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Port-Hamiltonian Sliding Mode Observer Design for a Counter-current Heat Exchanger

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Abstract: This paper presents a sliding mode observer (SMO) for estimating temperatures in a heat exchanger. First a port-Hamiltonian formulation for a countercurrent heat exchanger is proposed. It is so as to guarantee convergence of the observer. It is shown that the Stokes-Dirac structure obtained by opening only the dissipation ports due to the convection phenomenon, is conservative. Secondly, a SMO based on an interconnected structure of port-Hamiltonian systems is designed. The convergence of the dynamics of the estimation error is proven. The simulation results illustrate the effectiveness of this estimation strategy.

Keywords: Heat exchanger, SMO, port-Hamiltonian systems, state estimation.

1. INTRODUCTION

The dynamics of many industrial chemical and biochemical systems can be described by models that take into account the phenomena of transport (diffusion and convection) and reaction, mathematically translated in partial differential equations (PDEs). This work is motivated by a large-scale application: heat exchangers. These systems are devices that allow the exchange of heat between two fluids. Their dynamics is described by hyperbolic (Maidi et al. (2009), Chen (2014), Malinowski and Chen (2016), Aulisa et al. (2016)) or parabolic PDEs (Burns and Cliff (2014), Burns and Kramer (2015)).
Research on the control of heat exchangers has been developed, while taking into account their distributed nature. At least, these control laws are synthesized on the assumption that the state of the system is measured at each point of space. This hypothesis is purely theoretical. In practice, it is difficult or even impossible in the case of distributed parameter systems to access the complete state of the system. Indeed, the dimension of the state space of these systems is infinite, while that of the observations is finite (in other words, the measurement is accessible only on certain subsets of the domain). Hence there is a need to develop state observers that can provide an estimate of the variables needed for the synthesis of control laws.
In this work we focus on the synthesis of an observer by sliding mode. Indeed, this observer, unlike that of Kalman, has the advantage of being insensitive to modeling errors as well as to uncertainties in the system parameters. The specificity of our approach is to put forward the structure of the system using a port-Hamiltonian formulation (Duindam et al. (2009), Jacob and Zwart (2012), Macchelli et al. (2015)) of the heat exchanger. This will in particular be helpful to emphasize the convergence properties of the observer.
This formalism exists in the literature, yet not very largely used at the present time. Estay (2012) for example finds a Hamiltonian formulation, based on the ideal simple model, including the temperatures. It is important to emphasize that these considerations are made for the sole purpose of illustrating certain theoretical results. Another approach, but this time including enthalpy balances, was developed in Zitte et al. (2018). It is a port-Hamiltonian representation for a network of three heat exchangers in series. However, this representation remains as insufficient as in the case of Estay (2012), in that it does not include the transport phenomena.
We begin this paper with a review of the Port-Hamiltonian structure of PDE-governed systems. In Section III we develop a port-Hamiltonian representation of a counter-current heat exchanger, while opening the ports due to the phenomenon of convection. It is shown that the obtained geometric structure of Stokes-Dirac is conservative. In section IV we extend the results by Meghnous et al. (2013) of a port-Hamiltonian formalism of an SMO for systems with a lumped parameter model to distributed parameter systems. Section V presents the results of temperature estimation from the data taken at the laboratory.

2. PORT-HAMILTONIAN WITH INFINITE DIMENSIONAL SYSTEMS

In what follows we are interested in the class of 1D systems defined on $z \subset \Omega$ by the following partial differential equation (PDE):

$$\begin{cases} \frac{\partial x}{\partial t}(z,t) = (\mathcal{J}-\mathcal{R})\frac{\partial H}{\partial x} + \mathcal{G}u(z,t) \\ y(z,t) = \mathcal{G}^* \frac{\partial H}{\partial x} \end{cases} \quad (1)$$

where $x(z,t) \in L^p(\Omega, \mathbb{R}^n)$ is the state of the system, $\mathcal{J}$ is an skew-symmetric differential operator and $\mathcal{R}$ is a positive definite symmetric matrix ($\in \mathbb{R}^{m\times m}$, m representing the number of dissipative ports) that translates the dissipative aspect of the physical system. The input vector $\mathcal{G}$ is considered constant here and its adjoint denoted by $\mathcal{G}^*$ is therefore equivalent to $\mathcal{G}^\top$. The pair (u, y) denotes the respectively the distributed inputs-outputs variables.
The Hamiltonian of the system (energy function) is defined by:

$$H(z) = \frac{1}{2}\int_\Omega \mathcal{H} dz = \frac{1}{2}\int_\Omega x^\top Q x dz \quad (2)$$

