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A Fundamental Bound on Performance of Non-Intrusive Load Monitoring Algorithms with Application to Smart-Meter Privacy ⋆

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Abstract: We consider non-intrusive load monitoring by a sophisticated adversary that knows the load profiles of the appliances and wants to determine their start-finish times based on smart-meter readings. We prove that the expected estimation error of non-intrusive load monitoring algorithms is lower bounded by the trace of the inverse of the cross-correlation matrix between the derivatives of the load profiles of the appliances. This is an interesting observation illustrating that the derivatives of the load profiles are more important than the profiles themselves for non-intrusive load monitoring (i.e., small rapidly-changing loads are easier to identify than large, yet slowly-varying ones). This fundamental bound on the performance of non-intrusive load monitoring adversaries is used to develop privacy-preserving policies. Particularly, we devise a load-scheduling policy by maximizing the lower bound on the expected estimation error of non-intrusive load monitoring algorithms.

Keywords: Non-intrusive load monitoring; Cross-correlation; Smart meter; Fisher information; Privacy.

1. INTRODUCTION

Non-intrusive load monitoring research is dedicated to development of algorithms for dis-aggregating overall energy consumption of households measured by smart meters to estimate timing of individual appliances, such as fridge or air-conditioning units (Hosseini et al., 2017; Zoha et al., 2012; Davies et al., 2019; Jin et al., 2011; Zhao et al., 2015; Rashid et al., 2019). The research is often motivated by the interest in providing consumers with energy-saving tips to lower bills or counter climate change, performing fault detection, and accommodating electricity grid transformations due to integration of renewable energy. Although an important area of research, little is done in understanding fundamental bounds on the achievable performance of non-intrusive load monitoring algorithms.

In this paper, we consider a sophisticated adversary that has access to the load profiles of the appliances within a house, e.g., it knows the specific brand and the model of appliances. The adversary is interested in estimating the start-finish times of the appliances based on frequent smart-meter readings, measuring total energy consumption of the house. Our interest in this powerful adversary originates from our desire to provide a lower bound on the abilities of all non-intrusive load monitoring algorithms with different levels of access to information (i.e., adversaries without exact knowledge of the load profiles can only perform worse in comparison). We prove that the *expected estimation error of the start times of the appliances by any non-intrusive load monitoring algorithm is lower bounded by the trace of the inverse of the cross-correlation matrix between the derivatives of the load profiles of the appliances.* This is an interesting observation showing the importance of the magnitude of the derivatives of the load profiles rather than the load profiles themselves for non-intrusive load monitoring, i.e., small rapidly-changing loads are easier to isolate than large, yet slowly-varying ones.

Energy time series, collected by smart meters, is known to leak private information of households, such as occupancy and appliance usage (McDaniel and McLaughlin, 2009). This sensitive data can be accessed by third-party data-analytic companies [1] through electricity retailers or utility companies. For instance, in Australia, an electricity retailer required its customers to consent to sharing their data with third parties in the United States before permitting them to use an online web portal (Chadwick et al., 2012). Therefore, non-intrusive load monitoring can be used for gaining further privacy-intrusive insights, e.g., entertainment or feeding habits. This is evident from the patents on the technology expressing its use in targeted advertising (Leeb and Kirtley Jr, 1996; Haghighat-kashani et al., 2015). Also, non-intrusive load monitoring has been proved to be commercially viable and attractive [1]. Similar technologies are currently being used (albeit based on smart meters for water) to track elderly behaviour and help those in distress (Corner, 2018) with similar appli-

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[1] See `http://bidgely.com` and `https://plotwatt.com/` as examples of commercially viable initiatives providing non-intrusive load monitoring services.

cations to monitoring patients (Chalmers et al., 2018). This technology is clearly a double-edged sword that can be used by both healthcare professionals and insurance agencies.

