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Decision Making Method for Nonparametric Diagnosis in Nonlinear Dynamic Systems

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Abstract: The problem of robust fault diagnosis in nonlinear dynamic systems under presence of disturbances is studied within the scope of analytical redundancy conception. Solution of the problem assumes residual generation by checking redundancy relations existing among system inputs and outputs measured over a finite time window followed by decision making through evaluation of the residuals. Nonparametric method is considered for residual generation. To make decision, the unified method is developed whose feature is it combines the threshold logic of decision making and the residuals comparing with the fault syndromes evaluated on-line. Joint application of both nonparametric method for residual generation and unified method for decision making gives the possibility to develop the universal diagnostic platform for different systems described by ordinary nonlinear differential equations of the same structure but distinguished by these equations coefficients values.

Keywords: Nonlinear systems, fault diagnosis, robustness, nonparametric method, decision making.

1. INTRODUCTION

This work is devoted to the problem of fault diagnosis in dynamic systems described by nonlinear ordinary differential equations whose coefficients (system parameters) may be unknown. Solution of this problem is considered within the framework of analytical redundancy conception Chow and Willsky (1984). According to this conception, fault diagnosis is based on checking relations that exist among system inputs and outputs measured over a finite time window. The process of diagnosis includes residual generation as a result of mismatch between the system behavior and its reference model behavior, followed by decision making through evaluation of the residuals.

Faults in the system are the first reason of mismatch between the system behavior and its reference model behavior. The second reason is influence of disturbances due to the modelling errors, uncontrollable external actions and measurement noise. As soon as disturbances and faults all act upon the residual, the robustness problem arises. It is common practice to distinguish active and passive approaches to the robustness problem solution. An active approach is concentrated on the stage of residual generation to make the residual insensitive or low sensitive to disturbances and, simultaneously, sensitive to faults, while the passive approach touches upon the decision making procedure Blanke et al. (2006); Chen and Patton (1999); Gertler (1998).

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Among the methods developed in the frames of active approache, there exists the method considered in Shumsky (2007). The feature of this method is that the knowledge of some constant system parameters is not required for redundancy relations checking. Thus, this method provides active robustness; moreover, it can be called as non-parametric one, see also Ding (2014); Lindner and Auret (2014).

Nevertheless, existence of time variable disturbance actions calls for the use the passive approach whose main tool is the threshold logic of decision making. To guarantee acceptable trade-off between the count of false alarms and missing of faults, the threshold should be adaptive, depended on intensity of disturbances.

This paper concentrates mainly on the use of the passive approach to robustness aimed at its application in the frames of nonparametric method. The main contribution of the paper is unified method for decision making whose distinguishing feature is it combines the threshold logic of decision making that involves adaptive threshold and the residuals comparing with the fault syndromes evaluated on-line.

It will be shown in the paper that application of proposed results leads to obtaining the single diagnostic procedure (platform) for different systems under diagnosis belonging to the same class. All the systems from one class are described by ordinary nonlinear differential equations of the same structure. Under this, the equations given for different systems from the same class may contain distinguishing values of their coefficients.

The rest of the paper is organised as follows. Firstly, in Section 2, the details of nonparametric method application for residual generation are considered. Then, in Section 3, the unified method for decision making is proposed. After this, an example is given to illustrate the features and results of proposed method application. Section 4 concludes the paper.

2. RESIDUAL GENERATION BY NONPARAMETRIC METHOD

2.1 The diagnostic models in use

Let the system under diagnosis is described by the model

$$dx(t)/dt = f(x(t), u(t), a), \quad y(t) = h(x(t), a) \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^p$, $y(t) \in R^l$ are the vectors of the states, inputs (controls), outputs (measurements) respectively and $a \in R^m$ is the vector of the coefficients including 1) the model constant coefficients corresponding to unknown system parameters and 2) the coefficients that take nonzero value only when appropriate fault occurs. For convenience, vector of the last coefficients is considered as subvector of a and is written as a^f.

It is assumed that the fault in the system results in abrupt or gradual deviation of appropriate coefficient from zero. It is also assumed only single fault in the system. Nonlinear vector functions $f(*)$ and $h(*)$ describe the system dynamics and sensors characteristics respectively. The symbol "star" is used instead of omitted function arguments.

