Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

An aero-engine U-control method based on LPV model

Jiajie Chen*. Jiqiang Wang*,
Zhongzhi Hu*, Georgi Marko Dimirovski †**

** Nanjing University of Aeronautics and Astronautics, Nanjing, Jiangsu, 210016, China.*
*** Dogus University, Faculty of Engineering, Acibadem, Istanbul 34722, Turkey*
† SS Cyril and Methodius University, School FEIT, Skopje 1000, N. Macedonia

Abstract: Due to the harsh working environment and various constraints, the engine control unit (ECU) has very limited computational ability and few control methods can be applied to the real-word ECU maturely. Developing advanced control methods with good performance as well as low computational complexity is the research focus in the control area for the next generation aero-engines. The study reported in this paper combines the LPV model of aero-engine process with U-model control concept, which simplifies solving difficulty and avoids complexity of traditional LPV variable-gain controller. The results of Matlab-Simulink simulations demonstrate clearly this U-control method based on LPV model can be successfully used for designing quality aero-engine speed control system. It ensures a good control performance while guaranteeing stable operation in the proximity of chosen equilibrium steady-state thus demonstrating a considerable application potential.

Keywords: Aero-engine; Dynamic inversion; LPV model; Nonlinear process dynamics; U-control.

1. INTRODUCTION

An operating aero-engine is known to imply a certain specific complex dynamical process which could be only described by a complex time-varying nonlinear thermodynamic system, whose dynamic performance varies with the change of engine thrust and flight conditions. Therefore it is very difficult directly to derive and develop adequate but suitable nonlinear control laws for such nonlinear systems. Previous studies of these authors include control design studies on finite-time regulation (Wang et al, 2016), Hammerstein-Wiener model based control (Wang et al, 2017), and Switched-LPV model based control (Zhu et al, 2018).

On the other hand, the engine control unit (ECU) has finite computational ability with around 20ms a calculation period (control algorithms cost ≤ 5ms) (1/ SAE International Group, 2012) due to the demanding working environment. Up to now, only PID and LQG control methods could be applied to the real-world ECU maturely; many advanced control algorithms are limited by the feasible ECU computing power. It is very important therefore to explore and find advanced control method with low computational complexity but considerably improved performance.

Because of this, many works are focused on linear control methods. The general linear control method is based on the linearized model at a certain steady-state operating point of the engine to design the controller. However, when the dynamic characteristics change greatly, it is difficult for a single controller to meet the control performance requirements of the engine under the condition of large-scale variation. To solve this problem, the most common solution is the variable gain controller. Traditional method designs linear controllers at a series of steady-state operating conditions respectively, and then switches controllers in a designed switching law to meet the system dynamic characteristics of a wide range. But for traditional variable gain controller, quantity of linear controllers is excessive and the switching law is difficult to guarantee the system stability and robustness. These years, more and more attention has been paid to the variable gain controller design method based on LPV model. This control method directly designs the controller according to the scheduling parameters in the LPV model without interpolation and switching between controllers. The widely used controller design method based on LPV model relies on Lyapunov function to solve linear matrix inequality (LMI) on the whole control envelope parameter trajectory, which has high complexity and difficulty in solving.

In this paper, combined with U-control theory, a new controller design method based on LPV model is proposed. U-model, which is a polynomial structure that comprises of time-varying system parameters, was originally developed by Zhu (13/ Zhu and al, 2002) in 2002. U-model can represent a wide range of nonlinear systems. After more than ten years of development, the nonlinear control method based on U-model has been applied to predictive control (4/ Du and al, 2014), internal model control (8/ Shafiq and al, 2005), adaptive control (12/ Wu and al, 2011), (6/ Hasan and al, 2017) and so on. Besides, the dedicated 1st U-control symposium was successfully held in Wuhan, Hubei, China in 2019 (e.g. see 5/).

Most recent advances into the theory and applications of the principle of U-model based control can be found in Geng et al (2019) on Smith-predictor control of time-delay nonlinear processes, in Zhu et al (2019) on U-neural network-enhanced control, and in Zhu et al (2018) on switched LPV systems H_{inf} tracking control of aero-engine. However, to the best of our awareness, up to now the U-control concept has not been directly extended to the area of aero-engine process control as yet.