The privacy and security concerns associated with smart meter data has motivated the development of privacy-preserving polices for smart meters based on additive noise, renewable resources, rechargeable batteries, and air conditioning units; see (Ács and Castelluccia, 2011; Yao and Venkitasubramaniam, 2017; Cho et al., 2019; Liu et al., 2017; Sankar et al., 2012; Eibl and Engel, 2014; Farokhi and Sandberg, 2017; Giaconi et al., 2017; Liu and Cheng, 2017; Farokhi and Sandberg, 2020) and references there-in. In this paper, however, we take a different approach to smart meter privacy using load scheduling. Particularly, we use the aforementioned *fundamental bound on estimation error of non-intrusive load monitoring algorithms to develop privacy-preserving policies.* We devise load-scheduling policies by maximizing the trace of the inverse of the cross-correlation matrix between the derivatives of the load profiles of the appliances. This enables us to select a schedule for operating the appliances that renders the problem of non-intrusive load monitoring difficult for even the most sophisticated adversaries. Previous attempts at load scheduling for privacy, e.g., (Liu and Cheng, 2017), have not considered the performance of non-intrusive load monitoring adversaries. They further tried to keep the smart meter readings constant by load scheduling with the aid of batteries. However, in this paper, we do not use batteries. We also use the previously-discussed fundamental bound on the estimation error of non-intrusive load monitoring algorithms as a measure of privacy.

2. NON-INTRUSIVE LOAD MONITORING

Consider a house with $n \in \mathbb{N}$ appliances. Appliance $i \in [n] := \{1, \dots, n\}$ has a load signature of $f_i : \mathbb{R} \to \mathbb{R}$, i.e., its unique energy consumption pattern. In this paper, these functions are known by the non-intrusive load monitoring algorithm. The load's signature is such that $f_i(t) = 0$ for all $t < 0$ (before starting to work) and $t > T$ (after finishing its work) for some large enough T. Appliance $i \in [n]$ is scheduled to start at $\tau_i \in \mathbb{R}$. Therefore, the total consumption of the house at any given time $t \in \mathbb{R}$ is $\sum_{i\in[n]} f_i(t - \tau_i)$.

We assume that a non-intrusive load monitoring algorithm can access noisy measurements of the total consumption at discrete times $(t_\ell)_{\ell\in[k]} \subseteq [0, T]$. The noise models measurement noise, privacy-preserving additive noises (e.g., differential-privacy noise), and consumption of small loads that the non-intrusive load monitoring algorithm is not interested in identifying. The measurement y_ℓ at time t_ℓ is then given by

$$y_\ell = \sum_{i\in[n]} f_i(t_\ell - \tau_i) + w_\ell, \tag{1}$$

where $w := (w_\ell)_{\ell\in[k]}$ is a sequence of i.i.d.[2] noises. The non-intrusive load monitoring algorithm is interested in estimating $\tau := (\tau_i)_{i\in[n]} \in \mathbb{R}^n$ from the measurements $y := (y_\ell)_{\ell\in[k]} \in \mathbb{R}^k$ using a family of arbitrary estimators denoted by $\hat{\tau}_i : \mathbb{R}^k \to \mathbb{R}$ for all $i \in [n]$. We are interested for finding a lower bound on

$$\sum_{i\in[n]} \pi_i \mathbb{E}\{(\hat{\tau}_i(y) - \tau_i)^2\} = \mathbb{E}\{\|\Pi^{1/2}(\hat{\tau}(y) - \tau)\|_2^2\}, \tag{2}$$

where $\hat{\tau} := (\hat{\tau}_i)_{i\in[n]}$ and $\Pi := \mathrm{diag}(\pi_1, \dots, \pi_n)$. We make the following standing assumption.

Assumption 1. (Regularity). $p(w)$ is continuously differentiable; $p(w) = 0, \forall w \in \partial\,\mathrm{supp}(p)$.

The regularity condition in Assumption 1 is a basic assumption that is common in signal processing results, such as the Cramér-Rao bound (Shao, 2003, p. 169). This assumption holds for Gaussian and Laplace distributions, and many other density functions. In fact, any density function with an unbounded support automatically satisfies this condition. We can prove the following fundamental bound on the performance of unbiased non-intrusive load monitoring algorithms.

Theorem 1. For any unbiased estimator $\hat{\tau}$, i.e., $\mathbb{E}\{\hat{\tau}\} = \tau$, we get

$$\mathbb{E}\{\|\Pi^{1/2}(\hat{\tau}(y) - \tau)\|_2^2\} \geq \frac{1}{\mathcal{I}^w} \mathrm{trace}(\Pi R_{\mathrm{d}}(\tau)^{-1}),$$

where $\mathcal{I}^w = \mathbb{E}_w\{(p'(w)/p(w))^2\}$ is the Fisher information of the additive noise and $R_{\mathrm{d}}(\tau)$ is the discrete cross-correlation matrix function for the derivatives of the load signatures with entry in i-the row and j-the column defined as

$$[R_{\mathrm{d}}]_{ij}(\tau_i, \tau_j) := \sum_{\ell\in[k]} f_i'(t_\ell - \tau_i) f_j'(t_\ell - \tau_j).$$