where $Q \in \mathbb{R}^{n\times n}$ is a positive definite matrix, $\mathcal{H}$ is the energy density and $\frac{\partial H}{\partial x}$ is the variational derivative of H (Macchelli and Melchiorri (2004)). The systems thus defined are *Hamiltonian systems with ports* (Duindam et al. (2009), Jacob and Zwart (2012)) [1] . Such systems are non-dissipative if $\mathcal{R} = 0$, and dissipative in the opposite case.
Note that the dynamics of port Hamiltonian systems is provided by an interconnection structure associated with the Hamiltonian called the Stokes-Dirac structure. This structure is defined from power variable pairs, called port variables and a balanced power product. Given the linear spaces $\mathbb{F}$ and $\mathbb{E}$, whose elements are respectively the flow $f \in L^2(\Omega, \mathbb{R}^m)$ and the effort $e \in X^1(\Omega, \mathbb{R}^m)$ variables, the space of bond variables as the Hilbert space $\mathcal{B} = \mathbb{F} \times \mathbb{E}$. The balanced power product is given by:

$$\begin{array}{c} \langle \cdot|\cdot \rangle : \mathbb{F}\times\mathbb{E} \to \mathbb{R} \\ \forall (f,e) \to p = \langle e|f \rangle \end{array} \quad (3)$$

where $\langle \cdot|\cdot \rangle$ is the balanced power product. In close relation with this power product there exists a bilinear form defined by:

$$\ll (f', e'), (f, e) \gg := \langle e'|f \rangle + \langle e|f' \rangle \quad (4)$$

where (f', e') and (f, e) belong to $\mathcal{B}$.

Definition 1. (Duindam et al. (2009)) A Dirac structure on $\mathcal{B} := \mathbb{F}\times\mathbb{E}$ is a subspace $D \subset \mathcal{B}$, such that $D = D^\perp$, where $\perp$ denotes the orthogonal complement with respect to the bilinear form (4).

3. PORT HAMILTONIAN FORMULATION OF THE HEAT EXCHANGER

We consider a counter-curent heat exchanger configurable, as shown in Fig. 1, in which the temperatures of two fluids $T_1(z,t)$ and $T_2(z,t)$ are non-homogeneous, i.e. they depend on the space position over the entire length of the exchanger. It is assumed that the thermal exchange coefficients α_j ($j = 1, 2$ is an index which symbolizes each fluid), the specific masses ρ_j, the superficial fluid velocities v_j as well as the associated specific heat c_{p_j} of each fluid are constant. The system does not exchange heat with the external environment, and the fluids are incompressible and monophasic. Similarly, it is considered that the heat exchanger dynamics is characterized by a convection phenomenon.
Under these hypotheses we obtain the standard model of the system energy balances (Maidi et al. (2009), Burns and Kramer (2015), Burns and Cliff (2014), Aulisa et al. (2016)) described by two partial differential equations:

$$\begin{cases} \frac{\partial T_1(z,t)}{\partial t} = -v_1 \frac{\partial T_1(z,t)}{\partial z} + q_1(z,t) \\ \frac{\partial T_2(z,t)}{\partial t} = v_2 \frac{\partial T_2(z,t)}{\partial z} + q_2(z,t) \end{cases} \quad (5)$$

with $q_j = \alpha_j(T_k(z,t) - T_j(z,t))$ with $k \neq j$. We complete the model with Dirichlet boundary equations:: $T_1(0,t) = T_{10}(t)$ and $T_2(L,t) = T_{20}(t)$.

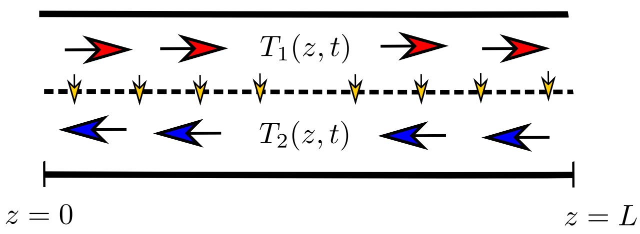

Fig. 1. A counter-current heat exchanger

The aim of this section is to find a port-Hamiltonian representation in a infinite dimension context. For that, as defined in Zitte et al. (2018) and Estay (2012) for heat exchangers, we choose the global entropy of the system as Hamiltonian:

$$ds = \sum_{j\to 1}^{2} \frac{1}{T_k} q_j = \alpha_1 \frac{1}{T_2} dT_1 - d\alpha_1 + \alpha_2 \frac{1}{T_1} dT_2 - d\alpha_2 \quad (6)$$

