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Maximal Permissiveness of Modular Supervisory Control via Multilevel Structuring ⋆

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Abstract: Modular supervisory control is motivated by the gain in complexity of control synthesis of supervisors. Sufficient conditions for maximal permissiveness of supervisors include mutual controllability and mutual normality. In this paper, we show how these conditions can be weakened. Namely, we can relax the requirement that the conditions hold for all pairs of components by putting the tuples of plants that do not satisfy the given condition for maximal permissiveness into different groups on an intermediate level of abstraction.

Keywords: Supervisory control, mutual controllability, multi-level structure.

1. INTRODUCTION

A modular discrete-event system (DES) is defined as the parallel composition of two or more discrete-event systems. Most current-day engineering systems are networked systems, i.e. modular DES Hering de Queiroz and Cury (2000). Examples of modular DES with a large number of components are an MRI scanner of Theunissen et al. (2014) that we treated in our relaxed coordination control framework in Komenda et al. (2015), or controllers for theme-park vehicles studied by Forschelen et al. (2012). Modular computation of supervisors has also been combined with the hierarchical computation (reduced supervisors) by Schmidt et al. (2006).

To ensure that the joint action of locally computed supervisors is maximally permissive, the conditions of mutual controllability and mutual relative observability have been proposed by Wong and Lee (2002) and Komenda et al. (2019). The known results in these papers then say that if the conditions hold for all pairs of components, the maximal permissiveness of locally computed supervisors is fulfilled.

In this paper, we relax the requirement that the conditions need to hold for all pairs of components, by bringing a modular DES into a multi-level tree structure that corresponds to a hierarchical structure of partial synchronous products of (subsets of) components. Multilevel hierarchical structures have already been used in supervisory control by Goorden et al. (2017) to achieve significant complexity savings. Our approach is, however, different, because it is based on structural conditions that do not depend on specifications and the main issue is maximal permissivenesses, while the authors of Goorden et al. (2017) deal with general design issues and form a multi-level structure based on the coupling between the local components and various control specifications.

We introduce the notions of *multi-level mutual controllability* (MMC) and *multi-level mutual relative observability* (MMRO) as sufficient conditions to achieve, with help of the multi-level structuring of a modular DES, the same behavior as with the monolithic maximally permissive supervisor. The conditions are weaker than mutual controllability and mutual relative observability for flat modular systems. Our main result states that a sufficient condition for MMC, respectively MMRO, is that in all levels of the hierarchy for every subsystem in one group (of sibling subsystems) there exists a subsystem in other groups on the same level such that the former subsystem is controllable, respectively relatively observable, wrt (with respect to) the latter subsystem, and vice versa. It is based on a simple formulation of sufficient conditions for controllability, respectively relative observability of a synchronous product of languages wrt another synchronous product of languages, namely that for each individual language from the first product there exists a language from the second product wrt which it is controllable, respectively relative observable. Such a condition is then weaker than the standard condition requiring these properties to be satisfied for all pairs of local components. This standard assumption is particularly restrictive in systems with a large number of local components.

The controllability of a product of specifications wrt a modular plant has been investigated in the literature, see, e.g., Akesson et al. (2002); Brandin et al. (2004). Our approach is based on the observation formulated above, applied to the concept of mutual controllability that concerns only the plant (and not also specifications as in the above works), and can be extended to the properties under partial observations, namely to relative observability.

Compared to our previous work in Komenda et al. (2016), we use a general (multi-level) structure as in Goorden et al. (2017). Moreover, we consider supervisors under partial observations, where maximal permissiveness of modular closed-loops in terms of supremal relatively observable sublanguages has been recently investigated by Komenda et al. (2019), and, in this paper, the sufficient condition for maximal permissiveness (mutual observability) is weakened (not required to hold among all pairs of local plants) by organizing local subsystems into

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a tree structure. The main message of this paper is that using multi-level hierarchy does not only help to deal with complexity issues of large modular systems, but also helps to weaken sufficient conditions for maximal permissiveness.

The paper has the following organization. Section 2 recalls necessary definitions and concepts and results on which the paper is based. In Section 3 a multi-level framework for large modular discrete-event systems is proposed. In Section 4 our main results are presented, in particular weaker and computationally efficient sufficient conditions for maximal permissiveness of modular control synthesis. Conclusion is stated in Section 5.

2. PRELIMINARIES

For $n \geq 1$, let $\mathbb{Z}_n = \{1, 2, \ldots, n\}$. An alphabet E is a finite nonempty set. A language over E is a subset of the set E^* of all finite sequences over E. The prefix closure of a language L is $\overline{L} = \{w \in E^* | \exists\, v \in E^*\ wv \in L\}$. If $L = \overline{L}$, L is prefix-closed. In the sequel, we consider only prefix-closed languages.

A *generator* is a quadruple $G = (Q, E, f, q_0)$, where Q is a finite set of states, E is an alphabet, $f\colon Q \times E \to Q$ is a partial transition function, and $q_0 \in Q$ is the initial state; as usual, f can be extended to the domain $Q \times E^*$ by induction. The *language generated* by G is the set $L(G) = \{s \in E^* \mid f(q_0, s) \in Q\}$.