Proof. The conditional probability density of observing y given τ is equal to

$$p(y|\tau) = \prod_{\ell\in[k]} p\left(y_\ell - \sum_{i\in[n]} f_i(t_\ell - \tau_i)\right).$$

Differentiating logarithm of the conditional density $p(y|\tau)$ results in

$$\frac{\partial \log(p(y|\tau))}{\partial \tau_i} = \sum_{j\in[k]} \frac{p'\left(y_j - \sum_{i\in[n]} f_i(t_j - \tau_i)\right)}{p\left(y_j - \sum_{i\in[n]} f_i(t_j - \tau_i)\right)} f_i'(t_j - \tau_i).$$

Following this, we can compute the entry in i-th row and q-th column of the Fisher information matrix as

[2] i.i.d. stands for independently and identically distributed.

$$
\begin{aligned}
\mathcal{I}_{iq} :=& \mathbb{E}_y \left\{ \frac{\partial}{\partial \tau_i} \log(p(y|\tau)) \frac{\partial}{\partial \tau_q} \log(p(y|\tau)) \right\} \\
=& \mathbb{E}_w \left\{ \left(\sum_{j\in[k]} \frac{p'(w_j)}{p(w_j)} f_i'(t_j - \tau_i) \right) \right. \\
& \left. \times \left(\sum_{j\in[k]} \frac{p'(w_j)}{p(w_j)} f_q'(t_j - \tau_q) \right) \right\} \\
=& \sum_{j_1, j_2 \in [k]} \mathbb{E}_w \left\{ \frac{p'(w_{j_1})}{p(w_{j_1})} \frac{p'(w_{j_2})}{p(w_{j_2})} \right\} \\
& \times f_i'(t_{j_1} - \tau_i) f_q'(t_{j_2} - \tau_q) \\
=& \mathcal{I}^w \sum_{j\in[k]} f_i'(t_j - \tau_i) f_q'(t_j - \tau_q) \\
=& \mathcal{I}^w [R_\mathrm{d}]_{iq}(\tau_i, \tau_q).
\end{aligned}
$$

Therefore, the Fisher information matrix is equal to

$$
\mathcal{I} := \mathbb{E}_y \left\{ \nabla_\tau p(y|\tau) \nabla_\tau p(y|\tau)^\top \right\} = \mathcal{I}^w R_\mathrm{d}(\tau).
$$

Finally, we get

$$
\begin{aligned}
&\mathbb{E}\{\|\Pi^{1/2}(\hat{\tau}(y) - \tau)\|_2^2\} \\
&\quad = \mathrm{trace}(\mathbb{E}\{\Pi^{1/2}(\hat{\tau} - \tau)(\hat{\tau} - \tau)^\top \Pi^{1/2}\}) \\
&\quad = \mathrm{trace}(\Pi^{1/2}\mathbb{E}\{(\hat{\tau} - \tau)(\hat{\tau} - \tau)^\top\}\Pi^{1/2}) \\
&\quad \geq \mathrm{trace}(\Pi^{1/2}(\mathcal{I}^w R_\mathrm{d}(\tau))^{-1}\Pi^{1/2}) \\
&\quad = \mathrm{trace}(\Pi^{1/2}R_\mathrm{d}(\tau)^{-1}\Pi^{1/2})/\mathcal{I}^w \\
&\quad = \mathrm{trace}(\Pi R_\mathrm{d}(\tau)^{-1})/\mathcal{I}^w.
\end{aligned}
$$

This concludes the proof. ■

If we assume that w_ℓ is a zero-mean Gaussian random variable with variance $\sigma_w^2 > 0$ for all $\ell \in [k]$, we can simplify the bound in Theorem 1 by noting that $\mathcal{I}^w = \sigma_w^{-2}$.

Note that the bound in Theorem 1 is only valid for unbiased estimators. We generalize this bound to unbiased estimators in the next theorem.