Design of the reference model needs in some transformations of the initial one. Consider them.

Firstly, transformation of the model (1) to the model without internal feedbacks

$$\begin{gathered} dx^{(1)}(t)/dt = f^{(1)}(y(t), u(t), a), \\ dx^{(2)}(t)/dt = f^{(2)}(x^{(1)}(t), y(t), u(t), a), \\ \vdots \\ dx^{(N)}(t)/dt = \\ f^{(N)}(x^{(N-1)}(t), x^{(N-2)}(t), ..., x^{(1)}(t), y(t), u(t), a), \end{gathered} \tag{2}$$

$$y_*(t) = h_*(x^{(N)}(t), x^{(N-1)}(t), ..., x^{(1)}(t), a) \tag{3}$$

is executed. The model (2) dynamics is split into N parts and $x^{(i)}$ is the state vector of the part number i, $1 \le i \le N$. Every part depends on the system input and output vectors as well as the state vectors of previous parts. The output vector of this model is formed as some function of the parts state vectors, see (3). Transformation of the model (1) to the model (2), (3) is fulfilled under assumption about existing the links between state vector of the model (1) and state vectors of the model (2) parts

$$x^{(i)}(t) = \varphi^{(i)}(x(t)), 1 \le i \le N, \tag{4}$$

as well as between the output vectors of the models (1) and (3)

$$y_*(t) = \psi(y(t)) \tag{5}$$

where $\varphi^{(i)}(*), 1 \le i \le N$ and $\psi(*)$ are some differentiated vector functions.

To avoid the repetition of known results, note that solution to the above model transformation task was given on the base of differential geometric tools in Shumsky (2007, 2012).

Secondly, the transition from the model (2), (3) to the system model in input-output form is executed. To realize such transition, sometimes one may need in the system functions representation in polynomial form. This demand can be satisfied by explanation of the system functions into a series. Obtained errors are considered as modelling ones.

Describe the features of such transition. To simplify the next writings, introduce operator I according to relation

$$\begin{gathered} I(f^{(j)}(x^{(j-1)}(t), x^{(j-2)}(t), ..., x^{(1)}(t), y(t), u(t), a) = \\ = \int_{t_0}^{t} f^{(j)}(x^{(j-1)}(\tau), x^{(j-2)}(\tau), ..., x^{(1)}(\tau), y(\tau), u(\tau), a)d\tau + \\ x^{(j)}(t_0) = x^{(j)}(t). \end{gathered} \tag{6}$$

Due to (6) one can write

$$\begin{gathered} x^{(1)}(t) = I(f^{(1)}(*)), \\ x^{(2)}(t) = I(f^{(2)}(I(f^{(1)}(*)), y(t), u(t), a)), \\ \vdots \\ x^{(N)}(t) = \\ I(f^{(N)}(I(f^{(N-1)}(*)), ..., I(f^{(1)}(*), y(t), u(t), a). \end{gathered} \tag{7}$$

After substitution (7) into (3) one can obtain input-output description of the system in the form

$$\psi(y(t)) = (A + A^f)W(t_0, t) \tag{8}$$

where $W(t_0, t)$ and A, A^f are the column vector and the matrices such that 1) elements of $W(t_0, t)$ are the functionals depended on the measurable system inputs and outputs at time interval $[t_0, t]$; 2) elements of the matrix A are polynomials of all the model (2) states at some initial instant of time t_0 and of the constant coefficients (components of the vector a) such that these coefficients are not subjected to faults action; 3) elements of the matrix A^f are polynomials of the vector a^f components and may be of the rest components of the vector a and states of the model (2) at time instant t_0. It is important that for healthy system case equality

$$A^f = 0 \tag{9}$$

holds. The model (8) is considered as the reference one. This model will be used for analytical relations construction whose checking should allow to detect and isolate the faults.

2.2 Residual generation

Let $t_0, t_1, ..., t_L$, $t_i = t_0 + i\,{}_\Delta t, 1 \le i \le L$, be some discrete time readings and ${}_\Delta t$, is sampling period. The choice of ${}_\Delta t$ is depended on the system dynamics (speed of response) and computational power of the diagnostic device. The less ${}_\Delta t$, the less time expenses to detect and isolate the faults.