In this research, a LPV model is established to represent the aero-engine nonlinear process and then, based on the U-control theory, the dynamic inversion of the LPV model can be transformed into a U-model root solver. In addition, a fixed linear controller can be designed by linear control theory in series to realize the nonlinear close-loop control. The whole LPV-U controller is composed of the fixed linear controller and the U-model root solver representing the dynamic inverse of the LPV model. The simulation results show that this LPV-U controller has been successfully applied to the dynamic control of engine low pressure shaft speed from idle state to maximum state under the ground condition. This U-control method based on LPV model simplifies the solving difficulty and complexity of the variable gain controller.

The structure of this article is organized as follows. Section 2 introduces the concept and idea of U-control theory. Section 3 introduces the principle of aero-engine U-control method based on a LPV model. Section 4 presents the simulation results obtained in MATLAB/Simulink computing platform. Section 5 summarizes this new method and makes a look forward to future outlook.

2. THEORY OF U-CONTROL CONCEPT

U-control method is a method of linear controller design for nonlinear objects and plant processes based on the U-model structure. The U-model structure can be seen as an extension of NARMAX model. Consider a single input and single output (SISO) discrete-time causal polynomial U-model given with a triplet of

$$\left(y(k),\lambda(k),u(k-1)\right):y(k)=\sum_{j=0}^{J}\lambda_j\left(Y_{k-1},U_{k-2},\Theta\right)u^j\left(k-1\right),$$

where $y(k)\in\mathbb{R}$ and $u(k-1)\in\mathbb{R}$ are the output and input, respectively, at the sampling time instance $k\in\mathbb{Z}^+$. Quantity $\lambda_j(k)\in\mathbb{R}$ is a time varying parameter absorbing all the other remaining delayed inputs and outputs

$$(Y_{k-1}=\left[y\left(k-1\right),\ldots,y\left(k-y_m\right)\right]\in\mathbb{R}^{y_m},$$

$$U_{k-1}=\left[u\left(k-2\right),\ldots,u\left(k-u_m\right)\right]\in\mathbb{R}^{u_m}),$$

and the coefficients Θ associated with the input $u^j\left(k-1\right)$, j being the degree of $u\left(k-1\right)$.

The inversion is to obtain the input $u(k)$ from for a given output $\overset{(M)}{y}(k)$ which is solved by means of the equation

$$\overset{(N)}{u}(k-1)\in\overset{(M)}{y}(k)-\sum_{j=0}^{J}\lambda_j\left(Y_{k-1},U_{k-1},\Theta\right)\left(\overset{(N)}{u}(k-1)\right)^j=0. \quad (1)$$

Based on the U-model structure, the control parameter can be directly calculated in the root solver (Matlab), which serves as the dynamic inversion of the controlled nonlinear system. The dynamic inversion based on U-model is connected in series to the linear controller within the feedback (FB) loop as generic part of the controller. Thus, only one more design of linear invariant controller is needed to realize the controller design of the nonlinear system.

The closed-loop system structure of the U-control is depicted in Figure 1 (of course, it is based on negative feedback principle) below. In there, there clearly shown the following components:

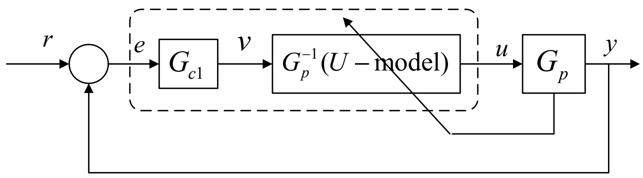


Fig. 1 Overall system architecture of U-control strategy

Model G_{c1} denotes a linear invariant controller, and G_P^{-1} is the dynamic inverse of the plant G_p . Components $G_{c1}=\left.\frac{G}{1-G}\right|_{G_p=1}$ and G_p^{-1} are designed separately, and then interconnected into the proposed new controller $G_c=G_{c1}G_p^{-1}$.