NB: For simplicity in the following, we omit the term (z,t) of the state variables.
For α_1 and α_2 constant the time derivative of ds along the system path (5) is written:

$$\begin{aligned} \frac{ds}{dt} &= \int_\Omega \left(\frac{\alpha_2}{T_2}\frac{\partial T_1}{\partial t} + \frac{\alpha_1}{T_1}\frac{\partial T_2}{\partial t} \right) dz \\ &= \int_\Omega \left(\frac{\alpha_2}{T_2}(-v_1 \partial_z T_1 + q_1) + \frac{\alpha_1}{T_1}(v_2 \partial_z T_2 + q_2) \right) dz \end{aligned} \quad (7)$$

We consider the flux vector $\mathcal{F} = (F_{T_1} \quad F_{T_2})^\top = (\partial_t T_1 \quad \partial_t T_2)^\top$ given by the derivatives to the state variables T_j, as well as the effort vector $\mathcal{E} = (\, E_{T_1} \; E_{T_2}\,)^\top = \begin{pmatrix} \frac{1}{T_2} & \frac{1}{T_1} \end{pmatrix}^\top$ given by the variational derivative of the Hamiltonian with respect to T_1 and T_2.
In the context of tubular reactors, in the presence of only the phenomena of *diffusion-reaction*, Zhou et al. (2017)

[1] In many papers, the variational derivative is denoted by δ, but it degenerates to the partial derivative basically denoted by ∂.

have shown that, in order to obtain a skew-symmetric structure which reflects the conservation of the total energy of the system, it is necessary to extend the variables of effort and flow by adding variables due to physical phenomena. Therefore, by integrating equation (7) by part to include the flow and effort variables due to the convection phenomenon, we simultaneously show the flow and effort variables at the system boundaries.
The new expressions of the flow and effort variables are given in Table 1.

Table 1. Pairing of flow-effort

Flux	Effort	Flow space	Effort space
$f_{T_1} = T_1$	$e_{T_1} = -\partial_z \frac{1}{T_2}$	$F^{\partial}_{T_1} = f_{T_1}\vert_{\Omega}$	$E^{\partial}_{T_1} = E_{T_1}\vert_{\Omega}$
$f_{T_2} = T_2$	$e_{T_2} = -\partial_z \frac{1}{T_1}$	$F^{\partial}_{T_2} = f_{T_2}\vert_{\Omega}$	$E^{\partial}_{T_2} = E_{T_2}\vert_{\Omega}$
$u_1 = -q_1$	$y_1 = \frac{1}{T_2}$		
$u_2 = -q_2$	$y_2 = \frac{1}{T_1}$		

Let $\mathbb{F}$ be the flow space (for example, the space of integrable squares functions $L^2(\Omega, \mathbb{R}^4)$) and $\mathbb{E}$, the effort space (in this case a Sobolev space $W^{1,2}(\Omega, \mathbb{R}^4)$).

Therefore, one extends the vectors of effort port $(\mathcal{E}, \mathcal{E}_\partial) \in \mathbb{E}$ and flux $(\mathcal{F}, \mathcal{F}_\partial) \in \mathbb{F}$ in the domain:

$$\begin{cases} \mathcal{E} = (\, E_{T_1} \;\; E_{T_2} \;\; f_{T_1} \;\; f_{T_2} \,)^\top \\ \mathcal{F} = (\, F_{T_1} \;\; F_{T_2} \;\; e_{T_1} \;\; e_{T_2} \,)^\top \end{cases} \quad (8)$$

with the following boundary variables:

$$\mathcal{E}_\partial = \begin{pmatrix} E^{\partial}_{T_1} \\ E^{\partial}_{T_2} \end{pmatrix}, \text{ and } \mathcal{F}_\partial = \begin{pmatrix} F^{\partial}_{T_1} \\ F^{\partial}_{T_2} \end{pmatrix} \quad (9)$$

Thus the integral energy balance (7) is written according to the flows and efforts such as:

$$\int_\Omega (E_{T_1} F_{T_1} + E_{T_2} F_{T_2})\, dz = -\int_\Omega (\alpha_2 v_1 e_{T_1} f_{T_1} - \alpha_1 v_2 e_{T_2} f_{T_2} + u_1 y_1 + u_2 y_2)\, dz - \left[\alpha_2 v_1 E^{\partial}_{T_1} F^{\partial}_{T_1} - \alpha_1 v_2 E^{\partial}_{T_2} F^{\partial}_{T_2}\right]_\Omega \quad (10)$$