In healthy system case, taking into account (9) one can write for time interval (moving time window) $T_L = [t_0, t_L]$

$$Y(t_L) = AV(t_L), \tag{10}$$

where the matrices

$$Y(t_L) = \big(\psi(y(t_1))|\psi(y(t_2))|\ldots|\psi(y(t_L))\big), \tag{11}$$

$$V(t_L) = \big(W(t_0, t_1)|W(t_0, t_2)|\ldots|W(t_0, t_L)\big). \tag{12}$$

Let the size of moving time window is taken such that

$$rankV(t_{L-1}) = rankV(t_L). \tag{13}$$

In this case, the matrix $V(t_L)$ is singular one and has nonzero kernel. Due to this, the rule for analytical relations checking can be taken in the form

$$r(t_L) = Y(t_L)v(t_L), \quad v(t_L) \in \ker V(t_L). \tag{14}$$

Indeed, according to (10) from (14) one has

$$r(t_L) = AV(t_L)v(t_L) = 0. \tag{15}$$

To reduce time expenses for fault detection and isolation to minimum, the size of mowing time window should be as minimal as possible. Given below algorithm for residual generation satisfies this demand.

Algorithm 1. (Residual generation)

(1) Take L=1. Compute $V(t_1) = W(t_0, t_1)$.
(2) Take L=L+1. Compute $V(t_L)$ according to (12).
(3) Check if equality (13) holds. If no, go to step 2.
(4) Compute $Y(t_L)$ according to (11).
(5) Compute the residual according to (14).
(6) End.

Notice, computational expanses of this algorithm may be slightly decreased by excluding the step 3. Indeed, under value $L = L_{\max}$, where $L_{\max}$ plus 1 exceeds the number of the matrix A columns, the matrix $V(t_L)$ is singular. But in this case, the minimal size of moving time window is not under guarantee.

3. UNIFIED METHOD OF DECISION MAKING

3.1 Threshold logic

Under absence of faults and disturbances the rule (14) gives zero residual vector. At a practice, due to disturbances action (whose description the reference model does not contain) the residual vector will be nonzero. Under this, the question arises is the disturbances action the single reason of nonzero residual vector generation? To give answer on this question the threshold logic of decision making is involved. Let evaluation of the residual vector results that the system is healthy at time interval up to time instant t_{L-1}.

Introduce relation for the threshold computation in the form

$$\Pi(t_L) = \Pi_0 + \max_{\tau \in T_*} (r(\tau)^T (r(\tau))^{1/2}, \quad T_* = [t_0, t_{L-1}]. \tag{16}$$

It is assumed to take the constant Π_0 from simulation results or full-scale test to exclude the false alarm, see example.

If the norm of the residual vector does not exceed the threshold

$$(r^T(t_L)r(t_L))^{1/2} \leq \Pi(t_L), \tag{17}$$

the decision is made about the healthy system. Given in (16) way for threshold computation does not take into account intensity of disturbances at instant t_L. Therefore, inequality (17) can be violated even if there are no faults in the system. So, the final decision needs in additional test (see below). If this test does not prove the fault presence, at the next position of time window the threshold will be increased.

Notice, relation (16) allows considering the possible changes of disturbances intensity under diagnosis. It provides threshold adaptation to real diagnostic conditions.

3.2 Comparing with the fault syndroms

Let the fault results in nonzero value of parameter a_j^f at the instant t_L such that inequality (17) is violated. Under this, the matrix A keeps its value, while $A^f \neq 0$. Therefore, (15) holds and, as a result, the value of the residual vector at instant of time t_L satisfies equality

$$r(t_L) = A^f W(t_0, t_L) v_L(t_L) \tag{18}$$

where $v_L(t_L)$ is the last component of the vector $v(t_L)$. Under assumption that the fault value is sufficiently small, use the linear approximation

$$A^f = (\partial A^f / \partial a_j^f)_{| \, a^f = 0} \, a_j^f. \tag{19}$$

In this case, due to (18) and (19), under absence of disturbances the residual vector direction coincides with direction of the vector

$$\sigma_j(t_L) = (\partial A^f / \partial a_j^f)_{| \, a^f = 0} \, W(t_0, t_L). \tag{20}$$

The vector $\sigma_j(t_L)$ is named "syndrome of the fault numbered j" and relation (20) is used for on-line syndrom computation.