3. DESIGN OF AEROENGINE U-CONTROL LAW BASED ON LPV MODEL

This study paper investigates on a Geared Turbofan (GTF) engine, as shown in the Figure 2. GTF engine represents the next generation of high-efficiency engines (Chapman and al, 2017). In the traditional turbofan engine, the fan is directly driven by the low pressure shaft, so the fan LPC and LPT cannot work in their own best rotational speeds at the same time.

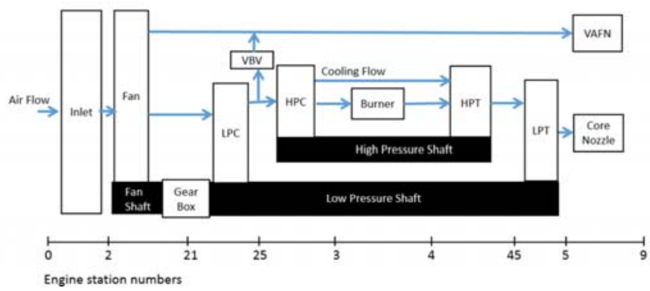


Fig. 2 The structure of GTF engine

By using a gear box, the GTF engine solves this problem of potential speed contradiction. The fan can work in ideal low speed and the LPT can keep high speed rotating, which reduces the engine noise and fuel consumption. Besides, there is available a starting background model (Chapman et al, 2014) and background guidance to U-control design due to Q. M. Zhu et al (2016, 2019). Therefore, carrying out research on designing the control system for this advanced aero-engine is extremely necessary and timely task.

The engine model used in this research is AGTF30 non-linear component-level model provided by the Toolbox for the Modeling and Analysis of Thermodynamic Systems, T-MATS (Chapman et al, 2014). The LPV model of GTF engine is established based on this model off-line in classical Jacobian linearization (Reberga and al, 2005). Mathematical expression of the engine non-linear model, naturally, is:

$$\dot{x} = f(x,u), \qquad y = g(x,u) \tag{2}$$

where $f(\cdot), g(\cdot)$ are assumed continuously differentiable. On the grounds of the physical considerations an equilibrium steady-state point $(x_0, u_0) \in \{(x,u) \mid f(x,u)=0\}$ is being selected. Thereafter, Taylor expansion is being applied by ignoring the terms of the second order and above, and thus obtain:

$$\begin{aligned} f(x,u) &\approx f(x_0,u_0) + \frac{\partial f}{\partial x}\Big|_{(x_0,u_0)} \Delta x + \frac{\partial f}{\partial u}\Big|_{(x_0,u_0)} \Delta u \\ g(x,u) &\approx g(x_0,u_0) + \frac{\partial g}{\partial x}\Big|_{(x_0,u_0)} \Delta x + \frac{\partial g}{\partial u}\Big|_{(x_0,u_0)} \Delta u \end{aligned} \tag{3}$$

Namely, the linearized model which can describe the dynamic process at neighbourhood around of the equilibrium steady-state is as follows

$$\Delta\dot{x} = A\Delta x + B\Delta u\,, \ \Delta y = Cx + D\Delta u. \tag{4}$$

To study the dynamic performance of engine low pressure shaft speed from idle state to maximum state under the ground condition. The input *u* is the fuel flow, and the output *y* represents high and low pressure shaft rotational speeds. In order to avoid the ill-condition of the system matrix, the input and output parameters are normalized. The normalized linear models of 14 steady-state points are taken to establish the discrete LPV model. The normalized high pressure rotational speed was selected as the scheduling parameter, and the simulation step length is 0.02. The LPV model is

$$\begin{aligned} \Delta\dot{X}(k) &= A(\theta(k))\Delta X(k) + B(\theta(k))\Delta u(k) \\ \Delta Y(k) &= \Delta X(k) \end{aligned} \tag{5}$$

In here symbols denote: $\Delta X \in R^n$ is the state vector $\begin{bmatrix}\Delta PN_L & \Delta PN_H\end{bmatrix}^T$; $\Delta Y \in R^n$ is the output vector $\begin{bmatrix}\Delta PN_L & \Delta PN_H\end{bmatrix}^T$; $\Delta u \in R$ is the control fuel flow ΔPW_f ; $\theta \in R$ is the scheduling parameter and normalized high pressure rotational speed PN_H .