The starting point for the definition of a port Hamiltonian system is the identification of an appropriate space of power variables related to the geometry of the system (Macchelli and Melchiorri (2004)). From the $\mathbb{E}$ and $\mathbb{F}$ port spaces, we define the link space (which is the cartesian product of the variable spaces) as follows:

$$\mathcal{B} := \{(\mathcal{E}, \mathcal{E}_\partial, \mathcal{F}, \mathcal{F}_\partial) \in \mathbb{E} \times \mathbb{F}\} \quad (11)$$

We provide this variable space with a bilinear product that corresponds to the thermal power:

$$\left\langle \begin{pmatrix} \mathcal{F} \\ u_j \\ \mathcal{F}_\partial \end{pmatrix}, \begin{pmatrix} \mathcal{E} \\ y_j \\ \mathcal{E}_\partial \end{pmatrix} \right\rangle : = \int_\Omega \left(\mathcal{E}^\top \mathcal{F} + \sum_{i=1}^{j} u_i y_i \right) dz + [\mathcal{E}_\partial^\top \mathcal{F}_\partial]_\Omega \quad (12)$$

Proposition 1. The linear sub-space $\mathcal{D} \subset \mathbb{F} \times \mathbb{E}$ defined by:

$$\mathcal{D} = \left\{ \left(\begin{pmatrix} \mathcal{F} \\ u \\ \mathcal{F}_\partial \end{pmatrix}, \begin{pmatrix} \mathcal{E} \\ y \\ \mathcal{E}_\partial \end{pmatrix} \right) \in \mathbb{F} \times \mathbb{E} \text{ such as} \right.$$

$$\begin{cases} \mathcal{F} = \mathcal{J}\mathcal{E} + (q_1 \;\; q_2 \;\; 0 \;\; 0)^\top \\ y = (1 \;\; 1 \;\; 0 \;\; 0)\, \mathcal{E} \end{cases} \quad (13)$$

where $\mathcal{J} = -\partial_z \begin{pmatrix} 0 & 0 & \alpha_2 v_1 & 0 \\ 0 & 0 & 0 & -\alpha_1 v_2 \\ -\alpha_2 v_1 & 0 & 0 & 0 \\ 0 & \alpha_1 v_2 & 0 & 0 \end{pmatrix}$, with the boundary conditions

$$\left. \begin{pmatrix} \mathcal{E}_\partial \\ \mathcal{F}_\partial \end{pmatrix} = \begin{pmatrix} \alpha_2 v_1 & 0 & 0 & 0 \\ 0 & -\alpha_1 v_2 & 0 & 0 \\ 0 & 0 & \alpha_2 v_1 & 0 \\ 0 & 0 & 0 & -\alpha_1 v_2 \end{pmatrix} \mathcal{E}|_\Omega \right\}$$

is a Stokes-Dirac structure with respect to the symmetric pairing (11) defined from the bilinear product (12).

Proof 1. For the sake of simplicity and without loss of generality, we normalize the convection velocities as well as the heat exchange coefficients at 1.
We first show that $\mathcal{D} \subset \mathcal{D}^\perp$.
To do this we consider two pairs of flow and effort variables belonging to $\mathcal{D}$: $(\mathcal{E}, \mathcal{F})$ and $(\mathcal{E}', \mathcal{F}')$, whose bilinear product is:

$$\langle \mathcal{E}' | \mathcal{F} \rangle + \langle \mathcal{E} | \mathcal{F}' \rangle = \int_\Omega (\mathcal{E}'^\top \mathcal{F} + \mathcal{E}^\top \mathcal{F}' + u_1 y_1' + u_2 y_2' + u_1' y_1 + u_2' y_2)\, dz + \mathcal{E}'^\top_\partial \mathcal{F}_\partial + \mathcal{E}^\top_\partial \mathcal{F}'_\partial \quad (14)$$

Let us use the definitions of the flow and effort vectors given in (8) and (9). By replacing the flows by their corresponding laws, and integrating by part, we obtain:

$$\langle \mathcal{E}' | \mathcal{F} \rangle + \langle \mathcal{E} | \mathcal{F}' \rangle = 0 \quad (15)$$

which shows that $\mathcal{D} \subset \mathcal{D}^\perp$.
The second step of the proof is to show that $\mathcal{D}^\perp \subset \mathcal{D}$. For this we introduce the assumption that a pair of effort and flow variable $(\mathcal{E}', \mathcal{F}') \in \mathcal{D}$.
The Stokes-Dirac structure as defined in (13) shows that there is no constraint on the choice of the effort variables. First let us take some functions that cancel each other out at the domain's boundary.
First of all let us calculate $\langle \mathcal{E}' | \mathcal{F} \rangle$:

$$\langle \mathcal{E}' | \mathcal{F} \rangle = \int_\Omega (E'_{T_1} F_{T_1} + E'_{T_2} F_{T_2} + f'_{T_1} e_{T_1} + f'_{T_2} e_{T_2}) dz \quad (16)$$