The disturbances influence results in not full coincidence of the residual and syndrome vectors direction. In this case, conclusion about the fault numbered j is made on the base of angle distance between the vectors $r(t_L)$ and $\sigma_j(t_L)$. It is proposed to use absolute value of cosine of the angle between these vectors given by following relation

$$\alpha_j(t_L) = \frac{|r^T(t_L)\, \sigma_j(t_L)|}{(r^T(t_L)\, r(t_L))^{1/2}\, (\sigma_j^T(t_L)\, \sigma_j(t_L))^{1/2}} \tag{21}$$

as the measure of a such distance. The closer $\alpha_j(t_L)$ is to one, the greater confidence in appropriate fault.

In general case, the matrix $(\partial A^f / \partial a_j^f)_{| \, a^f = 0}$ may contain unknown coefficients and immediately unobservable components of the vectors $x^{(j)}(t_0), 1 \leq j \leq N$. Two stages procedure is proposed to evaluate these coefficients and states: 1) compute the matrix A by direct solution of the equation (10); 2) express the coefficients and states looking for from the matrix A coefficients.

Assume that the size L of mowing time window is minimal such that equality (13) holds. In this case the matrix $V(t_0, t_{L-1})$ is nonsingular one and depends on the vectors of healthy system inputs and outputs measured at time interval $T_* = [t_0, t_{L-1}]$. If the matrix $V(t_0, t_{L-1})$ is not square one, then the following representation

$$A\, V(t_0, t_{L-1}) = [A_1 \,| A_2] \begin{bmatrix} V_1(t_0, t_{L-1}) \\ \hline V_2(t_0, t_{L-1}) \end{bmatrix}$$

is true, where $V_1(t_0, t_{L-1})$ is square matrix. Under this, from (10) one obtains

$$A_1 = [Y(t_0, t_{L-1}) - A_2 V_2(t_0, t_{L-1})]\, V_1^{-1}(t_0, t_{L-1}). \tag{22}$$

To solve equation (22) for the matrix A_1 one needs the matrix A_2 redefining. Let unknown system parameters included into the matrix A_2 take their values from some known intervals. In this case it is reasonable to take the middle of interval to redefine appropriate coefficient. If it is necessary to redefine the values of some system states at instant t_0, one can use an additional state observer for their estimation.

Obviously, if $V(t_0, t_{L-1})$ is square matrix, no redefining is required. To find the matrix A, it is sufficient to take $A_1 = A$, $V_1(t_0, t_{L-1}) = V(t_0, t_{L-1})$ and $A_2 V_2(t_0, t_{L-1}) = 0$ in (22).

3.3 The unified method realization

In given above relations (14), (16), (20) and (21) one uses "internal" time of the interval T_L corresponding to the real position of moving time window. In order to lock this "internal" time to the system time t, it is sufficient to take in these relations

$$t_L = t, \quad t_i = t - (L-i)\,{}_\Delta t, \quad 0 \le i \le L-1. \tag{23}$$

Consider feature of given above decision making methods joint use. Firstly, realization of the threshold logic method assumes the possibility to compute on-line both the residual vector and threshold. The residual vector computation is possible if measurements of input and output vectors are available at time interval T_L. This demand is always satisfied if $t \ge L_{\max\Delta} t$. Also, computation of the threshold is possible if the residual values are available at time interval T_* whose length does not exceed $(L_{max} - 1)_\Delta t$.

Secondly, it is necessary to take value $\alpha_{\min}$ sufficiently close to one, such that fulfilling inequality

$$\alpha_j(t) \ge \alpha_{\min} \tag{24}$$

allows making final conclusion about appropriate fault in the system. The next algorithm describes main steps of the unified method application.