After each simulation step *T*, the updating formula of real rotational speed from *k* to *k+1* is as follows:

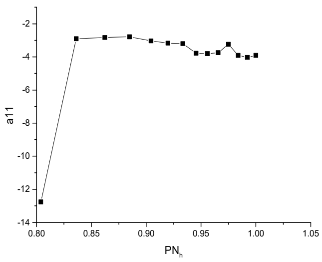
$$Y(k+1) = X(k+1) = X(k) + \Delta\dot{X}(k)\cdot T \tag{6}$$

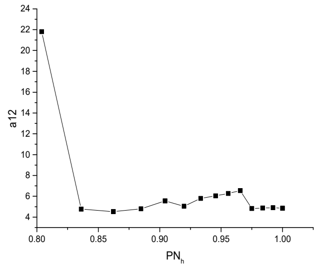
In this study, for the SISO system from fuel flow to low pressure rotational speed, the variation relationship between low pressure rotational speed and the fuel flow in each simulation step is as follows

$$\begin{aligned} \Delta P\dot{N}_L(k) &= a_{11}(\theta(k))\Delta PN_L(k) + a_{12}(\theta(k))\Delta PN_H(k) \\ &\quad + b_1(\theta(k))\Delta PW_f(k) \end{aligned}$$

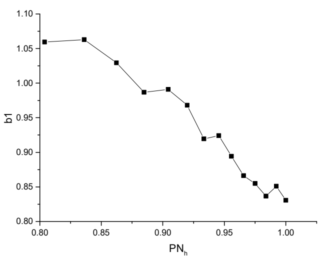
(7)

The aero-engine envelope is studied under the ground state from the idle state to the maximum state. In this aero-engine LPV model, linear interpolation scheduling method is adopted. The relationship between matrix coefficients and scheduling parameters is shown as follows:





(a) $a_{11}(\theta)$ (b) $a_{12}(\theta)$



(c) $b1(\theta)$

Fig. 3 The relationship between matrix coefficients and scheduling parameters

Also convert LPV model into the 1-order U-model structure, and there is

$$\Delta P\dot{N}_L(k) = \lambda_1 \Delta PW_f(k) + \lambda_0 \tag{8}$$

where symbols denotes:

$\lambda_1 = b_1(\theta(k))$;

$\lambda_0 = a_{11}(\theta(k))\Delta PN_L(k) + a_{12}(\theta(k))\Delta PN_H(k)$. λ_1 and λ_2 are time-varying coefficients.

Thus, the dynamic inversion of LPV model can be expressed as

$$\Delta PW_f(k) = \left(\Delta P\dot{N}_L(k) - \lambda_0\right) / \lambda_1 \tag{9}$$

Therefore, the real control parameter is

$$\begin{aligned} W_f(k) &= W_{fst}(\theta(k)) + \Delta W_f(k) \\ &= W_{fst}(\theta(k)) + \Delta P W_f(k) \cdot W_{fdesign} \end{aligned} \quad (10)$$

In this equation, there: $W_{fst}(\theta(k))$ is the fuel flow baseline corresponding to scheduling parameter θ. $W_{fdesign}$ is the fuel flow design value which is used for normalizing fuel flow values.

The aero-engine U-control structure based on LPV model is shown in the Figure 4. The linear controller can be simply designed as:

$$v(k) = K(a_1 e(k) + a_2 e(k-1)) \quad (11)$$

Combined with (9), the LPV-U controller structure is:

$$u(k) = W_{fst}(\theta(k)) + \frac{K(a_1 e(k) + a_2 e(k-1)) - \lambda_0}{\lambda_1} \cdot W_{fdesign} \quad (12)$$

In the above analysis, time-varying coefficients in U-model and LPV-model are updated synchronously. The principle of LPV-U controller is quite simplified because it employs only linear equations.