A *controlled generator* is a triple (G, E_c, Γ), where G is a generator over E, $E_c \subseteq E$ is a set of *controllable events*, $E_u = E \setminus E_c$ is the set of *uncontrollable events*, and $\Gamma = \{\gamma \subseteq E \mid E_u \subseteq \gamma\}$ is the *set of control patterns*. A *supervisor* for a controlled generator (G, E_c, Γ) is a map $S\colon L(G) \to \Gamma$. The *closed-loop system* associated with a controlled generator (G, E_c, Γ) and a supervisor S is the minimal language $L(S/G)$ such that $\varepsilon \in L(S/G)$ and, for every $s \in L(S/G)$ with $sa \in L(G)$ and $a \in S(s)$, $sa \in L(S/G)$. Intuitively, the supervisor disables some transitions of G, but never a transition labeled by an uncontrollable event.

A (prefix-closed) language $K \subseteq E^*$ is *controllable* wrt a language $L \subseteq E^*$ and E_u if $KE_u \cap L \subseteq K$. Controllability is preserved by language unions, which means that the supremal sublanguage of K that is controllable wrt L and E_u, always exists, and we denote it by $\sup \mathrm{C}(K, L)$.

Let G be a generator over an alphabet E. Given a specification $K \subseteq L(G)$, the aim of supervisory control is to find a supervisor S such that $L(S/G) = K$. Such a supervisor exists if and only if K is controllable wrt $L(G)$ and E_u, cf. Cassandras and Lafortune (2008); Wonham and Cai (2019).

A *projection* $P\colon A^* \to B^*$, for $B \subseteq A$, is a homomorphism defined as $P(a) = \varepsilon$, for $a \in A \setminus B$, and $P(a) = a$, for $a \in B$. The *inverse image* of P, denoted by $P^{-1}\colon B^* \to 2^{A^*}$, is defined as $P^{-1}(w) = \{s \in A^* \mid P(s) = w\}$. These definitions can be naturally extended to languages.

In supervisory control with partial observations, an additional property, called observability, is required to achieve a given specification language in the closed-loop system. We use a stronger property, relative observability, introduced by authors of Cai et al. (2015), which is stronger than observability, but weaker than normality. Moreover, it preserves language unions. Let $K \subseteq C \subseteq L$ be languages. Language K is *C-observable* wrt L and E_o if for every $w \in K$ and $w' \in C$ with $P(w) = P(w')$, and for every $a \in E$, if $wa \in K$ and $w'a \in L$, then $w'a \in K$. Note that there is a second condition in the original definition of relative observability that concerns the prefix closures, which is not relevant to our study that only concerns prefix-closed languages. The supremal sublanguage of K, C-observable wrt L, exists and we denote it by $\sup \mathrm{RO}(K, C, L)$.

The preservation under synchronous composition has recently been extended by Komenda et al. (2019) to the case of supremal relatively observable sublanguages. We studied two concepts of mutual relative observability depending on whether the reference language C is equal to the specification or to the plant language. It is more natural in the context of this paper to have purely structural conditions that depend on the plant only, i.e., we consider a stronger version of relative (C-)observability, namely with $C = L$ instead of $C = K$. Thus, in this paper, we use $C = L$, and hence we simplify the notation of $\sup \mathrm{RO}(K, L, L)$ to $\sup \mathrm{RO}(K, L)$, meaning the supremal sublanguage of K that is L-observable wrt L. Note that relative observability is still weaker than normality even in the case $C = L$.

The synchronous product of languages $L_i \subseteq E_i^*$, $i \in \mathbb{Z}_n$, is defined as $\|_{i=1}^n L_i = \cap_{i=1}^n P_i^{-1}(L_i)$, where $P_i\colon (\cup_{j=1}^n E_j)^* \to E_i^*$ are projections to local event sets. In terms of generators, $L(\|_{i=1}^n G_i) = \|_{i=1}^n L(G_i)$, cf. Cassandras and Lafortune (2008).

In the sequel, if a modular plant $G = \|_{i=1}^n G_i$ is considered, we denote the local plant languages $L(G_i)$ by L_i, and the overall language $L(G)$ by L, i.e. we have $L = \|_{i=1}^n L_i$.

We recall that a language $K \subseteq E^*$ is decomposable wrt (local) event subsets E_i, $i = 1, \ldots, n$, if $K = \|_{i=1}^n P_i(K)$. It is well known that K is decomposable wrt E_i, $i = 1, \ldots, n$ if and only if there exists local languages $K_i \subseteq E_i^*$ such that $K = \|_{i=1}^n K_i$.

Partial observation is denoted by projection $Q\colon E^* \to E_o^*$ with $E_o \subseteq E$, the set of observable events. The standard definition is that controllable and observable event sets are universal and locally (un)observable and (un)controllable event sets are simply given by the intersection with E_i.