By replacing $\mathcal{F}$ by the laws defined in the structure of Stokes-Dirac (13) one obtains the following expression:

$$\langle \mathcal{E}' | \mathcal{F} \rangle = \int_\Omega (E'_{h_1}(-\partial_z f_{h_1}) + E'_{h_2}(-\partial_z f_{h_2})) + f'_{h_1}(-\partial_z e_{h_1}) + f'_{h_2}(-\partial_z e_{h_2})) dz \quad (17)$$

By partially integrating this equation and considering the boundaries variables, this equation becomes:

$$\langle \mathcal{E}' | \mathcal{F} \rangle = \int_\Omega (f_{T_1} \partial_z E'_{T_1} + f_{T_2} \partial_z E'_{T_2} + E_{T_1} \partial_z f'_{T_1} + E_{T_2} \partial_z f'_{T_2}) dz \quad (18)$$

$\langle \mathcal{E} | \mathcal{F}' \rangle$ can be deduced from the relation (15) which must be verified, ie $\langle \mathcal{E} | \mathcal{F}' \rangle = -\langle \mathcal{E}' | \mathcal{F} \rangle$. Similarly we also have:

$$\langle \mathcal{E} | \mathcal{F}' \rangle = \int_\Omega (E_{T_1} F_{T_1'} + E_{T_2} F'_{T_2} + f_{T_1} e'_{T_1} + f_{T_2} e'_{T_2}) dz \quad (19)$$

It is deduced by identification that the flow vector $\mathcal{F}'$ must satisfy:

$$\begin{cases} F'_{T_1} = -\partial_z f'_{T_1} \\ F'_{T_2} = -\partial_z f'_{T_2} \end{cases} \text{ and } \begin{cases} e'_{T_1} = -\partial_z E'_{T_1} \\ e'_{T_2} = -\partial_z E'_{T_2} \end{cases} \quad (20)$$

which corresponds to the structure defined by Proposition 1.

Now take the case where boundary variables are different from zero. For that, it is enough to replace the relations of the flow vector $\mathcal{F}'$ obtain in (20) in the symmetric product (15). So we have:

$$\begin{aligned} \langle \mathcal{E}'|\mathcal{F}\rangle + \langle \mathcal{E}|\mathcal{F}'\rangle = & \int_\Omega (E'_{T_1}(-\partial_z f_{T_1}) + E'_{T_2}(-\partial_z f_{T_2}) \\ & + f'_{T_1}(-\partial_z e_{T_1}) + f'_{T_2}(-\partial_z e_{T_2}))dz \\ & + \int_\Omega (E_{T_1}(-\partial_z f'_{T_1}) + E_{T_2}(-\partial_z f'_{T_2}) \\ & + f_{T_1}(-\partial_z e'_{T_1}) + f_{T_2}(-\partial_z e'_{T_2}))dz \\ & + \mathcal{E}'^\top_\partial \mathcal{F}_\partial + \mathcal{E}^\top_\partial \mathcal{F}'_\partial \end{aligned} \quad (21)$$

After a step of integration by part one notes that:

$$\begin{aligned} \langle \mathcal{E}'|\mathcal{F}\rangle + \langle \mathcal{E}|\mathcal{F}'\rangle = & -[\mathcal{E}'^\top_\partial \mathcal{F}_\partial + \mathcal{E}^\top_\partial \mathcal{F}'_\partial] + [\mathcal{E}'^\top_\partial \mathcal{F}_\partial + \mathcal{E}^\top_\partial \mathcal{F}'_\partial] \\ = & \ 0 \end{aligned} \quad (22)$$

We also conclude that $\mathcal{D}^\perp \subset \mathcal{D}$.
So $\mathcal{D}$ is a Stokes-Dirac structure. ■

The structure of Stokes-Dirac (13) is written in compact form as:

$$\mathcal{F} = (\mathcal{J} - \mathcal{R})\mathcal{E} \quad (23)$$

with $\mathcal{R} = \begin{pmatrix} 0 & 0 & -\alpha_1 & \alpha_2 \\ 0 & 0 & \alpha_2 & -\alpha_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

4. SLIDING MODE OBSERVER FOR PORT-HAMILTONIAN SYSTEMS

In this section, we focus on the design of a sliding-mode observer based on the Hamiltonian representation given in the previous section [2].