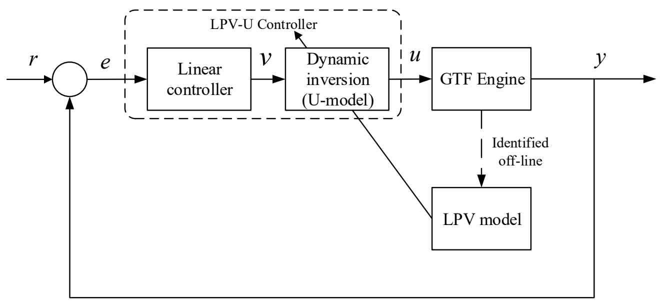


Fig. 4 The developed U-control system structure based on LPV model for aero-engine

In order to confirm the stability of this LPV-U control system, we consider the model "mismatch" error. A more realistic model the plant can be described as

$$G_p' = G_p + \Delta \quad (13)$$

where $G_p^{-1} G_p' \neq 1$. Next, defining the uncertainty of the model as follows:

$$E = \frac{\Delta}{G_p} \quad (14)$$

The disturbance occurring in the flight environment is impossible to be much too large, hence $|E| < 1$ is undisputed. Therefore, the following conditions are satisfied:

a/. $G = \dfrac{G_{cl}}{1 + G_{cl}}$ is stable

b/. $|G(j\omega) \cdot E(j\omega)| < 1$

Therefore, on the grounds of the celebrated *small gain theorem*, the U-controller close loop system can be keep stable in the proximity of the operating steady-state equilibrium.

4. RESULTS OF SIMULATION INVESTIGATIONS

To demonstrate further the controlling capabilities and the stability of this LPV-U controller in closed, a large envelope based dynamic simulations under two flight conditions was carried out on the *Matlab/Simulink* platform. The dynamic performance of engine low pressure shaft speed from idle under the ground condition state to maximum state under cruise operation is investigated. The change curve of fuel flow calculated by U-controller is shown in the figure. The change curve of low pressure rotational speed N_L of GTF engine component-level model and its relative error with LPV model are shown in below.

4.1 Ground condition (Ma=0, H=0)

The dynamic performance of engine low pressure shaft speed from idle state to maximum state under the ground condition is showed in fig.4. The change curve of fuel flow calculated by LPV-U controller is shown in the Figure 5.

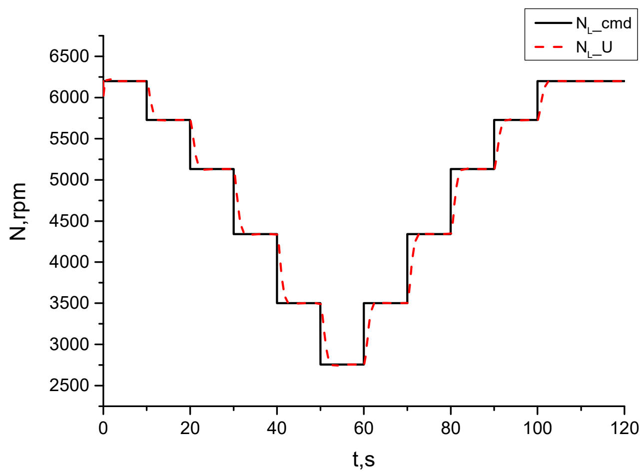


Fig. 4 The of N_L dynamic performance under the ground condition

Figure 4 shows that the dynamic performance of N_L has no steady-state error. Furthermore, the overshoot is less than 1% and the rising time is less than 5s, both of which can satisfy the control requirement of aero-engine speed control system. In addition, Figure 5 shows that the change curve of fuel flow, computed by the LPV-U controller, is indeed smooth and remains in accordance with the change rule of actual fuel system installed.