Theorem 2. For any estimator $\hat{\tau}$, we get

$$
\begin{aligned}
&\mathbb{E}\{\|\Pi^{1/2}(\hat{\tau}(y) - \tau)\|_2^2\} \\
&\qquad \geq \frac{1}{\mathcal{I}^w} \mathrm{trace}\left(\Pi \frac{\partial \mu(\tau)}{\partial \tau} R_\mathrm{d}(\tau)^{-1} \frac{\partial \mu(\tau)}{\partial \tau}^\top \right) \\
&\qquad\quad + \|\Pi^{1/2}\mu(\tau)\|_2^2,
\end{aligned}
$$

where $\mu(\tau) := \mathbb{E}\{\hat{\tau}(y)\}$.

Proof. The proof follows from that

$$
\begin{aligned}
&\mathbb{E}\{\|\Pi^{1/2}(\hat{\tau}(y) - \tau)\|_2^2\} \\
&\quad \geq \mathrm{trace}\left(\Pi^{1/2} \left(\frac{\partial \mu(\tau)}{\partial \tau} (\mathcal{I}^w R_\mathrm{d}(\tau))^{-1} \frac{\partial \mu(\tau)}{\partial \tau}^\top \right.\right. \\
&\qquad \left.\left. + \mu(\tau)\mu(\tau)^\top \right) \Pi^{1/2} \right) \\
&\quad = \frac{1}{\mathcal{I}^w} \mathrm{trace}\left(\Pi \frac{\partial \mu(\tau)}{\partial \tau} R_\mathrm{d}(\tau)^{-1} \frac{\partial \mu(\tau)}{\partial \tau}^\top \right) \\
&\qquad + \|\Pi^{1/2}\mu(\tau)\|_2^2.
\end{aligned}
$$

This concludes the proof. ■

We can further simplify the lower bound in Theorem 2.

Corollary 1. For any estimator $\hat{\tau}$, we get

$$
\mathbb{E}\{\|\Pi^{1/2}(\hat{\tau}(y) - \tau)\|_2^2\} \geq \frac{c_1(\tau)}{\mathcal{I}^w} \mathrm{trace}(\Pi R_\mathrm{d}(\tau)^{-1}) + c_2(\tau),
$$

where

$$
\begin{aligned}
c_1(\tau) &:= \lambda_{\min}\left(\Pi^{-1/2} \frac{\partial \mu(\tau)}{\partial \tau}^\top \Pi \frac{\partial \mu(\tau)}{\partial \tau} \Pi^{-1/2} \right) \geq 0, \\
c_2(\tau) &:= \|\Pi^{1/2}\mu(\tau)\|_2^2 \geq 0.
\end{aligned}
$$

Proof. Note that

$$
\begin{aligned}
&\mathrm{trace}\left(\Pi \frac{\partial \mu(\tau)}{\partial \tau} R_\mathrm{d}(\tau)^{-1} \frac{\partial \mu(\tau)}{\partial \tau}^\top \right) \\
&\quad = \mathrm{trace}\left(\Pi^{-1/2} \frac{\partial \mu(\tau)}{\partial \tau}^\top \Pi \frac{\partial \mu(\tau)}{\partial \tau} \Pi^{-1/2} \right. \\
&\qquad \left. \times \Pi^{1/2} R_\mathrm{d}(\tau)^{-1} \Pi^{1/2} \right) \\
&\quad \geq \lambda_{\min}\left(\Pi^{-1/2} \frac{\partial \mu(\tau)}{\partial \tau}^\top \Pi \frac{\partial \mu(\tau)}{\partial \tau} \Pi^{-1/2} \right) \\
&\qquad \times \mathrm{trace}(\Pi^{1/2} R_\mathrm{d}(\tau)^{-1} \Pi^{1/2}),
\end{aligned}
$$

where the inequality follows from the inequality on traces of positive semi-definite matrices in (Kleinman and Athans, 1968). Combining this inequality with the fact that $\mathrm{trace}(\Pi^{1/2}R_\mathrm{d}(\tau)^{-1}\Pi^{1/2}) = \mathrm{trace}(\Pi R_\mathrm{d}(\tau)^{-1})$ concludes the proof. ■

Note that $R_\mathrm{d}(\tau)$ in Theorems 1 and 2 is a function of the sampling times $(t_\ell)_{\ell\in[k]}$. However, as k increases, this dependence disappears. This is discussed in the following corollary.