Algorithm 2. (Decision making)

(1) Start when $t = L_{\max\Delta} t$.
(2) Take $i = L_{\max} - 1$.
(3) Applying Algorithm 1 compute $r(t)$.
(4) If $i \ge 1$ take $i = i - 1$, $t = t +_\Delta t$ and go to step 3.
(5) Take $t = t +_\Delta t$.
(6) Applying Algorithm 1 compute $r(t)$.
(7) Applying relation (16) compute $\Pi(t)$.
(8) If inequality (16) holds go to step 5.
(9) For every j compute $\sigma_j(t)$ and $\alpha_j(t)$ according to (20) and (21) respectively.
(10) If inequality (24) does not hold for each j go to step 5.
(11) If for some j and all $i, i \ne j$, inequality $\alpha_j(t) > \alpha_i(t)$ holds, make a decision about j^{th} fault in the system.

Give necessary comments to above algorithm. Fulfilling steps (1)-(4) provides the possibility of the residual and threshold computation at the initial stage of diagnosis. Steps (5)-(10) present the diagnostic stage accomplished by the fault detection, while at step (11) solution to fault isolation problem is given. Notice, if for some faults with numbers i and j approximate equality $\alpha_i(t) \simeq \alpha_j(t)$ takes place, above faults are indistinguishable within the framework of the proposed diagnostic method.

4. ILLUSTRATIVE EXAMPLE

4.1 The reference model design

Consider the features of the proposed results application on the base of three tank system model, see Patton et. al. (1989). The model of three tank system used in this paper has a form

$$\begin{gathered} dx_1(t)/dt = a_1 u_1(t) - (a_2 - a_2^f)\sqrt{x_1(t) - x_2(t)}, \\ dx_2(t)/dt = (a_2 - a_2^f)\sqrt{x_1(t) - x_2(t)} - \\ (a_3 - a_3^f)\sqrt{x_2(t) - x_3(t)}, \\ dx_3(t)/dt = (a_3 - a_3^f)\sqrt{x_2(t) - x_3(t)} - \\ (a_4 - a_4^f)\sqrt{x_3(t)}, \\ y_1(t) = x_2(t), \quad y_2(t) = x_3(t) \end{gathered} \tag{25}$$

where $a_j, 1 \le j \le 4$, are the constant coefficients depended on the system parameters. It is assumed that these coefficients are unknown. The coefficients $a_j^f, 2 \le j \le 4$, are given for faults in the sysyem and result in appropriate pipes blocking. In spite of the model given in Patton et. al. (1989), the model (25) assumes that liquid is drained through pipe at the bottom of the third tank.

Transformation to the model without internal feedbacks results in

$$\begin{gathered} dx_1^{(1)}(t)/dt = a_1 u_1(t) - (a_3 - a_3^f)\sqrt{y_1(t) - y_2(t)}, \\ dx_2^{(1)}(t)/dt = (a_3 - a_3^f)\sqrt{y_1(t) - y_2(t)} - \\ (a_4 - a_4^f)\sqrt{y_2(t)}, \\ dx_1^{(2)}(t)/dt = (a_2 - a_2^f)\sqrt{x_1^{(1)}(t) - 2\,y_1(t)} - \\ (a_3 - a_3^f)\sqrt{y_1(t) - y_2(t)}, \\ y_1(t) = x_1^{(2)}(t), \quad y_2(t) = x_2^{(1)}(t) \end{gathered} \tag{26}$$

under $\varphi^{(1)}(x(t)) = \begin{pmatrix} x_1(t) + x_2(t) & x_3(t) \end{pmatrix}^T$, $\varphi^{(2)}(x(t)) = x_2(t)$, $\psi(y(t)) = \begin{pmatrix} y_1(t) & y_2(t) \end{pmatrix}^T$.

