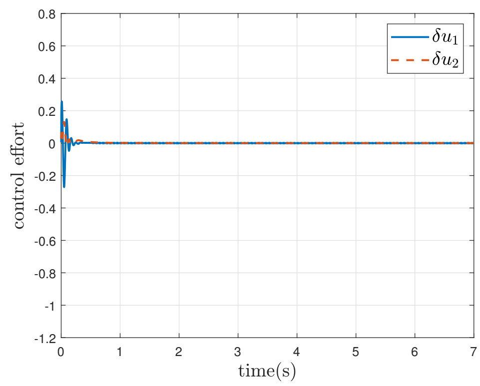

Fig. 5. Input to MagLev system stabilized at true equilibrium position.

6. CONCLUSIONS AND FUTURE WORK

In this work, we investigated the state-derivative and output-derivative feedback control for the stabilization of dynamic systems with unknown or uncertain equilibrium states, and in the presence of control input bounding constraints. The derivative feedback controller stabilizes the system at the true equilibrium of the system, which results in zero steady state control action.

The presented stability conditions for the derivative feedback system is verified by designing a controller for a magnetic levitation test-bed. Experimental results showed that the proposed control synthesis method can stabilize the magnetic plates to their true equilibrium, in the presence of unknown magnetic equilibrium. The importance of incorporating the bounded control conditions in the control synthesis was highlighted through experimental results by comparing the response of our proposed solution to the responses with alternative control laws. Future work will aim to extend the results of this paper to incorporate robustness to dynamic uncertainties.

REFERENCES

Abdelaziz, T.H.S. (2010). Optimal control using derivative feedback for linear systems. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, 224(2), 185–202.

Abdelaziz, T.H.S. and Valášek, M. (2004). Pole-placement for siso linear systems by state-derivative feedback. *Proceedings of the Institution of Electrical Engineers*, 151(4), 377–385.

Abdelaziz, T.H.S. and Valášek, M. (2005). State derivative feedback by lqr for time-invariant systems. *IFAC Proceedings Volumesl*, 38(1), 435–440.

Arthur, K.M., Basu, H., and Yoon, S.Y. (2018). Control of compressor surge in systems with uncertain equilibrium states. In *American Control Conference*, 1758–1763.

Assunção, E., Teixeira, M.C.M., Faria, F.A., Silva, N.A.P.D., and Cardim, R. (2007). Robust state-derivative feedback lmi-based designs for multivariable linear systems. *International Journal of Control*, 80(8), 1260–1270.

Bazanella, A.S., Kokotovic, P.V., and Silva, A.S.E. (2000). On the control of dynamic systems with unknown operating point. *International Journal of Control*, 73(7), 600–605.

Boyd, S., Ghaoui, L.E., Feron, E., and Balakrishna, V. (1994). In *Linear matrix inequalities in systems control theory*. Studies in Applied Mathematics.

Chen, G. and Yu, X. (1999). On time-delayed feedback control of chaotic systems. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 46(6), 767–772.

da Silva, J.M.G. and Tarbouriech, S. (2005). Antiwindup design with guaranteed regions of stability: an lmi-based approach. *IEEE Transactions on Automatic Control*, 50(1), 106–111.

Faria, F.A., Assunção, E., Teixeira, M.C.M., Cardim, R., and da Silva, N.A.P. (2009). Robust state-derivative pole placement lmi-based designs for linear systems. *International Journal of Control*, 82(1), 1–12.

Ghaoui, L.E., Oustry, F., and AitRami, M. (1997). A cone complementarity linearization algorithm for static output-feedback and related problemss. *IEEE Transactions on Automatic Control*, 42(8), 1171–1176.

Gomes da Silva, J.M., Tarbouriech, S., and Garcia, G. (2003). Local stabilization of linear systems under amplitude and rate saturating actuators. *IEEE Transactions on Automatic Control*, 48(5), 842–847.

Grimm, G., Hatfield, J., Postlethwaite, I., Teel, A.R., Turner, M.C., and Zaccarian, L. (2003). Antiwindup for stable linear systems with input saturation: an lmi-based synthesis. *IEEE Transactions on Automatic Control*, 48(9), 1509–1525.

Hövel, P. and Schöll, E. (2005). Control of unstable steady states by time-delayed feedback methods. *Physical Review E*, 72(4), 046203.

Klai, M., Tarbouriech, S., and Burgat, C. (1993). Stabilization via reduced-order observer for a class of saturated linear systems. In *Proceedings of 32nd IEEE Conference on Decision and Control*, 1814–1819 vol.2. doi:10.1109/CDC.1993.325504.

Kokame, H., Hirata, K., Konishi, K., and Mori, T. (2001a). Difference feedback can stabilize uncertain steady states. *IEEE Transactions on Automatic Control*, 46(12), 1908–1913.

Kokame, H., Hirata, K., Konishi, K., and Mori, T. (2001b). State difference feedback for stabilizing uncertain steady states of non-linear systems. *International Journal of Control*, 74(6), 537–546.

Moon, Y.S., Park, P., Kwon, W.H., and Lee, Y.S. (2001). Delay-dependent robust stabilization of uncertain state-delayed systems. *International Journal of Control*, 74(14), 1447–1455.

Parks, T.R. (1999). Manual for model 730, magnetic levitation system.

Pyragas, K. (1992). Continuous control of chaos by self-controlling feedback. *Physics Letters A*, 170(6), 421–428.

Pyragas, K. (1995). Control of chaos via extended delay feedback. *Physics Letters A*, 206(5), 323–330.

Pyragas, K. (2001). Control of chaos via an unstable delayed feedback controller. *Physical Review Letters*, 86(11), 2265–2268.

Shigekuni, T. and Takimoto, T. (2013). Stabilization of uncertain equilibrium points by dynamic state-derivative feedback control. In *13th International Conference on Control, Automation and Systems*, 23–27.

Turner, M.C. and Tarbouriech, S. (2006). Anti-windup for linear systems with sensor saturation: sufficient conditions for global stability and l2 gain. In *Proceedings of the 45th IEEE Conference on Decision and Control*, 5418–5423.

Ulsoy, A.G. (2015). Time-delayed control of siso systems for improved stability margins. *Journal of Dynamic Systems, Measurement, and Control*, 137(3), 041014.

Zaheer, M.H., Rehan, M., Mustafa, G., and Ashraf, M. (2014). Delay-range-dependent chaos synchronization approach under varying time-lags and delayed nonlinear coupling. *ISA Transactions*, 53(6), 1716–1730.