Corollary 2. For any unbiased estimator $\hat{\tau}$, i.e., $\mathbb{E}\{\hat{\tau}\} = \tau$, we get

$$
\lim_{k\to\infty} k\mathbb{E}\{\|\Pi^{1/2}(\hat{\tau} - \tau)\|_2^2\} \geq \frac{1}{\mathcal{I}^w} \mathrm{trace}(\Pi R_\mathrm{c}(\tau)^{-1}),
$$

where $R_\mathrm{c}(\tau)$ is the continuous cross-correlation matrix function for the derivatives of the load signatures with entry on i-the row and j-the column defined as

$$
[R_\mathrm{c}]_{ij}(\tau_i, \tau_j) := \int_{-\infty}^{+\infty} f_i'(t - \tau_i) f_j'(t - \tau_j) \mathrm{d}t.
$$

Proof. The proof follows from the convergence of the Riemann integral. The bounds of the integral can be pushed from $[0, T]$ to $(-\infty, +\infty)$ due to the fact that the load profiles are equal to zero outside $[0, T]$. ■

3. SMART-METER PRIVACY BY LOAD SCHEDULING

In this section, we propose an approach for smart meter privacy based on load scheduling. Clearly, the lower bound on the performance of an adversary employing a non-intrusive load monitoring algorithm for prying into a household in Theorem 1 and Corollary 1 is a function of the scheduling time of the appliances τ. This motivate us to solve the following optimization problem for finding the optimal privacy-preserving scheduling:

$$
\max_{\tau\in\mathfrak{T}} \mathrm{trace}(\Pi R_\mathrm{d}(\tau)^{-1}), \tag{3}
$$

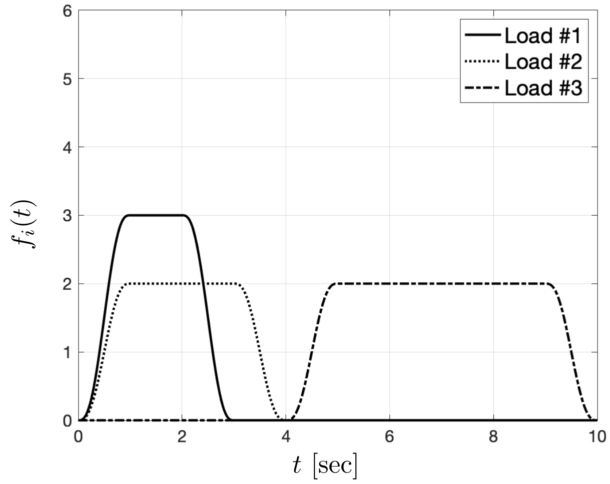

Fig. 1. Examples of three generic loads. The loads are shifted for optimum visibility; otherwise, the user wants to schedule all at the beginning.

where $\mathfrak{T}$ denotes the set of feasible schedules. The cost function of this optimization problem is however non-convex. Therefore, there are many local optima that can be recovered using numerical algorithms, such as the gradient descent. For this, we note that

$$\begin{aligned}\frac{\partial\,\mathrm{trace}(\Pi R_{\mathrm{d}}(\tau)^{-1})}{\partial\tau_i} &= \mathrm{trace}\left(\Pi\left(\frac{\partial R_{\mathrm{d}}(\tau)^{-1}}{\partial\tau_i}\right)\right)\\ &= -\,\mathrm{trace}\left(R_{\mathrm{d}}(\tau)^{-1}\Pi R_{\mathrm{d}}(\tau)^{-1}\left(\frac{\partial R_{\mathrm{d}}(\tau)}{\partial\tau_i}\right)\right),\end{aligned}$$

where

$$\left[\frac{\partial R_{\mathrm{d}}(\tau)}{\partial\tau_i}\right]_{ji} = \left[\frac{\partial R_{\mathrm{d}}(\tau)}{\partial\tau_i}\right]_{ij} = \begin{cases}-2\sum_{j\in[k]} f_i''(t_j-\tau_i)f_i'(t_j-\tau_i), & i=j,\\ -\sum_{j\in[k]} f_i''(t_j-\tau_i)f_j'(t_j-\tau_j), & i\neq j,\end{cases}$$

and

$$\left[\frac{\partial R_{\mathrm{d}}(\tau)}{\partial\tau_i}\right]_{q\ell} = 0, \quad \forall q,\ell\neq i.$$