Taking into account (6) and making substitution according to (7) one can write

$$\begin{gathered} x_1^{(1)}(t) = a_1 \int_{t_0}^{t} u_1(\tau)\,d\tau - \\ (a_3 - a_3^f) \int_{t_0}^{t} \sqrt{y_1(\tau) - y_2(\tau)}\,d\tau + x_1^{(1)}(t_0), \\ x_2^{(1)}(t) = (a_3 - a_3^f) \int_{t_0}^{t} \sqrt{y_1(\tau) - y_2(\tau)}\,d\tau - \\ (a_4 - a_4^f) \int_{t_0}^{t} \sqrt{y_2(\tau)}\,d\tau + x_2^{(1)}(t_0), \\ x_1^{(2)}(t) = (a_2 - a_2^f) \int_{t_0}^{t} \sqrt{x_1^{(1)}(t_0) + \mu(\tau)}\,d\tau - \\ (a_3 - a_3^f) \int_{t_0}^{t} \sqrt{y_1(\tau) - y_2(\tau)}\,d\tau + x_1^{(2)}(t_0) \end{gathered} \tag{27}$$

where

$$\mu(\tau) = a_1 \int_{t_0}^{\tau} u_1(\tau')\, d\tau' - (a_2 - a_2^f) \int_{t_0}^{\tau} \sqrt{y_1(\tau') - y_2(\tau')}\, d\tau' - 2\, y_1(\tau). \quad (28)$$

Using expansion of square root into a series one can write

$$\sqrt{1 + \frac{\mu(\tau)}{x_1^{(1)}(t_0)}} = 1 + \frac{1}{2}\frac{\mu(\tau)}{x_1^{(1)}(t_0)} - \frac{1}{8}\left(\frac{\mu(\tau)}{x_1^{(1)}(t_0)}\right)^2 - \ldots .$$

Involving only three terms, one obtains

$$x_1^{(2)}(t) = (a_2 - a_2^f)\times \sqrt{x_1^{(1)}(t_0)} \int_{t_0}^{t} \left(1 + \frac{1}{2}\frac{\mu(\tau)}{x_1^{(1)}(t_0)} - \frac{1}{8}\left(\frac{\mu(\tau)}{x_1^{(1)}(t_0)}\right)^2\right) d\tau - -(a_3 - a_3^f) \int_{t_0}^{t} \sqrt{y_1(\tau) - y_2(\tau)}\, d\tau + x_1^{(2)}(t_0). \quad (29)$$

Taking into account (27) and (28), (29), the matrices of the reference model (8) can be written as follows

$$A = \begin{pmatrix} x_1^{(2)}(t_0) & a_2\sqrt{x_1^{(1)}(t_0)} & \frac{a_1 a_2}{2\sqrt{x_1^{(1)}(t_0)}} & \frac{-a_2}{\sqrt{x_1^{(1)}(t_0)}} \\ x_2^{(1)}(t_0) & 0 & 0 & 0 \end{pmatrix} \Bigg| \begin{matrix} \frac{a_2}{2x_1^{(1)}(t_0)\sqrt{x_1^{(1)}(t_0)}} & -a_3 & \frac{-a_2^2}{2\sqrt{x_1^{(1)}(t_0)}} & 0 \\ 0 & a_3 & 0 & -a_4 \end{matrix} \Bigg| \begin{matrix} \frac{a_1^2 a_2}{8x_1^{(1)}(t_0)\sqrt{x_1^{(1)}(t_0)}} & \frac{a_1 a_2^2}{4x_1^{(1)}(t_0)\sqrt{x_1^{(1)}(t_0)}} \\ 0 & 0 \end{matrix} \Bigg| \begin{matrix} \frac{-a_2^3}{8x_1^{(1)}(t_0)\sqrt{x_1^{(1)}(t_0)}} & \frac{-a_2^2}{2x_1^{(1)}(t_0)\sqrt{x_1^{(1)}(t_0)}} \\ 0 & 0 \end{matrix} \Bigg),$$