Definition 1. Existence of universal uncontrollable and observable event sets. Consider a modular DES. There exists event subsets E_u, $E_o \subseteq E$ such that for all $i \in \mathbb{Z}_n$ the set of locally uncontrollable events is $E_i^u = E_i \cap E_u$ and set of locally observable events is $E_i^o = E_i \cap E_o$.

Local partial observations are then denoted by projections $Q_i\colon E_i^* \to E_i^{o*}$.

Since the number of states of a modular system $\|_{i=1}^n G_i$ is exponential in the number of components, the monolithic supervisors for the whole composed plant are considered as computationally unfeasible if the number of components is large. On the other hand, computationally attractive local control synthesis, i.e. computation of supervisors S_i for individual $G_i, i = 1, \ldots, n$ with local closed-loop S_i/G_i, suffers even in the prefix-closed case from lack of maximal permissiveness. Namely, it appears that local (modular) closed-loop $\|_{i=1}^n S_i/G_i$ is typically strictly included in the monolithic closed-loop $S/(\|_{i=1}^n G_i)$. Let us recall that under complete observations maximally permissive monolithic closed-loop $S/(\|_{i=1}^n G_i)$ equals the supremal controllable sublanguage.

We now recall the concepts of mutual controllability of Wong and Lee (2002) and mutual relative observability of Komenda et al. (2019) as sufficient conditions for maximal permissiveness of local control synthesis.

Definition 2. Languages $L_i \subseteq E_i^*$, $i = 1,\dots,n$ are *mutually controllable* wrt $(E_i^u \cap E_j)$ if, for every $i \neq j$, $P_i^{-1}(L_i)(E_i^u \cap E_j) \cap P_j^{-1}(L_j) \subseteq P_i^{-1}(L_i)$.

The original definition of mutual controllability of Wong and Lee (2002) was slightly different, namely controllability of L_i was required wrt $P_i P_j^{-1}(L_j) \subseteq E_i^*$. However, the above definition that is computationally more attractive (only inverse projections are used) is shown to be equivalent to mutual controllability in the extended version of Komenda et al. (2019) published on arxiv. The following result then follows from Wong and Lee (2002).

Theorem 3. Let $\|_{i=1}^n K_i \subseteq \|_{i=1}^n L_i$ with languages $K_i, L_i \subseteq E_i^*$ be such that the languages $L_1,\dots,L_n$ are mutually controllable. Then the local control synthesis is maximally permissive, that is, $\sup \mathrm{C}(\|_{i=1}^n K_i, \|_{i=1}^n L_i) = \|_{i=1}^n \sup \mathrm{C}(K_i, L_i)$

Relative observability, introduced by Cai et al. (2013, 2015) is weaker than normality , while it still preserves language unions. Let $K \subseteq C \subseteq L$. K is *C-observable* wrt L and E_o if $(\forall w \in K)(\forall w' \in C)\ P(w) = P(w') \Rightarrow (\forall a \in E)(wa \in K \wedge w'a \in L \Rightarrow w'a \in K)$. The supremal sublanguage of K, C-observable wrt L, exists and we denote it by $\sup \mathrm{RO}(K,C,L)$. In this paper, we use $C = L$ and we simplify $\sup \mathrm{RO}(K,L,L)$ to $\sup \mathrm{RO}(K,L)$. Thus, by $\sup \mathrm{RO}(K,L)$ we mean the supremal sublanguage of K that is L-observable wrt L.

The preservation under synchronous composition has recently been extended by Komenda et al. (2019) to the case of supremal relatively observable sublanguages. We have studied two concepts of mutual relative observability depending on whether the reference language C equals the specification or to the plant language. It is more natural in the context of this paper to have purely structural conditions that depend on the plant only, i.e., we consider a stronger version of relative (C-)observability, namely with $C = L$ instead of $C = K$. It is also possible to consider $C = K$, but then all our results depend also on K. We first recall the result by Komenda et al. (2019).

Note that for $n = 2$ the above definition means that L_1 and L_2 are mutually relatively observable if $P_1^{-1}(L_1)$ is $P_2^{-1}(L_2)$-observable wrt $P_2^{-1}(L_2)$ and $E_{o,1}$ and $P_2^{-1}(L_2)$ is $P_1^{-1}(L_1)$-observable wrt $P_1^{-1}(L_1)$ and $E_{o,2}$.

We recall the result by Komenda et al. (2019), which is based on relative observability of inversed projections of local plant languages wrt one another and with reference language C given also by these inverse projections.

Theorem 4. Given $K_i \subseteq L_i \subseteq E_i^*$, for $i = 1,2$. If $P_1^{-1}(L_1)$ is $P_2^{-1}(L_2)$-observable wrt $P_2^{-1}(L_2)$ and $P_2^{-1}(L_2)$ is $P_1^{-1}(L_1)$-observable wrt $P_1^{-1}(L_1)$, then

$$\sup \mathrm{RO}(K_1 \| K_2, L_1 \| L_2) \subseteq \sup \mathrm{RO}(K_1, L_1) \| \sup \mathrm{RO}(K_2, L_2).$$

Note that this inclusion means that local control synthesis is at least as permissive as the global control synthesis. The opposite inclusion is considered as obvious in modular control with prefix-closed specification languages. The extension of this result to $n \geq 2$ is straightforward and used below. We now define mutual relative observability.