4.1 Main idea

The synthesis of this observer is based on the theory of systems with variable structures introduced by Fillipov in the 60's and Utkin in the late 70's (Utkin (1993)). It consists in constraining the dynamics of the observation errors by using discontinuous functions so that they converge to a sliding surface. The specificity of this observer is that the correction term is a discontinuous function *sign* defined by:

$$sign(x) = \begin{cases} 1, & \text{if } x > 0 \\ -1, & \text{if } x < 0 \end{cases} \quad (24)$$

Before giving the equations of the SMO, we introduce some concepts on the observability of distributed parameter systems.

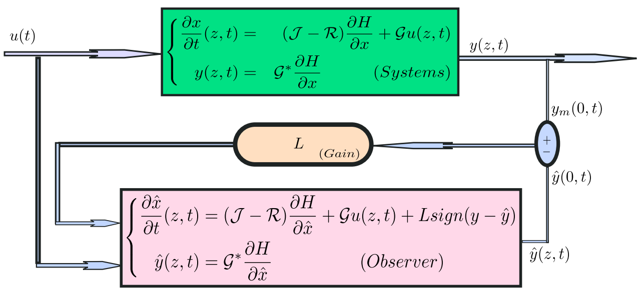

Fig. 2. SMO for distributed port-Hamiltonian systems

Let us consider a dynamical system

$$\frac{dx}{dt} = Ax(t) + Bu(t)$$

with the unbounded $A : D(A) \subset X \to X$ and the bounded operator $B : \mathbb{R}^2 \to X$, where X is an Hilbert space. Let us consider an output function $y(\cdot)$ defined as follows:

$$y(t) = (Cx)(t), \qquad t \geq 0$$

where $C : X \to \mathbb{R}^2$ is a bounded linear operator.
Let us denote the unobservable subspace by $\mathrm{NO}(C, A)$, and let G be an invariant subspace of X such that $0 \in G$.

Definition 2. (Curtain and Zwart (1995)(Chapter 4)) The pair (C, A) is observable if $\mathrm{NO}(C, A) = 0$.

Suppose that the system (1) is accessible to the measurement. The dynamical system defined by

$$\begin{cases} \dfrac{\partial \hat{x}}{\partial t}(z,t) = (\mathcal{J} - \mathcal{R})\dfrac{\partial H}{\partial \hat{x}} + \mathcal{G}u(z,t) \\ \qquad + Lsign(y(0,t) - \hat{y}(0,t)) \\ \hat{y}(z,t) = \mathcal{G}^* \dfrac{\partial H}{\partial \hat{x}} \end{cases} \quad (25)$$

is an observer for the system (1). L represents the gain, y and $\hat{y}$ the measured and estimated output, respectively.

4.2 Error dynamics

The objective is to find an observation gain L such that the estimated error whose dynamics is in the form of a port-Hamiltonian system, tends to zero. The observation error e is given by the difference between the real state and its estimated value $e = x - \hat{x}$. Its dynamic is given by:

$$\frac{\partial e}{\partial t} = (\mathcal{J} - \mathcal{R})\left(\frac{\partial H}{\partial x} - \frac{\partial H}{\partial \hat{x}}\right) - Lsign(y(0,t) - \hat{y}(0,t)) \quad (26)$$

From equation (1) it is difficult to express the dynamics of the error according to the state x. To circumvent this difficulty, we consider the following Hamiltonian function:

$$H(z) = \frac{1}{2}x^\top P x \quad (27)$$

with P a symmetric full rank matrix.
To simplify the reasoning and the demonstration of the convergence of the observation error dynamics, we assume that the output y of the system (1) is scalar; that is to say that the temperature of each fluid is measured at a single point. Therefore, starting from equation (27), and considering the state transformation $\Psi = Px$, the observer equations can be rewritten as follows:

$$\frac{\partial \hat{\psi}}{\partial t} = P(\mathcal{J} - \mathcal{R})\hat{\psi} + P\mathcal{G}u(z,t) + PLsign(y(0,t) - \hat{y}(0,t)) \quad (28)$$

[2] In the literature, a similar representation exists (Meghnous et al. (2013)), but for lumped parameter systems. As far as we are concerned, these results are extended in the case of systems governed by PDEs.

where $\hat{\psi}$ is the new estimated state variable.