$$A^f = \begin{pmatrix} 0 & -a_2^f\sqrt{x_1^{(1)}(t_0)} & \frac{-a_1 a_2^f}{2\sqrt{x_1^{(1)}(t_0)}} & \frac{a_2^f}{\sqrt{x_1^{(1)}(t_0)}} \\ 0 & 0 & 0 & 0 \end{pmatrix} \Bigg| \begin{matrix} \frac{-a_2^f}{2x_1^{(1)}(t_0)\sqrt{x_1^{(1)}(t_0)}} & a_3^f & \frac{-2a_2 a_2^f + (a_2^f)^2}{2\sqrt{x_1^{(1)}(t_0)}} & 0 \\ 0 & -a_3^f & 0 & a_4^f \end{matrix} \Bigg| \begin{matrix} \frac{-a_1^2 a_2^f}{8x_1^{(1)}(t_0)\sqrt{x_1^{(1)}(t_0)}} & \frac{a_1(-2a_2 a_2^f + (a_2^f)^2)}{4x_1^{(1)}(t_0)\sqrt{x_1^{(1)}(t_0)}} \\ 0 & 0 \end{matrix} \Bigg| \begin{matrix} \frac{-3a_2^2 a_2^f + 3a_2(a_2^f)^2 - (a_2^f)^3}{8x_1^{(1)}(t_0)\sqrt{x_1^{(1)}(t_0)}} & \frac{-a_2 a_2^f + (a_2^f)^2}{2x_1^{(1)}(t_0)\sqrt{x_1^{(1)}(t_0)}} \\ 0 & 0 \end{matrix} \Bigg),$$

$$W(t_0,t) = \Bigg(1 \;\; t - t_0 \;\; \int_{t_0}^{t}\int_{t_0}^{\tau} u_1(\tau')d\tau' d\tau \;\; \int_{t_0}^{t} y_1(\tau)d\tau \;\Big|\; \int_{t_0}^{t} y_1^2(\tau)d\tau \;\; \int_{t_0}^{t} \sqrt{y_1(\tau) - y_2(\tau)}d\tau \;\Big|\; \int_{t_0}^{t}\int_{t_0}^{\tau} \sqrt{y_1(\tau') - y_2(\tau')}d\tau' d\tau \;\Big|\; \int_{t_0}^{t} \sqrt{y_2(\tau)}d\tau \;\; \int_{t_0}^{t} \Big(\int_{t_0}^{\tau} u_1(\tau')d\tau'\Big)^2 d\tau \;\Big|\; \int_{t_0}^{t} \Big(\int_{t_0}^{\tau} u_1(\tau')d\tau' \int_{t_0}^{\tau} \sqrt{y_1(\tau') - y_2(\tau')}d\tau'\Big) d\tau \;\Big|\; \int_{t_0}^{t} \Big(\int_{t_0}^{\tau} \sqrt{y_1(\tau') - y_2(\tau')}d\tau'\Big)^2 d\tau \;\Big|\; \int_{t_0}^{t} y_1(\tau)\Big(\int_{t_0}^{\tau} \sqrt{y_1(\tau') - y_2(\tau')}d\tau'\Big) d\tau \Bigg)^T .$$

Analysis of the above matrices allows making following conclusions. Firstly, as soon the matrix $W(t_0, t)$ does not contain unknown coefficients, the residual generation procedure provided by Algorithm 1 is nonparametric (knowledge of the coefficients $a_j, 1 \le j \le 4$ is not required). Secondly, the matrices$(\partial A^f / \partial a_j^f)_{|\, a^f=0}$, $2 \le j \le 4$, are depended on the model coefficients. But it is easy to see that the faults resulting in distortion a_2^f (the first fault) and a_4^f (the third fault) influence only on the first and the second rows of the matrix A^f respectively, while contribution of the coefficient a_3^f (the second fault) to the both rows of the matrix A^f is the same in quantity but different in sign. Considering (20), one can take $\sigma_1 = (1\; 0)^T$, $\sigma_2 = (1\; -1)^T$ and $\sigma_3 = (0\; 1)^T$. Thus, decision making also does not need in the system parameters knowledge.

4.2 Simulation results

Above matrices were used to simulate the diagnostic process in three tank system involving the package MatLab. Under simulation of the healthy system, the system parameters were taken as follows $a_1 = 1\text{m}^{-2}$, $a_2 = a_3 = a_4 = 0,4427\text{m}^{1/2}/\text{sec}$. Residual generation and decision making were realized via Algorithm 1 and Algorithm 2 respectively. The results obtained for the healthy system case are given in Fig.1. The plots (a), (b) and (c) describe behavior of the variables $u(t), y_1(t)$ and $y_2(t)$ respectively, while the plot (d) describes behavior of the residual vector norm and adaptive threshold. Nonzero value of the residual norm is explained by approximate character of the reference model due to expansion of square root into series. The value of the threshold constant term $\Pi_0 = 2 \times 10^{-4}$ is taken from the condition of false alarm absence at instant $t = 300$sec.