Definition 5. The languages $L_1,\dots,L_n$ are *mutually relatively observable* if for all $i, j \in \{1,\dots,n\}$, $i \neq j$, $P_i^{-1}(L_i)$ is $P_j^{-1}(L_j)$-observable wrt $P_j^{-1}(L_j)$ and $E_{o,i}$.

We have the following consequence of Theorem 4.

Theorem 6. Let $\|_{i=1}^n K_i \subseteq \|_{i=1}^n L_i$ be such that the languages $L_1,\dots,L_n$ are mutually relatively observable. Then the local control synthesis is maximally permissive, that is, it holds for all specification languages that $\sup \mathrm{RO}(\|_{i=1}^n K_i, \|_{i=1}^n L_i) = \|_{i=1}^n \sup \mathrm{RO}(K_i, L_i)$

3. MULTILEVEL APPROACH TO SUPERVISORY CONTROL

A multi-level framework has been proposed by Goorden et al. (2017), where the authors consider a modular plant with a large number of specifications. From a different viewpoint, the global specification is decomposed into a set of specifications, each concerning only a small number of components. The multi-level structure is then designed based on the degree of interaction of different components, given by coupling of components via specifications. Otherwise stated, the degree of interaction between two components is defined as the number of specifications they share.

A Design Structure Matrix (DSM) is a measure of interaction between components in a large modular system and it is used for multi-level clustering that may be viewed as building a hierarchical block structure. The block structure of a design structure matrix depends on an optimization criterion chosen, see the example in Wilschut et al. (2017). It requires further research to select an appropriate optimization criterion for each example. Clustering algorithms first compute a permutation of the rows and columns of the original matrix such that strongly related components form a cluster (block in the matrix) and the blocks are further divided into smaller blocks.

In this paper, we use multi-level structuring of local plants in order to weaken structural conditions of modular supervisory control that guarantee maximal permissiveness, an issue that is not discussed in Wilschut et al. (2017), where computational advantage of multi-level structuring is illustrated without performance analysis consisting in comparing the resulting closed-loop with the one obtained by the monolithic supervisor. The multi-level tree structure of a modular system is based solely on the local plants and not on the specifications. For simplicity we consider a decomposable (local) specification, because it is known that problems with global specifications can be replaced by local ones using conditional decomposability Komenda et al. (2012) and its extension (weakening) using more levels in the hierarchy. We show that multi-level structuring helps to weaken the sufficient structural condition from the flat control architecture (purely modular).

3.1 Multi-level structuring of modular DES

We consider a large modular plant $G = \|_{i=1}^n G_i$ with its language $L = L(G) = \|_{i=1}^n L_i$ and a specification language $K \subseteq A^*$ that is decomposable wrt $(E_i)_{i=1}^n$, i.e. $K = \|_{i=1}^n P_i(K)$ or (equivalently Willner and Heymann (1991)) there exist local languages $K_i \subseteq E_i^*$ such that $K = K_1 \| \dots \| K_n$.

A multi-level tree structure over $\{1,2,\dots,n\}$ is obtained as follows, see Fig. 1. Let $\ell \leq n$ be the number of (hierarchical) levels in the tree. At the top level we have the top element of the tree, namely $(1,1)$, which corresponds to the whole set of indices of the subsystems $\{1,2,\dots,n\}$. This set is split into two or more subsets such that the union of the subsets is again

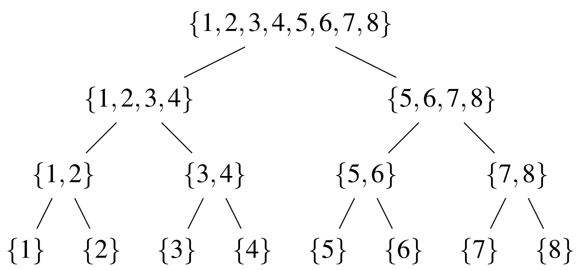

Fig. 1. Example of a tree structure of a multi-level clustering.

$\{1,2,\ldots,n\}$. This way we continue all the way to the bottom level, where we have n singleton sets, namely $\{1\},\ldots,\{n\}$. Element $(m,i) \in T$ of the tree then denotes the ith subset (of $\{1,2,\ldots,n\}$) from the left at the level m (counted from the top). For example, $(3,2)$ refers to the subindex set $\{3,4\}$ A simple example of such a tree structure is in Figure 1. Let $\mathrm{chi}(i,j) \subseteq \{1,\ldots,n\}$ denote the set of all children of node (i,j), and let $\mathrm{par}(i,j)$ denote the parent of node (i,j). We use chi^* for the transitive closure of $\mathrm{chi}(i,j)$ with an obvious meaning.