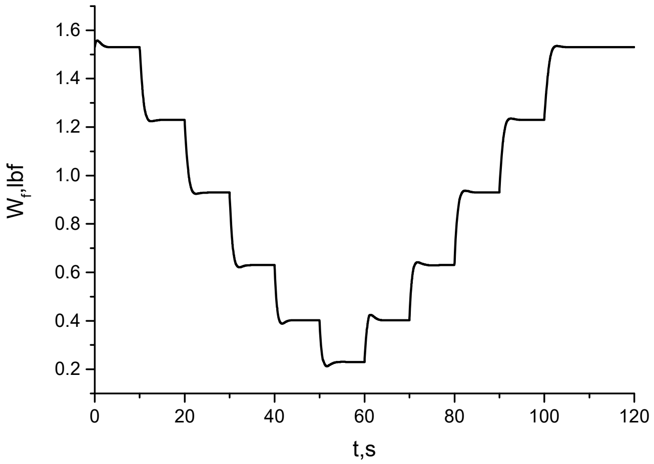


Fig. 5 The change curve of fuel flow under the ground condition

4.2 Cruise condition (Ma=0.8, H=35000ft)

In the cruise condition, the aero-engine process is simulated from normal cruise point (N_L = 6777rpm) → lower cruise point (N_L = 6551rpm) → normal cruise point → upper cruise point (N_L = 6915rpm) → normal cruise point transient state process. The command speed changing curve and LPV-U controller transient state the control effect is seen in Figure 6. The respective fuel flow changing curve is shown in Figure 7. Simulation results show that the steady-state error is zero, 0, which is likely to be approximately so in practice. Thus the algorithm of this U-controller can be preliminary verified as effective and viable.

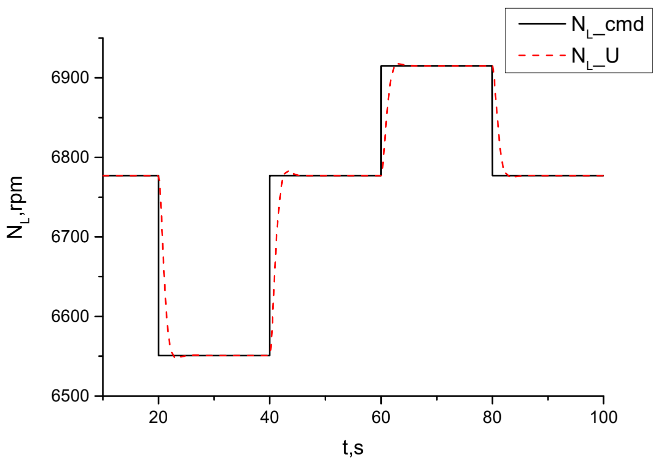


Fig. 6 Dynamic performance of N_L in the cruise condition

It can be seen from Figure 7 that the LPV model has more than 99% accuracy to describe the nonlinear aero-engine process. Figure 6 shows that the dynamic performance of N_L has no steady-state error, the overshoot is less than 1% and the rising time is less than 5s, which can satisfy the control requirement of the engine speed control system. Based on the simulation results, the LPV-U controller has good controlling performance when employed for aero-engine speed control system in a large operating envelope.

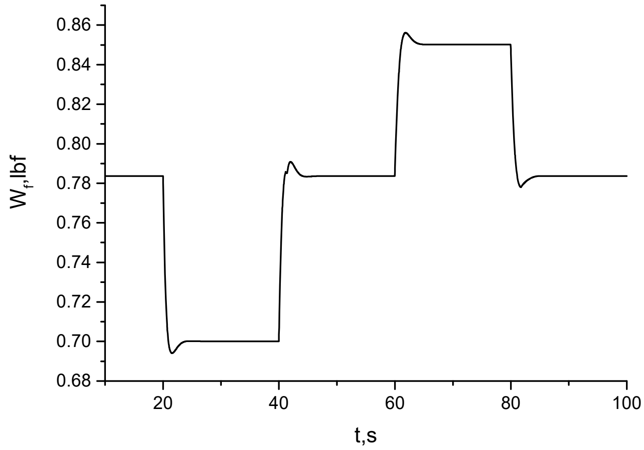


Fig. 7 The change curve of fuel flow in the cruise condition

In order to verify further the operating stability of the proposed LPV-U controller in the closed loop, atmospheric disturbance has been introduced added in the normal cruise condition (H=35000ft, Ma=0.8, N_L =6777rpm). In the computer simulation, atmospheric turbulence model due to G. Kopasakis (2010) is used. This atmospheric disturbance model is shown in Figure 8. The combination of sine curves of unit amplitude is used to obtain the adequate atmospheric disturbance shock, which acts on the inlet of the GTF engine model.