Therefore, we can use the projected gradient ascent to solve (3) by following

$$\tau^{k+1} = P_{\mathfrak{T}}\left[\tau^k + \mu_k\frac{\partial}{\partial\tau}\mathrm{trace}(\Pi R_{\mathrm{d}}(\tau)^{-1})\right], \qquad (4)$$

where $\mu_k > 0$ denotes the step size and $P_{\mathfrak{T}}[\cdot]$ denotes projection into the set $\mathfrak{T}$. For large k, we can replace $R_{\mathrm{d}}(\tau)$ with $R_{\mathrm{c}}(\tau)$ while noting that

$$\left[\frac{\partial R_{\mathrm{c}}(\tau)}{\partial\tau_i}\right]_{ji} = \left[\frac{\partial R_{\mathrm{c}}(\tau)}{\partial\tau_i}\right]_{ij} = \begin{cases}-2\int_{-\infty}^{\infty} f_i''(t-\tau_i)f_i'(t-\tau_i)\mathrm{d}t, & i=j,\\ -\int_{-\infty}^{\infty} f_i''(t-\tau_i)f_j'(t-\tau_j)\mathrm{d}t, & i\neq j,\end{cases}$$

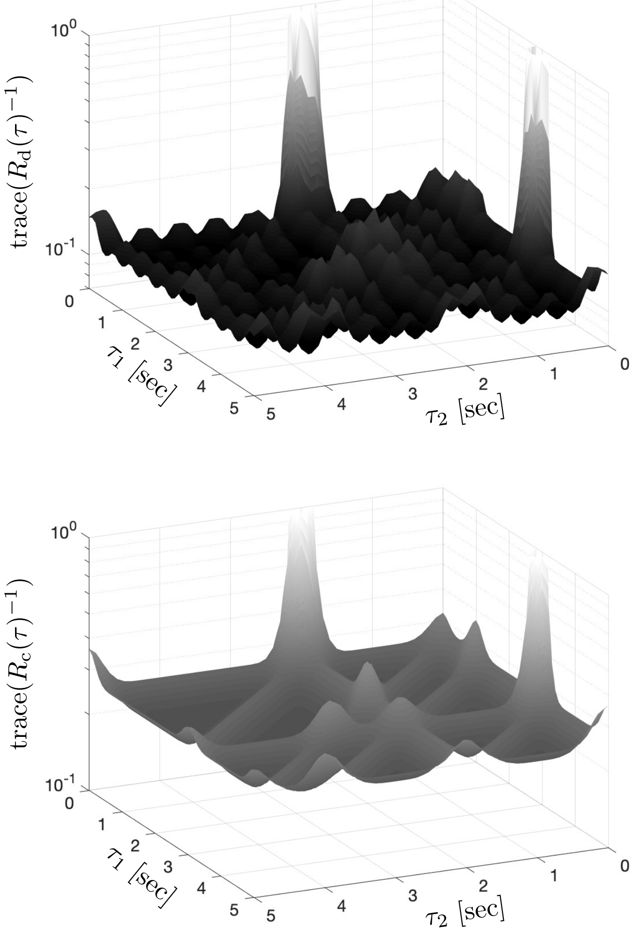

Fig. 2. Lower bound on the estimation error of any non-intrusive load monitoring algorithm in Theorem 1 [top] and Corollary 2 [bottom] versus scheduling delays of the first two loads.

and, similarly,

$$\left[\frac{\partial R_{\mathrm{c}}(\tau)}{\partial\tau_i}\right]_{q\ell} = 0, \quad \forall q,\ell\neq i.$$

4. NUMERICAL EXAMPLE

4.1 Illustrative Example

Here, we use an illustrative example to demonstrate the results of this paper. For this purpose, let us consider three generic loads depicted in Figure 1. These loads possess simple forms for ease of demonstration. Assume that the additive noise is Gaussian with zero mean and standard deviation $\sigma_w = 0.1$.