Proposition 2. Consider the observer given by equation (25), with the product of L and the *sign* function of observation error $y - \hat{y} = \mathcal{G}^*\tilde{\psi}$ which represents the correction term. If $Lsign(y-\hat{y}) = K\mathcal{G}^*\tilde{\psi}$ such that $(K\mathcal{G}^* + \mathcal{R})^\top + (K\mathcal{G}^* + \mathcal{R})$ is positive, then so the observation error $\tilde{\psi} = \psi - \hat{\psi}$ converges asymptotically to zero.

Proof 2. From Equation (28), define the dynamics of the observation error $\tilde{\psi} = \psi - \hat{\psi}$

$$\frac{\partial \tilde{\psi}}{\partial t} = P(\mathcal{J} - \mathcal{R})\tilde{\psi} - PLsign(y - \hat{y}) \tag{29}$$

Let $V = \tilde{\psi}^\top P^{-1}\tilde{\psi}$ the candidate Lyapunov function for the observer convergence study. From (29) the derivative of V with respect to time is written as:

$$\begin{aligned}\frac{\partial V}{\partial t} &= \tilde{\psi}^\top(\mathcal{J} - \mathcal{R})^\top\tilde{\psi} - L^\top sign(y - \hat{y})\tilde{\psi} \\ &\quad + \tilde{\psi}^\top(\mathcal{J} - \mathcal{R})\tilde{\psi} - \tilde{\psi}^\top Lsign(y - \hat{y})\end{aligned} \tag{30}$$

Given the fact that the operator $\mathcal{J} \in W^{1,p}(\Omega, \mathbb{R}) \subseteq L^p(\Omega, \mathbb{R})$ is skew-symmetric, $\tilde{\psi}^\top\mathcal{J}\tilde{\psi} = 0$. So the equation above becomes:

$$\frac{\partial V}{\partial t} = -\tilde{\psi}^\top(\mathcal{R}^\top + \mathcal{R})\tilde{\psi} - L^\top sign(y-\hat{y})\tilde{\psi} - \tilde{\psi}^\top Lsign(y-\hat{y}) \tag{31}$$

To guarantee the stability of the dynamics of the observation error, $\frac{\partial V}{\partial t} < 0$ must be used. For that, it is enough to choose the correction term such as:

$$Lsign(y - \hat{y}) = L\mathcal{G}^* sign(\psi - \hat{\psi}) \tag{32}$$

The function *sign* is a correction term, $s(\tilde{\psi}) = \psi - \hat{\psi}$ represents the sliding surface. To guarantee the convergence of the dynamics of the error it suffices that $s(\tilde{\psi}) = 0$; which means choosing $Lsign(y - \hat{y})$ such as:

$$Lsign(y - \hat{y}) = K\mathcal{G}^*\tilde{\psi} \tag{33}$$

where K is a gain.
So (31) becomes:

$$\frac{\partial V}{\partial t} = -\tilde{\psi}^\top\left[(K\mathcal{G}^* + \mathcal{R})^\top + (K\mathcal{G}^* + \mathcal{R})\right]\tilde{\psi} \tag{34}$$

If $(K\mathcal{G}^* + \mathcal{R})^\top + (K\mathcal{G}^* + \mathcal{R}) > 0$, so $\frac{\partial V}{\partial t} < 0$. This proves the convergence of the observation error. ■

Remark 1. If the operator $(K\mathcal{G}^* + \mathcal{R})^\top + (K\mathcal{G}^* + \mathcal{R}) > 0$ is non-positive, the asymptotic stability of the observation error can be studied using other theories such as the Lasalle's invariance principle (Luo et al. (1999)(Chapter V)).

5. APPLICATION

5.1 SMO for heat exchanger

It is assumed that the temperatures at the boundaries $T_1(L,t)$ and $T_2(0,t)$ of the heat exchanger are accessible for to the measurement. According to the system (5), the SMO is written as follows:

$$\begin{cases} \dfrac{\partial \hat{T}_1(z,t)}{\partial t} = -v_1\dfrac{\partial \hat{T}_1(z,t)}{\partial z} + \hat{q}_1(z,t) + \hat{\sigma}_1 \\ \dfrac{\partial \hat{T}_2(z,t)}{\partial t} = v_2\dfrac{\partial \hat{T}_2(z,t)}{\partial z} + \hat{q}_2(z,t) + \hat{\sigma}_2 \end{cases} \tag{35}$$

where $\hat{\sigma}_1 = Lsign(T_1(L,t) - \hat{T}_1(L,t))$ and
$\hat{\sigma}_2 = Lsign(T_2(0,t) - \hat{T}_2(0,t))$, with boundary conditions similar to those of the exchanger.