In terms of plant languages, such a tree over $\{1,2,\ldots,n\}$ induces the corresponding hierarchical tree of (partial) synchronous products of subsystems, namely the product of subsystems with indexes from $(m,i) \in T$, i.e. $G_{(m,i)} = \|_{j\in(m,i)} G_j$. At the lowest level, we have one element subsets of $\{1,\ldots,\mathrm{n}\}$, i.e. $\{i\}$, $i=1,\ldots,n$. These corresponds to local plants G_i, $i=1,\ldots,n$, that is, we have $G_{(\ell,i)} = G_i$ for $i=1,\ldots,n$. The systems are composed into groups $(\ell-1,i) \in T$, for $i=1,\ldots,n_{\ell-1}$ at the second lowest level consisting of the respective plants, i.e., G_i, $i \in (\ell-1,i)$. These groups are further merged into large groups at the next higher level until at the top level there is a singleton $(1,1) \in T$ with the meaning of the whole composition, i.e., $G_{1,1} = G_1 \| G_2 \| \ldots \| G_n$.

Since for every $(m,i) \in T$ we have simply $(m,i) = \{j = 1,\ldots,n : j \in (m,i)\}$, it is natural to define the tree structure of plants as partial composition, i.e. $L_{m,i} = \|_{j\in(m,i)} L_j$. Note that using this natural notation we have that for $L = \|_{i=1}^n L_i$ it automatically holds that $L_{m,i} = \|_{(m+1,i')\in \mathrm{chi}(m,i)} L_{m+1,i'}$ for $m=1,\ldots,\ell-1$. For instance, for $m=1$ this means that $L = L_{1,1} = \|_{(2,i)\in \mathrm{chi}(1,1)} L_{(2,i)}$.
We have for all levels $m=1,\ldots,\ell-1$, and for all $(m,i) \in T$

$$L_{m,i} = \|_{(m+1,i')\in \mathrm{chi}(m,i)} L_{m+1,i'}\,, \tag{1}$$

that is, $L_{m,i}$ is decomposable wrt the alphabets $(E_{m+1,i'})$, where $(m+1,i') \in \mathrm{chi}(m,i)$.

Similarly, it easily follows from decomposability of specification language K, i.e. existence of local languages $K_i \subseteq E_i^*$ that the tree structure over $\{K_1,\ldots,K_n\}$, induced by tree structure T over $\{1,2,\ldots,n\}$, defined by $K_{m,i} = \|_{j\in(m,i)} K_j$, satisfies $K_{m,i} = \|_{(m+1,i')\in \mathrm{chi}(m,i)} K_{m+1,i'}$ for $m=1,\ldots,\ell-1$. In particular, for $m=1$ we have

$$K = K_{1,1} = K_1 \| \ldots \| K_n = \|_{(2,i)\in \mathrm{chi}(1,1)} K_{2,i}.$$

We have for all $m=1,\ldots,\ell-1$, and for all $(m,i) \in T$ \\

$$K_{m,i} = \|_{(m+1,i')\in \mathrm{chi}(m,i)} K_{m+1,i'}\,, \text{ i.e.} \tag{2}$$

$K_{m,i}$ is decomposable wrt the alphabets $(E_{m+1,i'})_{(m+1,i')\in \mathrm{chi}(m,i)}$.

We use the following notation for alphabets, $E_{(i,j)}$ means the alphabet of $G_{i,j}$, i.e. the union of alphabets of the subsystems of which $G_{i,j}$ is formed. The notation for uncontrollable, respectively observable event subsets of $E_{(i,j)}$ are then $E^u_{(i,j)} = E_u \cap E_{(i,j)}$, respectively $E^o_{(i,j)} = E_o \cap E_{(i,j)}$ to be consistent with notation for flat modular systems. This structuring is illustrated in the following example.

Example 1. Let $G = G_1 \| G_2 \ldots \| G_8$ and $\ell = 4$. We can consider the following 4-level tree structure on the set $\{1,2,\ldots,8\}$ displayed in Fig. 1. We have $(4,i) = i$ for $i=1,2,\ldots,8$. At the second lowest level, we merge the lowest level nodes into four groups, namely $(3,1) = \{1,2\}$, $(3,2) = \{3,4\}$, $(3,3) = \{5,6\}$, and $(3,4) = \{7,8\}$. At the second highest level, we have only two groups $(2,1) = \{1,2,3,4\}$ and $(2,2) = \{5,6,7,8\}$. Finally, at the highest level, all nodes are merged into a single top node, i.e., $(1,1) = \{1,2,\ldots,8\}$. The corresponding partial modular plants are then: $G_{(4,i)} = G_i$ for $i=1,2,\ldots,8$. We have, for instance, $G_{(2,1)} = G_1 \| G_2 \| G_3 \| G_4$ with $E_{2,1} = E_1 \cup E_2 \cup E_3 \cup E_4$.