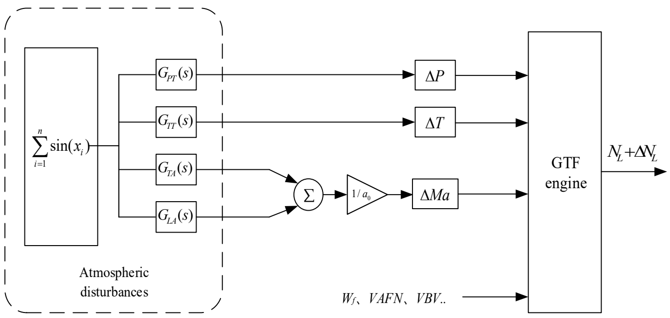


Fig.8 Kopasakis atmospheric disturbance model

The atmospheric disturbances added in the intel are shown in Figure 9. Under these disturbance circumstances, the performance of N_L appears as shown in Figure 10. It is apparently seen that the variation of N_L is less than 1.2% during the period of the atmospheric disturbances. When the disturbances have disappeared, the closed-loop control system operation returns back to the steady-state equilibrium. Thus the proposed new LPV-U control system can sustain stable steady-state operation even when possible atmospheric disturbances may occur.

In general, based on all these simulation results, it can be inferred the proposed LPV-U controller for aero-engine speed control system does guarantee good performance within in a large operating envelopes. Furthermore, in the proximity of the chosen equilibrium steady-state operating point, this new LPV-U controller meets the necessary stability requirement.

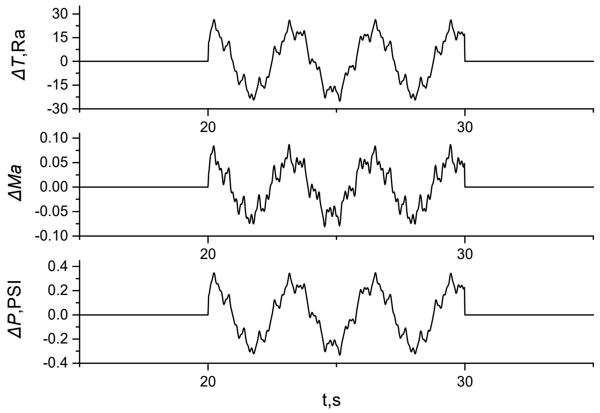


Fig. 9 Consequence of atmospheric disturbances added

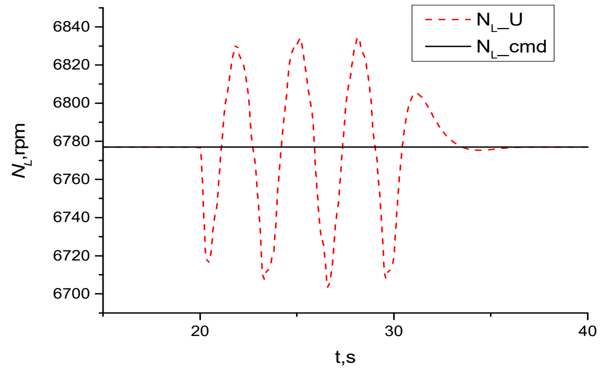


Fig. 10 Performance of N_L when atmospheric disturbances occur in normal cruise condition

5. CONCLUSIONS AND FUTURE RESEARCH

It is the first time that U-control method based on LPV model is used in aero-engine speed control. With U-control theory, the LPV nonlinear controller can be divided into a dynamic inversion of controlled nonlinear system based on LPV model and a simple linear controller. It simplifies greatly the traditional complicated LPV gain scheduling controller design process and has low computational complexity has great potential in real application of ECU. Obviously, this study is only a first small step for applying U-control method to aero-engine control system design. Future research work will be focused on the influence of disturbance uncertainties and model mismatch problem. Of course, an experiment of hardware-in-the-loop is needed for verification of this new controller based on the U-model control method.