Figures 2, 3 and 4 correspond to the first ($a_2^f = 0.1$ at $t = 350$sec), second ($a_3^f = 0.2$ at $t = 500$sec) and the third ($a_4^f = 0.1$ at $t = 700$sec) faults respectively. Under this, the plots denoted (a) describe behavior of the residual vector norm and adaptive threshold, while plots (b), (c) and (d) correspond to angle distance $\alpha_j, 1 \leq j \leq 3$ in a case when this distance exceeds the value $\alpha_{min} = 0.9$. If for some j holds $\alpha_j \leq \alpha_{min}$ it is shown $\alpha_j = 0$ at appropriate plot.

Comparing the plots (b), (c) and (d), obtained for different faults, allows making conclusion about possibility to isolate all the faults.

5. CONCLUSION

In present paper, an effective solution has been given to the problem of robust fault diagnosis in dynamic systems described by nonlinear ordinary differential equations with unknown constant coefficients. In contrast to former papers, proposed solution admits the influence of unstructured disturbances on the diagnostic process due to uncontrollable external actions, noise of measurements and approximations assumed under reference model design.

Proposed solution combines the nonparametric method of the residual generation and unified decision making method, as its complement. In some sense, proposed unified decision making method is also nonparametric one as soon it assumes unknown coefficients evaluation if necessary.

As a result, application of proposed results gives the possibility to develop the same diagnostic platform for different systems described by the ordinary differential equations of the same structure but distinguished by coefficient values. It allows speaking about practical significance of the paper results.

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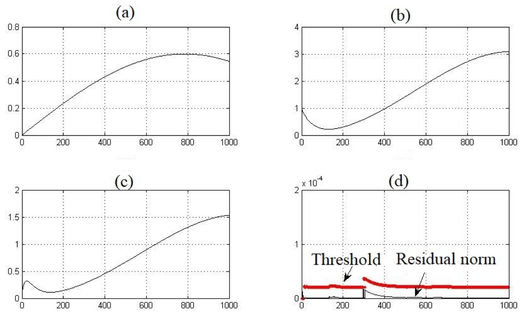

Fig. 1. The healthy system case

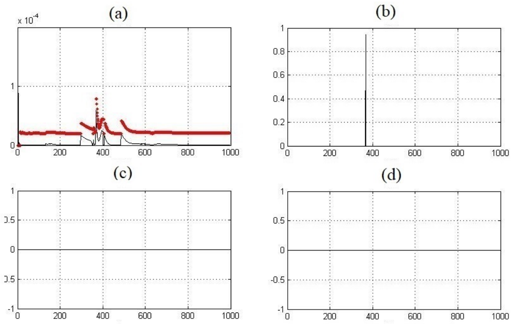

Fig. 2. The first fault in the system

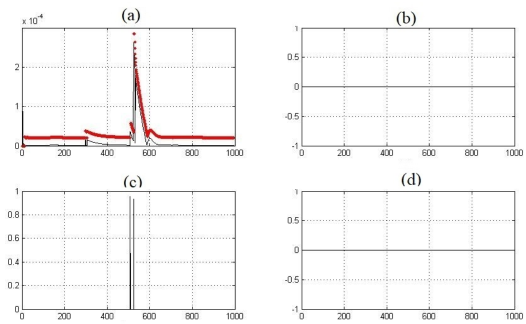

Fig. 3. The second fault in the system

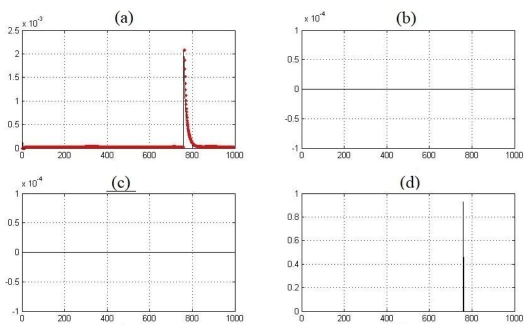

Fig. 4. The third fault in the system