4. COMPARISON OF MODULAR AND MONOLITHIC SUPERVISORS

In this section, we propose weaker and computationally efficient sufficient conditions for modular control synthesis to equal monolithic control synthesis based on multi-level structure of the last section. Consider a large modular DES $G = \|_{i=1}^n G_i$. It is interesting to look for weaker sufficient conditions for achieving equality $\sup \mathrm{RO}(K,L) = \|_{i=1}^n \sup \mathrm{RO}(P_i(K), L_i)$ than mutual relative observability, cf. Theorem 3. We show that the concepts of multilevel mutual controllability and multilevel mutual normality are such sufficient conditions, which are weaker then standard structural condition of mutual controllability and mutual normality.

Definition 7. A modular DES with a multilevel structure T is *multilevel mutually controllable* (MMC) if, for $m=1,\ldots,\ell-1$, every $(m,i) \in T$, and every $(m+1,j),(m+1,j') \in \mathrm{chi}(m,i)$ such that $(m+1,j) \neq (m+1,j')$, the languages $L_{(m+1,j)}$ and $L_{(m+1,j')}$ are mutually controllable wrt $E^u_{(m+1,j)} \cap E_{(m+1,j')}$. $\triangleleft$

Similarly, we have the following concept.

Definition 8. A modular DES with a multilevel structure T is *multilevel mutually relatively observable* (MMRO) if, for $m=1,\ldots,\ell-1$, every $(m,i) \in T$, and every $(m+1,j),(m+1,j') \in \mathrm{chi}(m,i)$ such that $(m+1,j) \neq (m+1,j')$, the languages $L_{(m+1,j)}$ and $L_{(m+1,j')}$ are mutually relatively observable as defined in Definition 5. $\triangleleft$

We are now ready to state the following result.

Theorem 9. Let $G = \|_{i=1}^n G_i$ and let $K = \|_{i=1}^n K_i$ be decomposable wrt local event sets $(E_i)_{i=1}^n$. If there exists a tree structure over local plants such that the modular plant is MMC, then for any decomposable specification K, $\sup \mathrm{C}(K,L) = \|_{i=1}^n \sup \mathrm{C}(K_i, L_i)$. $\Box$

For supremal relatively observable sublanguages we have the following result.

Theorem 10. Let $G = \|_{i=1}^n G_i$ and let $K = \|_{i=1^n} K_i$ be decomposable wrt local event sets $(E_i)_{i=1}^n$. If there exists a tree structure over local components such that the modular plant is MMRO, then for any decomposable specification K, $\sup \mathrm{RO}(K,L) = \|_{i=1}^n \sup \mathrm{RO}(K_i, L_i)$.

Theorems 9 and 10 can be generalized to the case of indecomposable specifications using multi-level conditional decomposition proposed by Komenda et al. (2016) for three levels ($\ell = 3$).

It can be easily extended to the general number of levels based on the concept of tree decomposition. Specifically, we can extend the formula of equation (2) to non decomposable specifications by replacing the partial products of local languages with projections of a global language to the alphabets of these products enriched by coordinator events whenever necessary. However, in this paper we deal with the special case of a decomposable specification language, hence our approach is based on an arbitrary tree structure that will satisfy MMC and MMRO conditions. This is useful in the case, where some local plant languages (say L_i and L_j,) are not mutually controllable or are not mutually observable, because these plants can be put into different groups on an intermediate level of abstraction and we will show (based on Propositions 11 and 12 below) that we do not always need L_i and L_j to be mutually controllable and mutually observable for MMC and MMRO to hold true.

We will propose in Theorem 13 computationally efficient conditions, which involve only local plants and no partial composition of local plants need to be computed. Theorem 13 is based on the following result, which gives a weaker sufficient condition for controllability of one synchronous product wrt another synchronous product. Note that a similar property is known from Brandin et al. (2004).

Proposition 11. Consider languages $K_j \subseteq E_j^*,\ j \in \mathscr{L}$, with $E_1 = \cup_{j\in\mathscr{L}} E_j$, and $L_m \subseteq E_m^*$, $m \in \mathscr{M}$, with $E_2 = \cup_{m\in\mathscr{M}} E_m$ for disjoint index sets $\mathscr{L}$ and $\mathscr{M}$. If (1) for every $j \in \mathscr{L}$, there exists $m \in \mathscr{M}$ such that $P_j^{-1}(K_j)$ is controllable wrt $P_m^{-1}(L_m)$ and $E_j^u \cap E_2$, and (2) for every $m \in \mathscr{M}$, there exists $j \in \mathscr{L}$ such that $P_m^{-1}(L_m)$ is controllable wrt $P_j^{-1}(K_j)$ and $E_m^u \cap E_1$, then $\|_{j\in\mathscr{L}} K_j$ is mutually controllable wrt $\|_{m\in\mathscr{M}} L_m$.

For relative observability the condition of Theorem 13 is based on the following result.