We have previously shown that the port-Hamiltonian model of the heat exchanger is a conservative structure. Therefore it can be argued that the port-Hamiltonian model of the SMO:

$$\hat{\mathcal{F}} = (\mathcal{J} - R)\hat{\mathcal{E}} + Lsign(y - \hat{y}) \tag{36}$$

is also a conservative structure ($\hat{\mathcal{F}}$ and $\hat{\mathcal{E}}$ are the same size and type as $\mathcal{F}$ and $\mathcal{E}$ respectively). We have shown in subsection 5.2 that the correction term $Lsign(y - \hat{y})$ was a function of the measured and estimated state; we can therefore conclude that it belongs to the dissipative part of the Dirac structure.

From equation (36) it is possible to use the same line of reasoning than that of section 5, and to show that for any pair of flow $\hat{\mathcal{F}}$ and effort $\hat{\mathcal{E}}$ variable vectors, the dynamics of the observation error tends to zero.

5.2 Implementation with laboratory data

We test the observer on data collected on a tubular heat exchanger of the control laboratory of the Polytechnic Faculty of the University of Lubumbashi, whose description is in Kazaku et al. (Jul 2018).

Here the boundary conditions of the observer are taken differently than that of the exchanger, so as to visualize the convergence; either: $\hat{T}_1(L,t) = \hat{T}_2(0,t) = 10°C$. The initial conditions of the observer are profiles determined by canceling the time derivatives of the equation system (35). The gain of the observer L is taken equal to 200. The convection speeds as well as the thermal exchange coefficients are taken as giving in Maidi et al. (2009). The observer is implemented by considering a finite difference approximation.

The sliding mode observer results for the 1 and 2 fluids are shown in Fig. 3.

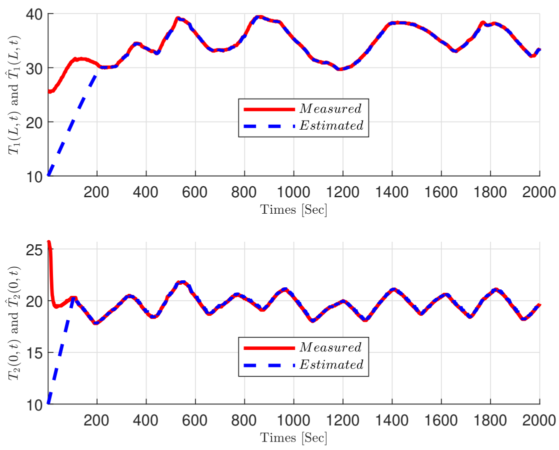

Fig. 3. Temperature profiles measured and estimated

We note that the dynamics of the system are well estimated despite the initialization and uncertainties of the observer parameters. To accelerate the convergence, one can increase the value of the gain. Fig. 4 shows the dynamics of the observation error system for different values of L.

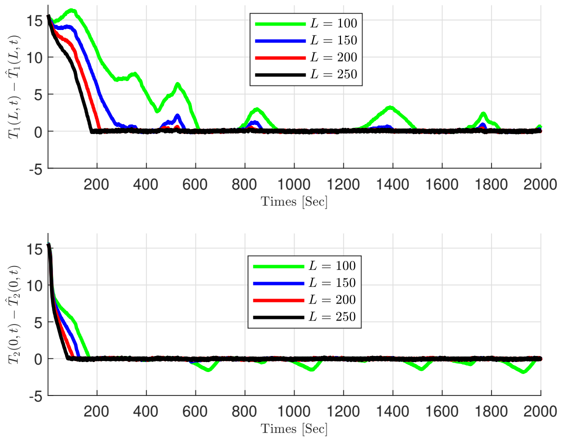

Fig. 4. Estimation error profile

In particular, faster convergence can be obtained by increasing the value of the gain L. Note that a detrimental effect might be the negative effect of the disturbances (like measurement noise) for high values of L when oscillations can be induced in the state estimation. It is then necessary to find a good compromise in the choice of the gain.

6. CONCLUSION

In this paper we have considered the problem of estimating the temperatures of a tubular heat exchanger in a context where the complete state of the system is inaccessible to the measurement. First, a Hamiltonian standard-port formulation (Burns and Cliff (2014), Maidi et al. (2009)) of counter-current heat exchangers is proposed. It has been shown that the Stokes-Dirac structure obtained from an extension of the set of variables conjugated to the power, is conservative. Second, a sliding mode observer based on the interconnected structure of Hamiltonian port systems was synthesized. We analyzed the convergence properties of the observation error using its structural properties. The numerical results based on experimental data illustrate the relevance of such an observation approach for this type of process.

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