Assume that the non-intrusive load monitoring algorithm has access to the total measurements at $\{0, 0.5, \ldots, 9.5, 10\}$. Figure 2 [top] illustrates the lower bound on the estimation error of any non-intrusive load monitoring algorithm in Theorem 1 versus scheduling delays of the first two loads τ_1, τ_2 when fixing $\tau_3 = 0$. Figure 2 [bottom] illustrates the lower bound on the estimation error of any non-intrusive load monitoring algorithm in Corollary 2 versus scheduling delays of the first two loads. Clearly, these two bounds

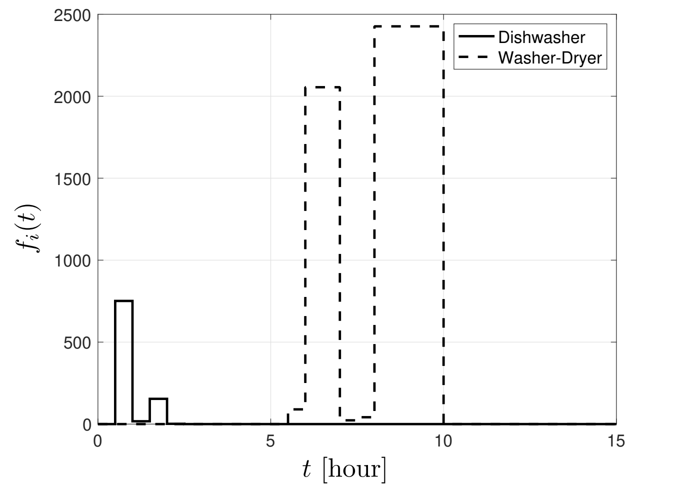

Fig. 3. Examples of two real loads in a house. Again, the loads are shifted for optimum visibility; otherwise, the user wants to schedule all at the beginning.

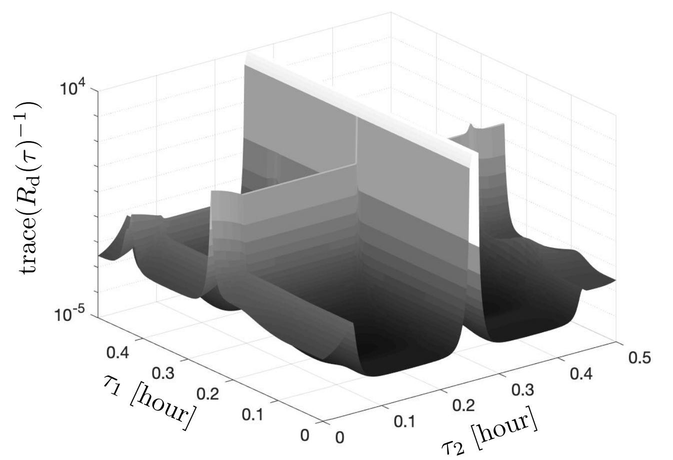

Fig. 4. Lower bound on the estimation error of any non-intrusive load monitoring algorithm in Theorem 1 versus scheduling delays of the first two loads.

are very close to each other due to the high number of measurements in the discrete case.

Furthermore, as expected, the lower bounds in Theorem 1 and Corollary 2 are non-convex in the decision variables of the scheduling problem τ and possess many local optima. However, there are two points for which the lower bounds become very large, in fact these points are the globally-optimal privacy-preserving schedules when fixing $\tau_3 = 0$. They corresponds to schedules for which its is impossible to determine if the second load is scheduled first or the third load. Therefore, the estimation error of any non-intrusive load monitoring algorithm is large. Therefore, these are the most privacy-preserving schedules.

4.2 Real Appliances

In this subsection, we consider two realistic household appliances: a dishwasher and a washer-dryer. The load profiles of these appliances is borrowed from (Farokhi and Cantoni, 2015). Let us assume that the additive noise is Gaussian with zero mean and given standard deviation σ_w. The non-intrusive load monitoring algorithm has access to the combined measurements at regular 30 min intervals. Figure 4 shows the lower bound on the estimation error of any non-intrusive load monitoring algorithm in Theorem 1 versus scheduling delays of the first two loads τ_1, τ_2. The optimum scheduling delay is given by $\tau_1 = 0.25$ and $\tau_2 = 0.2633$. This schedule combined with the fact that the non-intrusive load monitoring algorithm can only monitor the total consumption at half-hourly intervals render the task of distinguishing the start time of the dishwasher difficult for any non-intrusive load monitoring algorithm.

5. CONCLUSIONS AND FUTURE RESEARCH

We proved that the expected estimation error of non-intrusive load monitoring algorithms is lower bounded by the trace of the inverse of the cross-correlation matrix between the derivatives of the load profiles of the appliances. We developed privacy-preserving load scheduling policies by maximizing the lower bound on the expected estimation error of non-intrusive load monitoring algorithms. Future work can focus on experimental demonstration and validations of these results with off-the-shelf non-intrusive load monitoring algorithms.

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