Proposition 12. Consider languages $K_j \subseteq E_j^*,\ j \in \mathscr{L}$, with $E_1 = \cup_{j\in\mathscr{L}} E_j$, and $L_m \subseteq E_m^*$, $m \in \mathscr{M}$, with $E_2 \cup_{m\in\mathscr{M}} E_m$ for disjoint index sets $\mathscr{L}$ and $\mathscr{M}$. If (1) for every $j \in \mathscr{L}$ there exists $m \in \mathscr{M}$ such that $P_j^{-1}(K_j)$ is $P_m^{-1}(L_m)$-observable wrt $P_m^{-1}(L_m)$ and E_j^o, and (2) for every $m \in \mathscr{M}$ there exists $j \in \mathscr{L}$ such that $P_m^{-1}(L_m)$ is $P_j^{-1}(K_j)$-observable wrt $P_j^{-1}(K_j)$ and E_m^o, then $\|_{j\in\mathscr{L}} K_j$ and $\|_{m\in\mathscr{M}} L_m$ are mutually relatively observable.

The following example illustrates the previous result.

Example 2. Consider the languages L_1, L_2, L_3 defined by the respective generators of Fig. 2, where the set of uncontrollable events is $\{u_1, u_2\}$. It can be verified that the languages L_1 and L_2 are mutually controllable as well as the languages L_1 and L_3. However, the languages L_2 and L_3 are not mutually controllable, because the language $P_3^{-1} L_3$ is not controllable wrt $P_2^{-1}(L_2)$ and $\{u_1\}$. Consequently, we cannot use Theorem 3 to conclude that the locally computed supervisors are optimal.

However, notice that both L_1 and L_2 are controllable wrt L_3, and L_3 is controllable wrt L_1 and the set $\{u_1, u_2\}$. Then, by Proposition 11, $L_1 \| L_2$ and L_3 are mutually controllable.

Thus, if the specification can be decomposed wrt the structure $\{L_1, L_2\}$ and $\{L_3\}$ (it is implied, e.g., by $K = \|_{i=1}^3 P_i(K)$), we obtain optimality of the composition of locally computed supervisors. Indeed, if $K_i \subseteq L_i$, $i = 1,2,3$, are local specifications, then, by Theorem 3, we have that $\sup\mathrm{CN}(K_1\|K_2, L_1\|L_2) = \sup\mathrm{CN}(K_1, L_1) \parallel \sup\mathrm{CN}(K_2, L_2)$. Moreover, since $L_1\|L_2$ and

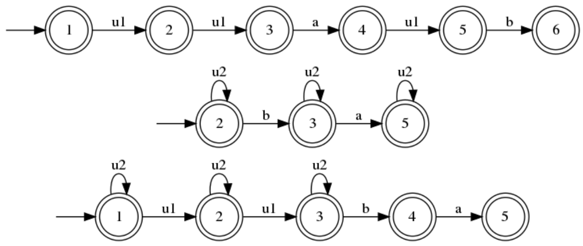

Fig. 2. Generators for languages L_1, L_2, and L_3, respectively

L_3 are mutually controllable, Theorem 3 implies that

$$\begin{aligned}
&\sup\mathrm{CN}((K_1\|K_2) \parallel K_3, (L_1\|L_2) \parallel L_3) \\
&= \sup\mathrm{CN}(K_1\|K_2, L_1\|L_2) \parallel \sup\mathrm{CN}(K_3, L_3) \\
&= \sup\mathrm{CN}(K_1, L_1) \parallel \sup\mathrm{CN}(K_2, L_2) \parallel \sup\mathrm{CN}(K_3, L_3) \quad \triangleleft
\end{aligned}$$

From Propositions 11 and 12 we can derive the following computationally efficient result, which formulates a sufficient condition for modular system to be MMC and MMRO.

Theorem 13. If for $m = 1, \dots, \ell - 1$, $(m,i) \in T$ and every $(m+1, j) \neq (m+1, j') \in \mathrm{chi}(m,i)$,

(1) for every $(\ell, k) \in (m+1, j)$, there exists $(\ell, k') \in (m+1, j')$ such that $L_{\ell,k}$ is controllable wrt $L_{\ell,k'}$ and $E^u_{(\ell,k)} \cap E_{(m+1,j')}$ and $L_{\ell,k'}$-observable wrt $L_{\ell,k'}$ and $E^o_{(\ell,k)}$, and
(2) for every $(\ell, k') \in (m+1, j')$, there exists $(\ell, k) \in (m+1, j)$ such that $L_{\ell,k'}$ is controllable wrt $L_{\ell,k}$ and $E^u_{(\ell,k')} \cap E_{(m+1,j)}$ and $L_{\ell,k}$-observable wrt $L_{\ell,k}$ and $E^o_{(\ell,k')}$,

then the conditions MMC and MMRO both hold true.

It should be clear that in view of Theorems 9 and 10 the previous result (Theorem 13) states sufficient conditions for maximal permissiveness that are weaker than known results, e.g. requiring structural conditions of mutual controllabilty to hold for every pair of local plants.