In addition, future research should also involve other types of models that can describe the aero-engine nonlinear process dynamics adaptively for further extension of the U-control theory application. Besides of models, it is important to study the use of alternative control principles based on using aero-engine models such as Hammerstein-Wiener representation (Wang et al, 2017) or nonlinear switched systems (Sun et al, 2019) and switched LPV systems (Zhu et al, 2019). It is believed therefore that this work has opened a whole new prospect for future research.

ACKNOWLEDGMENT

The authors thank to the anonymous reviewers for helping improve this manuscript in the final version.

REFERENCES

SAE International Group (2012). *Electronic Engine Control Specificationsand Standards*. Report No.: AIR4250B.

Chapman, J. W., Litt, S. L. (2017). Control Design for an Advanced Geared Turbofan Engine. Technical Report No.: NASA/TM-219569,

Chapman, J. W., May, R. D., Lavelle, T. M., et al. (2014). Toolbox for the Modeling and Analysis of Thermodynamic Systems (T-MATS) User's Guide. Technical Report.

Du, W., Zhu, Q.M., Wu, X. (2014). Support vector machine based U-model generalized predictive controller for nonlinear dynamic plants. In: *Proceedings of the 33th Chinese Control Conference. IEEE Press*, Nanjing, China.

Geng, X. P., Zhu, Q. M., Liu, T., and J. Na (2019). U-model based Smith predictor control for nonlinear processes with input delay. *Journal of Process Control* 73, 156–170, 2019.

Hasan, E., Ibrahim, R.B., Ali, S.S.A., et al. (2017). U-Model Based Adaptive Control of Gas Process Plant. *Procedia Computer Science*, **105**, 119-124.

Kopasakis, G. (2010), Atmospheric Turbulence Modeling for Aero Vehicles: Fractional Order Fits. Report No.: NASA/TM-216961, 2010.

Reberga, L., Henrion, D., Bernussou, J., and Vary, F. (2005). LPV modeling of a turbofan engine. *IFAC Proceedings Volumes (IFAC-PapersOnLine)* **38** (1), 526-531.

Shafiq, M., Haseebuddin, M. (2005). U-Model-Based Internal Model Control for Non-Linear Dynamic Plants. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, **219** (6), 449-458.

Sun, Y., Zhao, J., Dimirovski, G. M. (2019). Adaptive control for a class of state constrained high-order switched nonlinear systems with unstable subsystems. *Nonlinear Analysis: Hybrid Systems*, **32**, 91-105.

Wang, J., Ye, Z., Hu, Z., Wu, X., Dimirovski, G., Yue, H. (2017). Model-based acceleration control of turbofan engines with Hammerstein-Wiener representation. *International Journal of Turbo and Jet Engines,* 34 (2), article 2015-0061.

Wu, X., Liu, L., Zhu, Q. M., et al. (2011). U-model-based adaptive control for a class of stochastic non-linear dynamic plants with unknown parameters. *International Journal of Modelling Identification and Control*, **13** (3), 135-143.

Zhu, Q. M., Guo, L. Z. (2002). A pole placement controller for non-linear dynamic plants. *Proceedings of the Institution of Mechanical Engineers Part I: Journal of Systems and Control Engineering*, **216** (6), 467-476.

Zhu, Q. M., Zhao, D. Y., and Zhang, J. H. (2016). A general U-Block model based design procedure for nonlinear polynomial control systems, *Int. J. Systems Science*, **47** (14), 3465-3475, 2016.

Zhu, Q. M., Zhang, W. C., Zhang, J. H., and Sun, B. (2019). U-neural network-enhanced control of nonlinear dynamic systems. *Neurocomputing* **352**, 12–21, 2019.

Zhu, K., Zhao, J., Dimirovski, G. M. (2018). H_{inf} tracking control for switched LPV systems with application to aero-engines. *IEEE/CCA Journal of Automatica Sinica*, **5** (3), 699-705, 2018.