Example 3 illustrates how MMC and MMRO get weaker with the growing number of levels.

Example 3. Let $G = G_1 \| \dots \| G_8$ and consider multi-level structure of this modular plant depicted in Fig. 1.

The centralized closed-loop is given by $\|_{i=1}^8 L(S_i/G_i)$. Given a decomposable specification $K = \|_{i=1}^8 P_i(K)$, one computes maximally permissive local supervisors as $\|_{i=1}^8 \sup\mathrm{CN}(P_i(K), L_i)$, where $L_i = L(G_i)$. A sufficient condition for equality with monolithic $\sup\mathrm{C}(K, L)$ is according to Theorem 3 mutual controllability and for $\sup\mathrm{RO}(K, L)$ mutual relative observability of L_i wrt $L_{i'}$ for all $i \neq i' \in \{1, \dots, 8\}$. Hence, 56 conditions are required for both mutual controllability and mutual relative observability (all pairs of subsystems).

Now, using the multi-level coordination control architecture, we need (cf. Theorem 9) to check multi-level mutual controllability and multi-level relative observability. According to the definition of MMC we need to check that

- $L_{2,1} = L_1\|L_2\|L_3\|L_4$ and $L_{2,2} = L_5\|L_6\|L_7\|L_8$ are mutually controllable.
- for every $(2,i) \in \mathrm{chi}(1,1)$ and every $(3,j) \neq (3,j') \in \mathrm{chi}(2,i)$, $L_{3,j}$ and $L_{3,j'}$ are mutually controllable.
- for every $(3,i) \in T$ and every $(4,j) \neq (4,j') \in \mathrm{chi}(3,i)$, $L_{4,j}$ and $L_{4,j'}$ are mutually controllable.

However, we know from Theorem 13 that it suffices to check that for every $(\ell,k) \in (2,1)$, there exists $(\ell,k') \in (2,2)$ such that $L_{(\ell,k)}$ is controllable wrt $L_{(\ell,k')}$ and $E^u_{(\ell,k)} \cap E_{(m,j')}$. Otherwise stated, for the first condition to hold true it suffices that for all $i = 1,2,3,4$, there exists $j \in \{5,6,7,8\}$ such that L_i is controllable wrt L_j. This is stated in the first item ($m = 2$) of Theorem 13. Note that this follows directly from Proposition 11 on which Theorem 13 is based. We thus need in principle only $4 \times 2 = 8$ conditions of mutual controllability instead of requiring mutual controllability of all pairs in modular control.

The second item ($m = 3$) requires that for $(2,1) \in \text{chi}(1,1)$, $L_{3,1} = L_1 \| L_2$ and $L_{3,2} = L_3 \| L_4$ are mutually controllable, and for $(2,2) \in \text{chi}(1,1)$, $L_{3,3} = L_5 \| L_6$ and $L_{3,4} = L_7 \| L_8$ are mutually controllable. By Propositions 11 it suffices that for all $i \in \{1,2\}$, there exists $j \in \{3,4\}$ such that L_i is controllable wrt L_j and vice versa, for all $j \in \{3,4\}$ there exists $i \in \{1,2\}$ such that L_j is controllable wrt L_i. Similarly, it suffices that for all $i \in \{5,6\}$, there exists $j \in \{7,8\}$ such that L_i is controllable wrt L_j and vice versa.

Finally, recalling that $L_{4,j} = L_j = L(G_j)$, then the last item requires that L_1 and L_2 are mutually controllable, L_3 and L_4 are mutually controllable, L_5 L_6 are mutually controllable, and L_7 and L_8 are mutually controllable. Clearly for the second and third items we also need 8 conditions per item. Thus, we need in total $8 \times 3 = 24$ controllability conditions as opposed to $8 \times 7 = 56$ conditions required by modular architecture.

Similarly, if this example is extended to 16 components organized in a binary tree with 5 levels, we would obtain even more impressive weakening of MMC and MMRO conditions that guarantee maximal permissiveness in the multi-level coordination architecture compared to the pairwise mutual controllability and relative observability of centralized coordination, where in total $2 \times 16 \times 15 = 480$ conditions would be required. For this example we would need only 16 conditions for all $m = 2,3,4,5$, i.e. 64 conditions, which is already much less than 480 conditions needed for flat modular system. Note however that the interest is not in the number of conditions to be checked, but rather in the fact that we have weaker sufficient conditions for maximal permissiveness, because much less pairs need to be mutually controllable and mutually relatively observable.

The last example illustrates that replacing flat modular structure by adding more levels for the same system not only brings better complexity, but can be also used for proving that even for purely modular control addressed in this paper maximal permissiveness holds under weaker sufficient conditions.

5. CONCLUDING REMARKS

We have shown that multi-level(tree) architecture for large modular DES is not only useful for complexity savings as already documented in the literature, but also for weakening sufficient structural conditions that guarantee maximal permissiveness of locally computed (modular) supervisors. We plan to extend the results to the case of non decomposable specifications using multi-level conditional decomposability.

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